

***Al- Furat Al- Awsat Technical University
Najaf Technical Institute
Medical device Department***



Stage: First Stage

Academic Year: 2025-2026

Total Contact Hours: 120 Hours

Total Units: 8 Units

Weekly Hours: 4 Hours

Attendance: Classroom Lectures + Laboratory

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1. Course Description

Electrical Circuits and Measurements is a fundamental first-stage course covering electrical units, DC circuits, network analysis, AC circuits, resonance, electrical power, measuring instruments, oscilloscopes, and practical resistance measurement. The course combines theoretical analysis with laboratory practice.

2. Course Objectives

- 1-Apply fundamental electrical laws and principles to analyze electrical circuits.
- 2-Select appropriate analytical methods for DC and AC electrical circuits.
- 3-Understand fundamental electrical theories and apply their mathematical relationships.
- 4-Calculate voltage, current, resistance, impedance, and electrical power.
- 5-Analyze series, parallel, mixed, star, and delta resistor networks.
- 6-Apply Kirchhoff's laws and major network theorems.
- 7-Analyze sinusoidal AC quantities, phasors, RLC circuits, resonance, and power factor.
- 8-Understand the construction and operation of common electrical measuring instruments.
- 9-Perform practical resistance measurements using different methods.
- 10-Develop safe and systematic laboratory practices.

3. Course Learning Outcomes

- 1-Explain fundamental electrical quantities and SI units.
- 2-Analyze DC and AC electrical circuits mathematically.
- 3-Apply Kirchhoff's laws, Thevenin, Norton, Superposition, and Maximum Power Transfer methods.
- 4-Calculate impedance, admittance, phase angle, resonance frequency, bandwidth, and quality factor.
- 5-Calculate active, reactive, and apparent power and power factor.
- 6-Use common electrical measuring instruments appropriately.
- 7-Perform and interpret practical electrical measurements.
- 8-Compare theoretical and experimental results.

4. Teaching and Learning Strategies

- 1-Scientific lectures
- 2-Practical laboratory sessions
- 3-Circuit demonstrations
- 4-Mathematical problem solving
- 5-Individual and group activities
- 6-Daily oral and written quizzes
- 7-Monthly examinations
- 8-Scientific reports

5. 30-Week Course Vocabulary / Syllabus

Week	Topic
1	Electrical Units and Unit Conversion
2	Voltage, Current, Resistance and Ohm's Law
3	Direct-Current (DC) Circuits
4	Series Resistor Circuits
5	Parallel Resistor Circuits
6	Mixed Resistor Circuits
7	Star-Delta and Delta-Star Transformations
8	Applications of DC Resistor Networks
9	Kirchhoff's Laws
10	Maxwell's Method
11	Thevenin's Theorem
12	Norton's Theorem
13	Applications of Thevenin's and Norton's Theorems
14	Superposition Theorem
15	Voltage and Current Sources
16	Maximum Power Transfer Theorem
17	AC Quantities and RMS Values
18	Phasors and Complex Quantities
19	AC Circuits: R, L and C
20	Series and Parallel RLC Circuits
21	Complex Operator / j-Operator
22	Resonance Circuits
23	Network Theorems in AC Circuits
24	AC Power
25	Apparent Power and Power Factor
26	Maximum Power Transfer in AC Circuits
27	Electrical Measuring Instruments – Moving Coil
28	Moving-Iron Instruments
29	Dynamometer Instruments and Oscilloscope
30	Practical Resistance Measurement Methods

WEEK 1

Electrical Units and Unit Conversion

Learning Objectives

Identify SI electrical quantities and prefixes.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about electrical units and unit conversion?

State one basic principle related to this topic?

Name one practical application?

Lecture Explanation

V, A, Ω , W, J, C, Hz; $k=10^3$, $M=10^6$, $m=10^{-3}$, $\mu=10^{-6}$. Example: $4.7\text{ k}\Omega = 4700\ \Omega$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Electrical Units and Unit Conversion.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Electrical Units and Unit Conversion.

Learning Outcome

Students can explain the principles of electrical units and unit conversion, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 2

Voltage, Current, Resistance and Ohm's Law

Learning Objectives

Define V, I, R and apply Ohm's Law.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about voltage, current, resistance and ohm's law?

State one basic principle related to this topic?

Name one practical application?

Lecture Explanation

$V=IR$; $I=V/R$; $R=V/I$; $R=\rho L/A$; $R_T=R_0[1+\alpha(T-T_0)]$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Voltage, Current, Resistance and Ohm's Law.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Voltage, Current, Resistance and Ohm's Law.

Learning Outcome

Students can explain the principles of voltage, current, resistance and ohm's law, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 3

Direct-Current (DC) Circuits

Learning Objectives

Explain DC sources, loads and basic DC relationships.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about direct-current (dc) circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

For resistive loads $I=V/R$ and $P=VI=I^2R=V^2/R$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Direct-Current (DC) Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Direct-Current (DC) Circuits.

Learning Outcome

Students can explain the principles of direct-current (dc) circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 4

Series Resistor Circuits

Learning Objectives

Analyze series resistors and voltage division.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about series resistor circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$R_T = \Sigma R$; same current flows; $V_i = V_T R_i / R_T$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Series Resistor Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Series Resistor Circuits.

Learning Outcome

Students can explain the principles of series resistor circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 5

Parallel Resistor Circuits

Learning Objectives

Analyze parallel resistors and branch currents.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about parallel resistor circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$1/R_T = \Sigma(1/R_i)$; for two $R_T = R_1 R_2 / (R_1 + R_2)$; $I_i = V / R_i$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Parallel Resistor Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Parallel Resistor Circuits.

Learning Outcome

Students can explain the principles of parallel resistor circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 6

Mixed Resistor Circuits

Learning Objectives

Reduce and analyze mixed networks.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about mixed resistor circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Simplify the simplest series/parallel group first, then work backward for individual quantities.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Mixed Resistor Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Mixed Resistor Circuits.

Learning Outcome

Students can explain the principles of mixed resistor circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 7

Star-Delta and Delta-Star Transformations

Learning Objectives

Perform Y- Δ and Δ -Y transformations.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about star-delta and delta-star transformations?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$R_A = R_{AB} R_{CA} / (R_{AB} + R_{BC} + R_{CA})$, with analogous expressions for the other branches.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Star-Delta and Delta-Star Transformations.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Star-Delta and Delta-Star Transformations.

Learning Outcome

Students can explain the principles of star-delta and delta-star transformations, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 8

Applications of DC Resistor Networks

Learning Objectives

Integrate series, parallel, mixed and Y- Δ methods.
Apply the related equations or operating principles.
Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about applications of dc resistor networks?
State one basic principle related to this topic.
Name one practical application.

Lecture Explanation

Determine total resistance, current, branch currents, voltage drops and power.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Applications of DC Resistor Networks.
State the main equation or operating principle.
Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Applications of DC Resistor Networks.

Learning Outcome

Students can explain the principles of applications of dc resistor networks, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 9

Kirchhoff's Laws

Learning Objectives

Apply KCL and KVL.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about kirchhoff's laws?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$\Sigma I=0$ at a node; $\Sigma V=0$ around a closed loop.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Kirchhoff's Laws.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Kirchhoff's Laws.

Learning Outcome

Students can explain the principles of kirchhoff's laws, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 10

Maxwell's Method

Learning Objectives

Use loop currents and simultaneous equations.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about maxwell's method?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Assign loop currents and apply KVL to independent loops.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Maxwell's Method.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Maxwell's Method.

Learning Outcome

Students can explain the principles of maxwell's method, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 11

Thevenin's Theorem

Learning Objectives

Find V_{TH} and R_{TH} .

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about thevenin's theorem?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

A linear two-terminal network becomes V_{TH} in series with R_{TH} ; V_{TH} is open-circuit voltage.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Thevenin's Theorem.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Thevenin's Theorem.

Learning Outcome

Students can explain the principles of thevenin's theorem, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 12

Norton's Theorem

Learning Objectives

Find I_N and R_N .

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about norton's theorem?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

A linear two-terminal network becomes I_N in parallel with R_N ; $I_N = V_{TH}/R_{TH}$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Norton's Theorem.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Norton's Theorem.

Learning Outcome

Students can explain the principles of norton's theorem, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 13

Applications of Thevenin's and Norton's Theorems

Learning Objectives

Solve load problems with both equivalents.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about applications of thevenin's and norton's theorems?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Use the simplest equivalent circuit to determine load current and voltage.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Applications of Thevenin's and Norton's Theorems.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Applications of Thevenin's and Norton's Theorems.

Learning Outcome

Students can explain the principles of applications of thevenin's and norton's theorems, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 14

Superposition Theorem

Learning Objectives

Analyze linear circuits with multiple independent sources.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about superposition theorem?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Total response equals algebraic sum of responses from each source acting alone.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Superposition Theorem.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Superposition Theorem.

Learning Outcome

Students can explain the principles of superposition theorem, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 15

Voltage and Current Sources

Learning Objectives

Define and transform practical sources.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about voltage and current sources?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Source transformation: $I_S = V_S / R_S$, with equivalent resistance unchanged.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Voltage and Current Sources.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Voltage and Current Sources.

Learning Outcome

Students can explain the principles of voltage and current sources, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 16

Maximum Power Transfer Theorem

Learning Objectives

Determine optimum load in DC circuits.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about maximum power transfer theorem?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

For resistive DC networks $R_L = R_{TH}$; $P_{MAX} = V_{TH}^2 / (4R_{TH})$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Maximum Power Transfer Theorem.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Maximum Power Transfer Theorem.

Learning Outcome

Students can explain the principles of maximum power transfer theorem, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 17

AC Quantities and RMS Values

Learning Objectives

Analyze sinusoidal quantities and RMS.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about ac quantities and rms values?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$v(t) = V_m \sin(\omega t + \phi)$; $\omega = 2\pi f$; $T = 1/f$; $V_{RMS} = V_m/\sqrt{2}$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of AC Quantities and RMS Values.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of AC Quantities and RMS Values.

Learning Outcome

Students can explain the principles of ac quantities and rms values, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 18

Phasors and Complex Quantities

Learning Objectives

Represent and manipulate sinusoidal quantities as phasors.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about phasors and complex quantities?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Rectangular $a + jb$; polar $M \angle \theta$; $M = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}(b/a)$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Phasors and Complex Quantities.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Phasors and Complex Quantities.

Learning Outcome

Students can explain the principles of phasors and complex quantities, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 19

AC Circuits: R, L and C

Learning Objectives

Analyze pure R, L and C circuits.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about ac circuits: r, l and c?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$X_L = \omega L$; $X_C = 1/(\omega C)$; R: V and I in phase; L: I lags 90° ; C: I leads 90° .

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of AC Circuits: R, L and C.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of AC Circuits: R, L and C.

Learning Outcome

Students can explain the principles of ac circuits: r, l and c, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 20

Series and Parallel RLC Circuits

Learning Objectives

Calculate impedance, current and phase angle.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about series and parallel rlc circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Series: $Z = R + j(X_L - X_C)$; $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Series and Parallel RLC Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Series and Parallel RLC Circuits.

Learning Outcome

Students can explain the principles of series and parallel rlc circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 21

Complex Operator / j-Operator

Learning Objectives

Use complex impedance and admittance.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about complex operator / j-operator?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$Z_R=R$; $Z_L=j\omega L$; $Z_C=1/(j\omega C)$; $Y=1/Z$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Complex Operator / j-Operator.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Complex Operator / j-Operator.

Learning Outcome

Students can explain the principles of complex operator / j-operator, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 22

Resonance Circuits

Learning Objectives

Calculate resonance frequency, bandwidth and Q.
Apply the related equations or operating principles.
Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about resonance circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Series resonance when $X_L = X_C$; $f_0 = 1/(2\pi\sqrt{LC})$; $Q = f_0/BW$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Resonance Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Resonance Circuits.

Learning Outcome

Students can explain the principles of resonance circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 23

Network Theorems in AC Circuits

Learning Objectives

Apply Thevenin, Norton and Superposition to AC circuits.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about network theorems in ac circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Use complex impedance and phasors instead of only resistance.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Network Theorems in AC Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Network Theorems in AC Circuits.

Learning Outcome

Students can explain the principles of network theorems in ac circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 24

AC Power

Learning Objectives

Calculate active, reactive and apparent power.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about ac power?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$P=VI\cos\phi$; $Q=VI\sin\phi$; $S=VI$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of AC Power.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of AC Power.

Learning Outcome

Students can explain the principles of ac power, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 25

Apparent Power and Power Factor

Learning Objectives

Calculate apparent power and power factor.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about apparent power and power factor?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

$S=VI$; $PF=\cos\phi=P/S$; explain power-factor improvement.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Apparent Power and Power Factor.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Apparent Power and Power Factor.

Learning Outcome

Students can explain the principles of apparent power and power factor, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 26

Maximum Power Transfer in AC Circuits

Learning Objectives

Apply conjugate matching.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about maximum power transfer in ac circuits?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

For $Z_{TH}=R+jX$, maximum average power occurs when $Z_L=R-jX$.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Maximum Power Transfer in AC Circuits.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Maximum Power Transfer in AC Circuits.

Learning Outcome

Students can explain the principles of maximum power transfer in ac circuits, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 27

Electrical Measuring Instruments – Moving Coil

Learning Objectives

Explain construction and operation of moving-coil instruments.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about electrical measuring instruments – moving coil?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Moving-coil action produces deflection torque; commonly used for DC measurement.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Electrical Measuring Instruments – Moving Coil.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Electrical Measuring Instruments – Moving Coil.

Learning Outcome

Students can explain the principles of electrical measuring instruments – moving coil, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 28

Moving-Iron Instruments

Learning Objectives

Explain construction, operation and applications.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about moving-iron instruments?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Moving-iron instruments operate by magnetic force and can measure AC and DC.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Moving-Iron Instruments.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Moving-Iron Instruments.

Learning Outcome

Students can explain the principles of moving-iron instruments, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 29

Dynamometer Instruments and Oscilloscope

Learning Objectives

Explain dynamometer operation and basic oscilloscope use.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about dynamometer instruments and oscilloscope?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Dynamometer instruments use fixed/moving coils; oscilloscope displays waveforms and measures amplitude, period and frequency.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Dynamometer Instruments and Oscilloscope.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Dynamometer Instruments and Oscilloscope.

Learning Outcome

Students can explain the principles of dynamometer instruments and oscilloscope, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

WEEK 30

Practical Resistance Measurement Methods

Learning Objectives

Select and apply resistance-measurement methods.

Apply the related equations or operating principles.

Interpret theoretical and practical results.

Pre-Test – Diagnostic Assessment

What do you already know about practical resistance measurement methods?

State one basic principle related to this topic.

Name one practical application.

Lecture Explanation

Ohmmeter, voltmeter-ammeter, substitution, Wheatstone bridge, voltage divider and switching methods.

Worked Example

Solve a representative numerical or conceptual problem using the equations and principles presented in this lecture.

Learning Activity

Perform a guided circuit-analysis, calculation, demonstration, or laboratory activity related to the weekly topic.

Post-Test

Define the main concept of Practical Resistance Measurement Methods.

State the main equation or operating principle.

Solve one representative problem.

Homework

Complete assigned problems and prepare a short summary of Practical Resistance Measurement Methods.

Learning Outcome

Students can explain the principles of practical resistance measurement methods, perform relevant calculations, and apply the topic to appropriate electrical circuits or measurements.

6. Recommended References

Boylestad, R. L., Introductory Circuit Analysis.

Standard electrical circuits and measurements laboratory manuals.

Electrical measuring-instrument manuals and manufacturer datasheets.

Mastering SI Units, Variables, and Circuit Fundamentals Sets the Stage for Engineering Analysis

MODULE 01

Standardization & SI Units

Understand the International System of Units (SI) and convert quantities using engineering metric prefixes (pico to Tera).

MODULE 02

Core Circuit Variables

Define Charge (Q), Current (I), Voltage (V), and Power (P) with their respective standard units and relationships.

MODULE 03

Ohm's Law & Circuit States

Apply $V = I \times R$ to analyze Closed, Open, and Short circuit conditions under DC excitation.

MODULE 04

Resistance & Configurations

Evaluate physical resistance factors ($R = \rho L/A$) and differentiate Series vs. Parallel topologies.

Base Quantity	SI Unit Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Thermodynamic Temp.	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Derived Electrical SI Units

- Electric Charge (Q): Coulomb (C) [$1\text{ C} = 1\text{ A}\cdot\text{s}$]
- Electric Potential (V): Volt (V) [$1\text{ V} = 1\text{ J/C} = 1\text{ W/A}$]
- Resistance (R): Ohm (Ω) [$1\text{ }\Omega = 1\text{ V/A}$]
- Electric Power (P): Watt (W) [$1\text{ W} = 1\text{ J/s} = 1\text{ V}\cdot\text{A}$]
- Frequency (f): Hertz (Hz) [$1\text{ Hz} = 1\text{ s}^{-1}$]

Electric Charge (Q)

Symbol: Q | **Unit:** Coulomb (C)

Fundamental property of matter. An electron carries -1.602×10^{-19} C.

Governing Relation: $Q = \int I \, dt$

Electric Current (I)

Symbol: I | **Unit:** Ampere (A)

Time rate of charge flow through a cross-section.

Governing Relation: $I = dq / dt$

Electric Power (P)

Symbol: P | **Unit:** Watt (W)

Time rate of absorbing or supplying energy in a circuit.

Governing Relation: $P = V \times I = I^2 R$

Voltage / Potential (V)

Symbol: V | **Unit:** Volt (V)

Energy required to move a unit charge between two points.

Governing Relation: $V = dw / dq$

Powers of Ten Scale Electrical Quantities Across 15 Orders of Magnitude

Prefix Name	Symbol	Power of Ten	Decimal Value
Giga	G	10 ⁹	1,000,000,000
Mega	M	10 ⁶	1,000,000
kilo	k	10 ³	1,000
milli	m	10 ⁻³	0.001
micro	μ	10 ⁻⁶	0.000001
nano	n	10 ⁻⁹	0.000000001
pico	p	10 ⁻¹²	0.000000000001
femto	f	10 ⁻¹⁵	0.000000000000001

Unit Conversion Rules

1. Larger to Smaller Unit:
Multiply by 10⁺ⁿ (shift decimal right).

Example: 2.5 mA = 2.5 × 10⁻³ A = 2,500 μA
2. Smaller to Larger Unit:
Multiply by 10⁻ⁿ (shift decimal left).

Example: 4,700 Ω = 4.7 × 10³ Ω = 4.7 kΩ
3. Exponential Rule:
Difference between exponents = Δ exponent.

Ohm's Law Defines the Linear Relationship Between Voltage, Current, and Resistance

OHM'S LAW FORMULATIONS

$$V = I \times R \quad | \quad I = V / R \quad | \quad R = V / I$$

Where Voltage (V) is in Volts, Current (I) is in Amperes, and Resistance (R) is in Ohms.

Closed Circuit

Complete closed loop for current flow.

Finite current: $I = V / R$
Finite resistance $R > 0$.

Open Circuit

Break in conductor path ($R = \infty$).

Zero current: $I = 0 \text{ A}$
Voltage drop V across terminals.

Short Circuit

Zero resistance connection ($R = 0$).

Maximum current: $I \rightarrow \infty$
Zero voltage drop: $V = 0 \text{ V}$.

Material Resistivity, Length, and Area Directly Determine Electrical Resistance

Physical Resistance Equation

$$R = \rho \times (L / A)$$

- ρ (Resistivity): Material property in Ohm-meters ($\Omega \cdot m$).
- L (Length): Conductor length in meters (m). $R \propto L$.
- A (Cross-Sectional Area): Area in m^2 ($A = \pi d^2/4$). $R \propto 1/A$.
- Temperature (T): Resistance increases with temperature in metallic conductors.

Material	Resistivity ρ ($\Omega \cdot m$)	Classification
Silver	$10^{-8} \times 1.59$	Conductor
Copper	$10^{-8} \times 1.68$	Conductor
Aluminum	$10^{-8} \times 2.65$	Conductor
Silicon	$10^3 \times 2.30$	Semiconductor
Glass / Rubber	$10^{14} - 10^{16}$	Insulator

Topology Dictates Whether Current Is Shared or Voltage Is Constant

Series Circuit Topology

Single Path for Current Flow

- Current: Same current flows through all components ($I_{\text{total}} = I_1 = I_2 = I_3$).
- Voltage: Total voltage equals sum of individual drops ($V_{\text{total}} = V_1 + V_2 + V_3$).
- Equivalent Resistance: $R_{\text{eq}} = R_1 + R_2 + R_3 + \dots$
- Reliability: Single open-circuit breaks current flow across entire loop.

Parallel Circuit Topology

Multiple Branches across Voltage Source

- Voltage: Same voltage appears across every branch ($V_{\text{total}} = V_1 = V_2 = V_3$).
- Current: Total current equals sum of branch currents ($I_{\text{total}} = I_1 + I_2 + I_3$).
- Equivalent Resistance: $1 / R_{\text{eq}} = 1/R_1 + 1/R_2 + 1/R_3$
- Reliability: Open-circuit in one branch does not halt flow in other branches.

Applying Ohm's Law and Unit Conversions to Solve Circuit Parameters

EXAMPLE PROBLEM 1.1

A DC voltage source of $V = 12\text{ V}$ is connected across a resistor $R = 2.4\text{ k}\Omega$. Calculate:

(a) Circuit current I in milliamperes (mA). (b) Total power dissipated P in milliwatts (mW).

Step 1: Unit Conversion

Convert resistance to standard SI units:

$$R = 2.4\text{ k}\Omega = 2.4 \times 10^3\ \Omega = 2,400\ \Omega$$

Step 2: Calculate Current (I)

Apply Ohm's Law ($I = V / R$):

$$I = 12\text{ V} / 2400\ \Omega = 0.005\text{ A}$$

$$\text{In mA: } 0.005 \times 10^3 = 5.0\text{ mA}$$

Step 3: Calculate Power (P)

Apply Power relation ($P = V \times I$):

$$P = 12\text{ V} \times 0.005\text{ A} = 0.06\text{ W}$$

$$\text{In mW: } 0.06 \times 10^3 = 60.0\text{ mW}$$

Three Practice Problems Reinforce Core Concepts Before Next Lecture

Homework Problem 1

Unit Conversions & Prefixes

Convert the following quantities into standard base SI units:

- (a) 450 μA to Amperes.
- (b) 12.5 kV to Volts.
- (c) 3.3 M Ω to Ohms.

Homework Problem 2

Ohm's Law & Circuit Calculations

A 100 Ω resistor dissipates 4 Watt of power in a closed circuit.

- (a) Calculate the voltage V across the resistor.
- (b) Determine the current I flowing through it.

Homework Problem 3

Physical Resistance Factors

A copper wire ($\rho = 1.68 \times 10^{-8} \Omega \cdot \text{m}$) has a length $L = 50 \text{ m}$ and diameter $d = 1 \text{ mm}$.

- (a) Calculate the cross-sectional area A in m^2 .
- (b) Determine the total resistance R of the wire.

Electrical Current:

The continuous flow of free electronic constitutes an electric current. The unit of current is ampere (A) and is measured by Ammeter. It is denoted by the letter 'I'.

Ampere :If one coulomb charge cross over the area of cross section of the conductor per one second then the value of current flows through the conductor is called 'one Ampere'.

One Coulomb: $\times 10^{18}$ number of electrons is mention as one coulomb.

Voltage: To create the current flow in a conductor; the electrical pressure which is used to move the electrons is called voltage. It is denoted by the letter 'V'. the unit of voltage is 'volt' and is measured by voltmeter.

One Volt:

One volt means the force to move one coulomb of electrons in one second.

Resistance:

The property of conductor which opposes the flow of current through it is called resistance. It is denoted by the letter 'R'. the unit of resistance is ohms (Ω) and it is measured by Ohm meter.

Ohm:

When a conductor having 1V potential between the two end points; one ampere current will flowing through the conductor and the resistance value of the conductor is 1 Ohm (Ω).

The electrical unit:

Quantity	Symbol	Unit	Abbreviation
Current	I	Ampere	A
Voltage	V	volt	V
Resistance	R	Ohm	Ω

Charge	Q	Coulomb	C
Power	P	watt	W

Engineering prefix:

Power of 10	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Example1-Express the following in engineering prefix:

a) 10×10^4 volt. b) 0.1×10^{-3} watts. C) 250×10^{-7} ampere.

Solution:

a) 10×10^4 volt = 100×10^3 V = **100 kV** .

b) 0.1×10^{-3} watts = **0.1 milliwatts** = 0.1 mW .

c) 250×10^{-7} ampere = 25×10^{-6} A = **25 μ A** .

Example2- Convert 0.1MV to kV

Solution:

$$0.1MV = 0.1 \times 10^6 V = (0.1 \times 10^3) \times 10^3 = \mathbf{100kV}$$

Homework : Convert 1800kV to MV

Law of resistance:

The resistance of a conductor in a circuit depends upon the following states:

It depends upon the material.

Directly proportional to the length of the conductor.

Inversely proportional to the area of the cross-section of the conductor.

It also depends upon the temperature of the conductor.

Resistance calculation:

$$R = \rho \frac{L}{A}$$

Where:

R is the resistance (ohms)

ρ is specific resistance (resistivity) in (ohm. Meter).

L is length of the conductor (meter).

A is area of the cross section of a conductor (Sq.m).

Specific resistance:

The resistance that is offered by one cubic cm material is called specific resistance. The following table shows the specific resistance of material:

Materials		Specific resistance is ohm - meter
Gold	-	2.42×10^{-8}
Silver	-	1.63×10^{-8}
Copper	-	1.724×10^{-8}
Aluminium	-	2.83×10^{-8}
Rubber	-	8×10^7
Glass	-	10×10^{11}

Example-3: 1cm^2 cross section 50m long copper conductor has specific resistance 1.72×10^{-8} $\Omega\cdot\text{cm}$ find the resistance.

Solution:

$$L = 50m = 50 \times 100cm = 5000cm$$

$$A = 1 \text{ cm}^2$$

$$\text{specific resistance} = 1.72 \times 10^{-8} \Omega \cdot \text{cm}$$

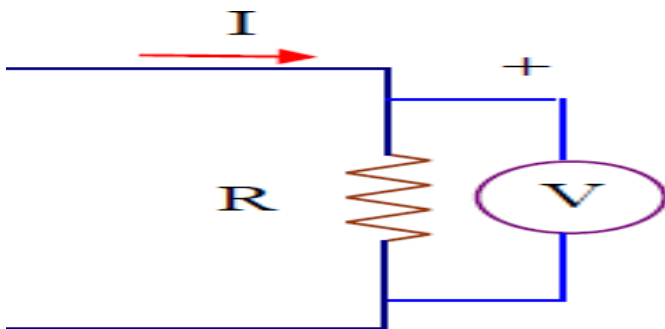
$$R = \rho \frac{L}{A} = 1.72 \times 10^{-8} \times \frac{5000}{1} = 0.0086 \Omega$$

Ohm's Law

A relationship was derived by the scientist Ohm; between the current; voltage and resistance of the circuit. It says;

"At a constant temperature; the current flowing through the circuit is directly proportional to the voltage and inversely proportional to the resistance".

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}} \quad \text{or} \quad I = \frac{V}{R}$$



$$R = \frac{V}{I}$$

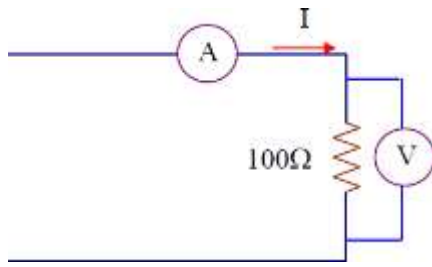
$$V = I \times R$$

When the resistance of a circuit is constant; if the voltage increases the current increases and if the voltage decreases the current decreases. If any two of the three values (I; V; R) are known the third value can be easily calculated.

Example-4: The supply voltage of the circuit is 50V (reading of voltmeter) and the resistance value is 100 Ω . Calculate the current flowing through this circuit (reading of ammeter).

Solution:

Voltage (V) = 50V



Resistance (R) = 100Ω

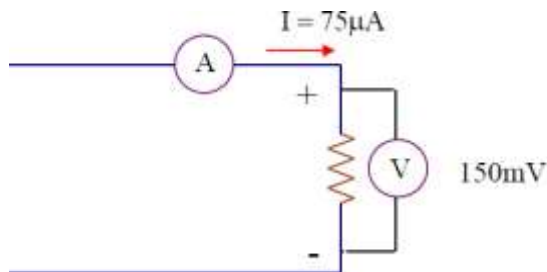
Current (I) = ?

According to Ohm's law

$$I = \frac{V}{R} = \frac{50}{100} = 0.5A$$

Example-5: For a given circuit the reading of voltmeter is 150mV. and the reading of ammeter is $75\mu A$. Calculate the resistance value of the circuit.

Solution:



$$R = \frac{V}{I} = \frac{150 * 10^{-3}}{75 * 10^{-6}} = 2 * 10^3 \Omega = 2K\Omega$$

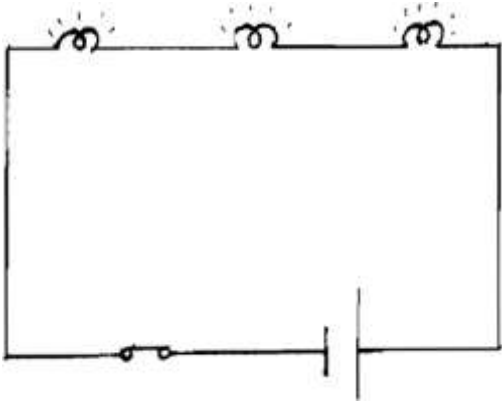
Electrical Circuits:

The circuit is defined as ; the current flows from the supply points through the load to complete path. The type of electrical circuit are:

Closed circuit. 2) Open circuit. 3) Short circuit.

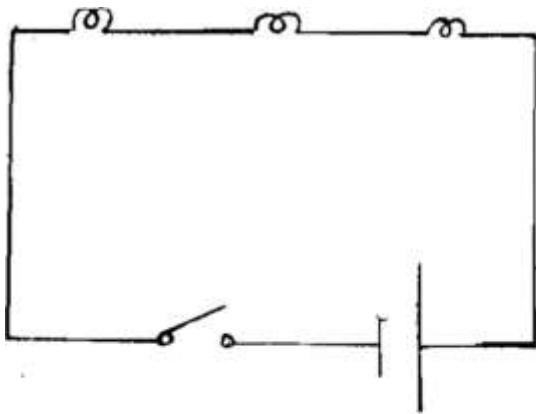
1. Closed circuit

When a load is connected between two terminals of an electrical supply in such a way; that the current should pass through the load is said to be closed circuit.



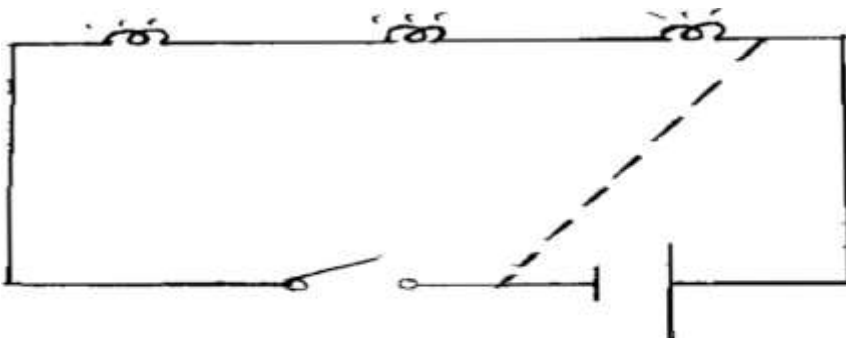
2. Open circuit

In a circuit if there is no way to the flow of current due to disconnection of wire or if the switch is Off state; then the circuit is said to be open circuit.



3. Short Circuit:

The wires contact each other when there are connected in supply; the short circuit will occurs when two terminals of the supply is connected directly without the load the current flow of the circuit is infinite because it has no resistance.



Classification of electric circuits

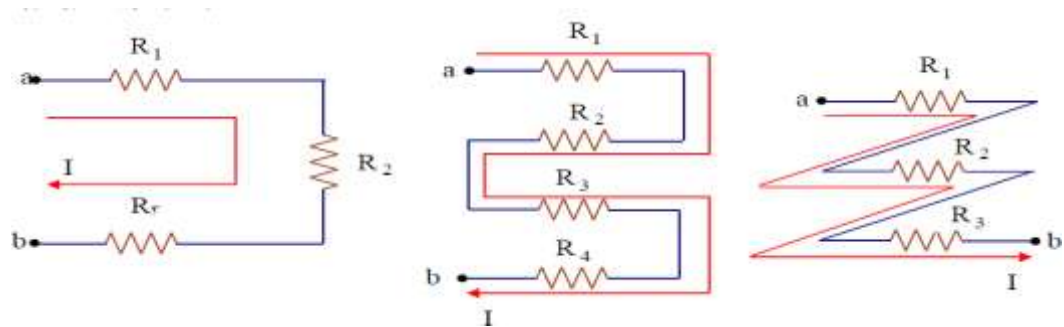
Series Circuit.

Parallel Circuit.

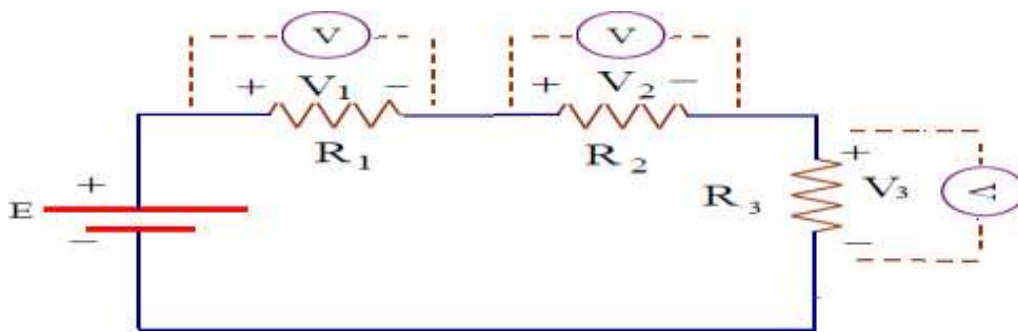
Series Parallel Circuit.

Mesh or Network Circuit.

Series Circuit:



"I" ampere current flows in all resistors" ,For agivin circuit (E)is electromotive force of power supply ,V is total voltage of source or (V_s)



Each resistor has a voltage drop across it as given by Ohm's law. Thus

$$V_1 = IR_1; V_2 = IR_2; V_3 = IR_3$$

The total drop in three resistors put together is:

$$E = V_1 + V_2 + V_3$$

$$= IR_1 + IR_2 + IR_3$$

$$= I(R_1 + R_2 + R_3)$$

$$\frac{E}{I} = R_1 + R_2 + R_3 \text{ Were } \left[\frac{V}{I} = R \right]$$

$$\therefore R = R_1 + R_2 + R_3 \text{ this is called (total resistance) } R_T$$

$$\therefore R_T = R_1 + R_2 + R_3$$

Voltage divider rule :

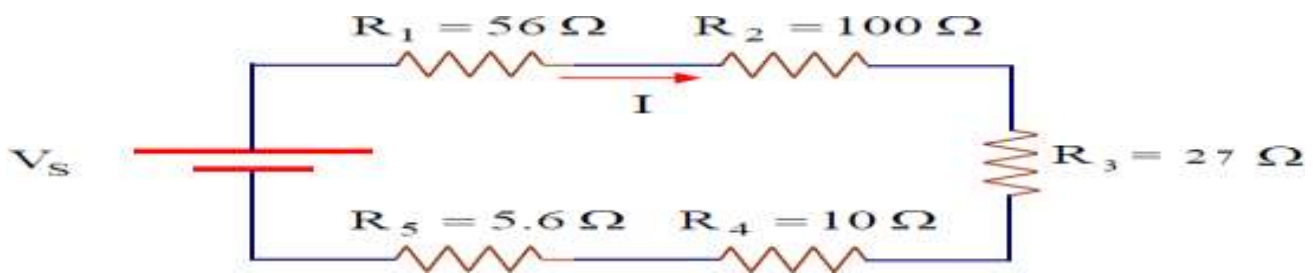
$$V_1 = E \times \frac{R_1}{R_1 + R_2 + R_3}$$

$$V_2 = E \times \frac{R_2}{R_1 + R_2 + R_3}$$

$$V_3 = E \times \frac{R_3}{R_1 + R_2 + R_3}$$

Example-6: the 56Ω ; 100Ω ; 27Ω ; 10Ω and 5.6Ω resistors are connected in series across a 24V supply. Find out the total resistance of the circuit and current that flows through the circuit.

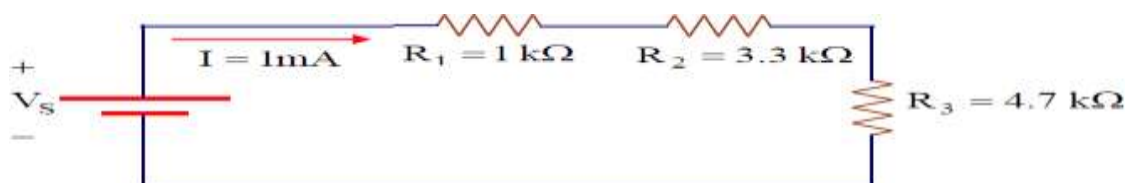
Solution:



$$R_T = R_1 + R_2 + R_3 + R_4 + R_5 = 56 + 100 + 27 + 10 + 5.6 = 198.6\Omega$$

$$I = \frac{V}{R_T} = \frac{24}{198.6} = 0.12\text{A}$$

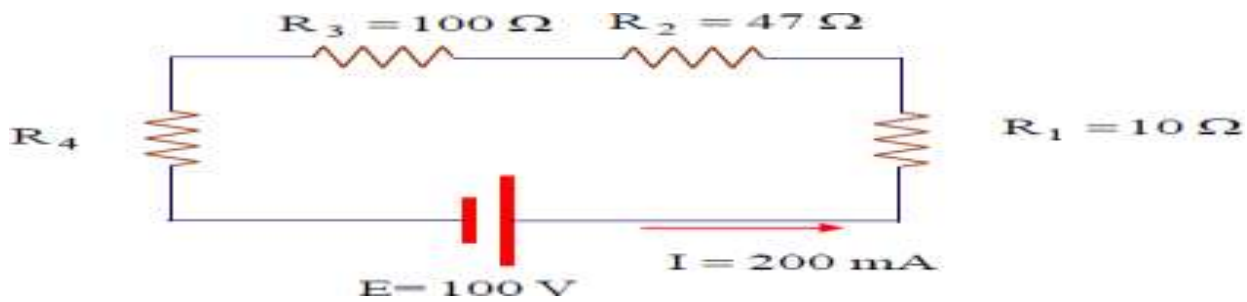
Example-7: Three resistors are connected in series. If the current passing through the circuit is 1mA , find the voltage source.



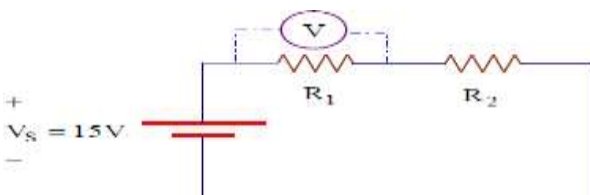
$$\text{Solution: } R_T = R_1 + R_2 + R_3 = (1 + 3.3 + 4.7) \times 1000 = 9000\Omega$$

$$V_S = I \times R_T = 1 \times 10^{-3} \times 9000 = 9 \text{ V}$$

Homework:- for giving circuit find (R_4) ?

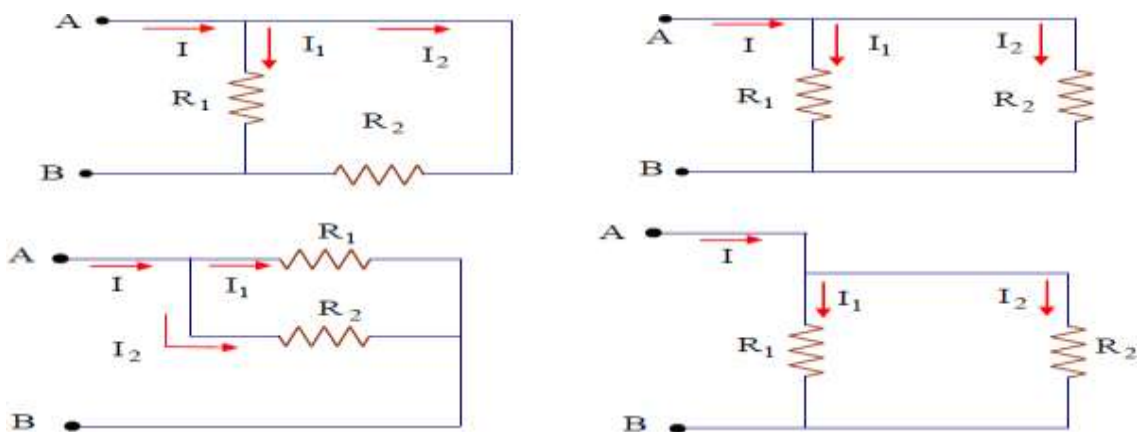


Homework :- for the circuit shown the reading of voltmeter is (4.69v) ; the resistor (R_2) is ($1.5 \text{ K}\Omega$), find (R_1) ?



Resistance in parallel Circuit:

When resistors are connected across one another so that the same voltage is applied between the end points of each; then they are said to be in parallel. The current in each resistor is different and the current (I) taken from the supply is divided among the resistors.



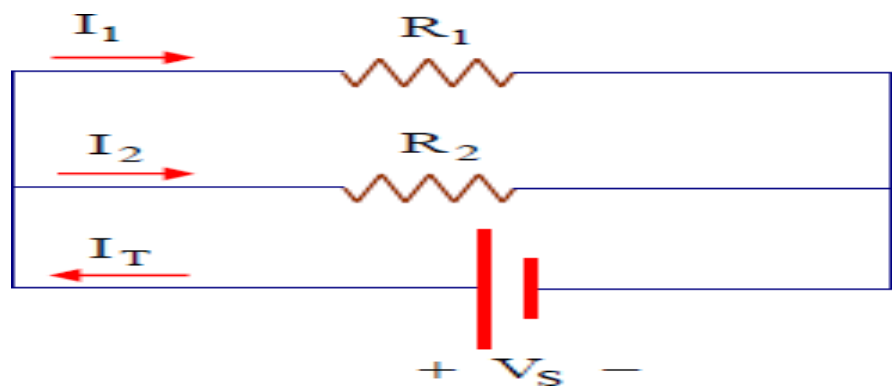
For two resistors R_1 and R_2 connected in parallel then

$$I_T = I_1 + I_2$$

Where $V_S = V_1 = V_2$ also $I_T = \frac{V_S}{R_T}$, $I_1 = \frac{V_1}{R_1}$, $I_2 = \frac{V_2}{R_2}$

$$\therefore \frac{V_S}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} \Rightarrow \frac{V_S}{R_T} = V_S \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$



Current divider rule :-

$$I_1 = \frac{V_1}{R_1} \text{ where } V_1 = V_S \text{ and } V_S = R_T \times I_T$$

$$\therefore R_T = \frac{R_1 \times R_2}{R_1 + R_2} \Rightarrow V_1 = \frac{R_1 \times R_2}{R_1 + R_2} \times I_T$$

$$\therefore I_1 = \frac{V_1}{R_1} \Rightarrow I_1 = \left(\frac{R_1 \times R_2}{R_1 + R_2} \times I_T \right) / R_1 = \frac{I_T \times R_2}{R_1 + R_2}$$

$$I_2 = \frac{I_T \times R_1}{R_1 + R_2}$$

In parallel circuit total current (I) is equal to some of the current I_1 ; I_2 and I_3 .

$$I_T = I_1 + I_2 + I_3$$

According to Ohm's law we can find the total resistance (R_T) as given below:

$$I_T = \frac{V}{R_T}$$

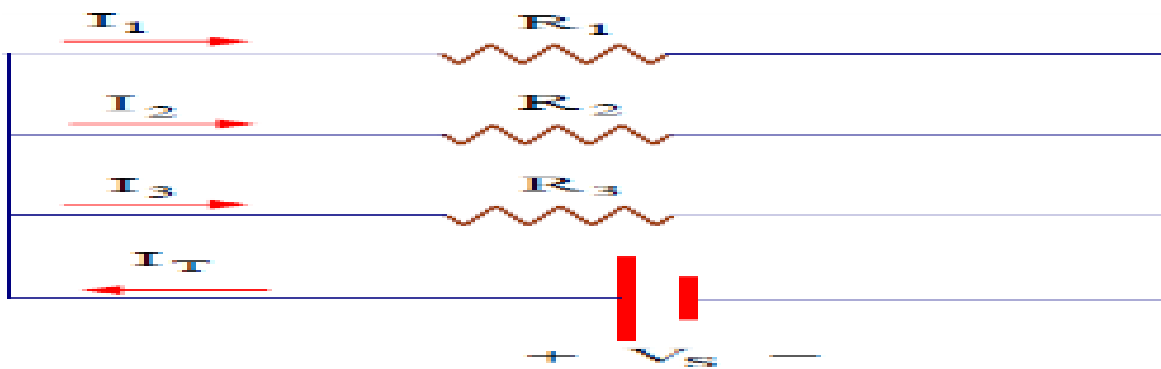
$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

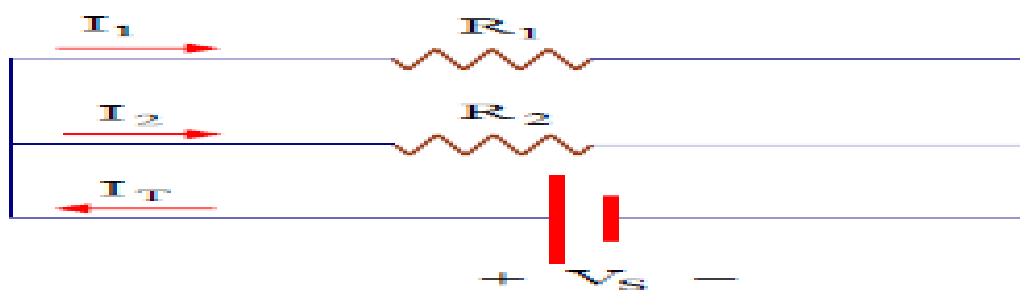
$$I_3 = \frac{V}{R_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

$$V = V_1 = V_2 = V_3 = V_S$$



Example-8: The resistor (6Ω) are connected in parallel with resistor (4Ω) across the voltage supply (24 v) , find the total current .



Solution

$$R_1 = 6\Omega, R_2 = 4\Omega \quad V = 24v$$

$$R_T = ? , I_T = ?$$

In parallel circuit

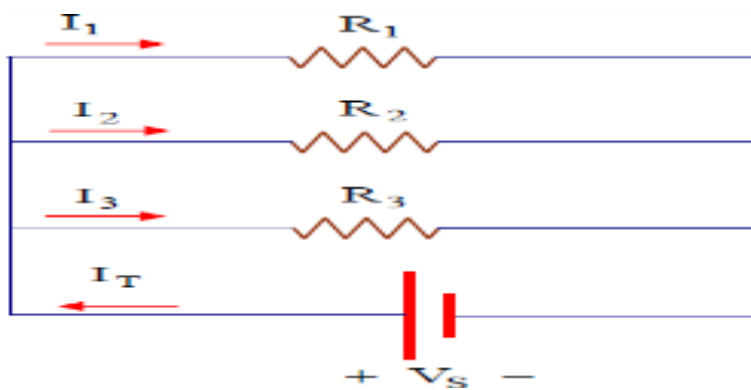
$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 4}{6 + 4} = \frac{24}{10} = 2.4 \Omega$$

According to Ohm's law

$$I_T = \frac{V}{R_T} = \frac{24}{2.4} = 10A$$

Example-9: Three resistors 10Ω ; 5Ω and 2Ω are connected in parallel. The total current flowing in the circuit is $2A$. find out the total resistance and supply voltage of the circuit.

Solution:



$$R_1 = 10\Omega \quad R_2 = 5\Omega \quad R_3 = 2\Omega$$

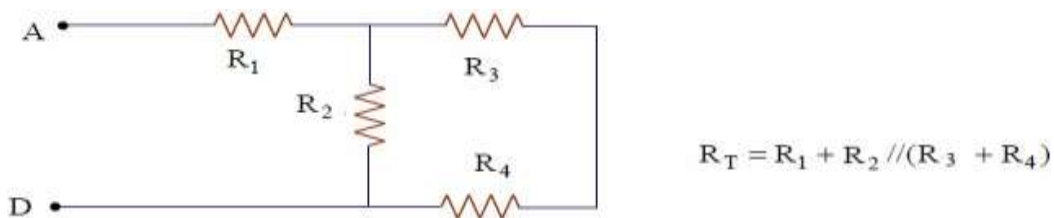
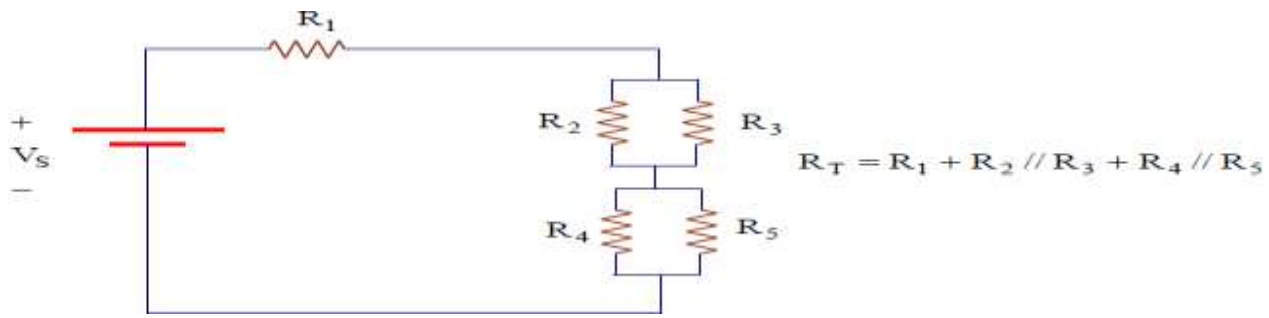
$$R_T = ? \quad I_T = 2A \quad V = ?$$

$$R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_2 + R_1 R_3} = \frac{10 \times 5 \times 2}{(5 \times 2) + (2 \times 10) + (10 \times 5)}$$

$$= \frac{100}{10 + 20 + 50} = \frac{100}{80} = 1.25\Omega$$

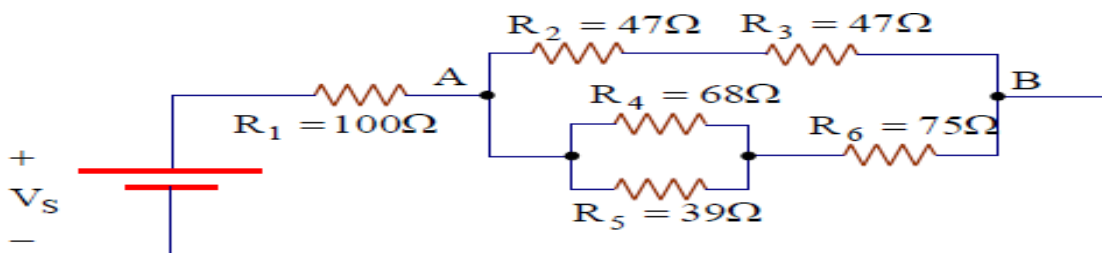
$$V = I_T \times R_T = 2 \times 1.25 = 2.5v$$

Resistance in series parallel circuit : In this circuit one and more resistors connected in series with one more resistors connected in parallel. It is a combination of series and parallel circuit. Here the total resistance (R_T) of the circuit is:



Example-10: , Find out the total resistance of the series parallel circuit.

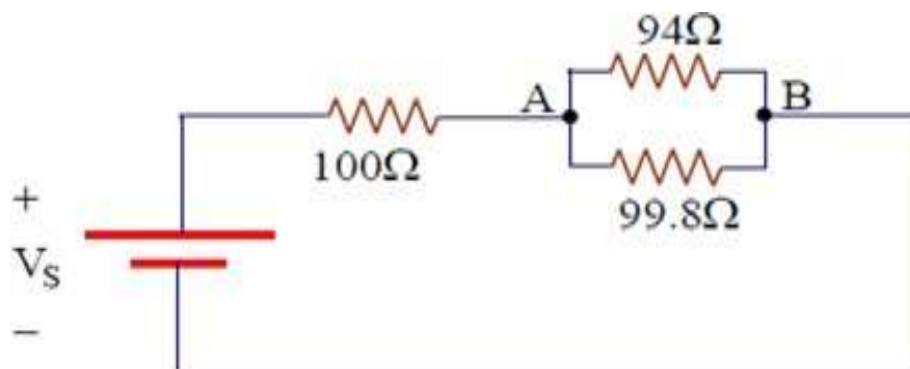
Solution:



$$R_{T456} = 75 + \frac{68 \times 39}{68 + 39} = 99.8\Omega$$

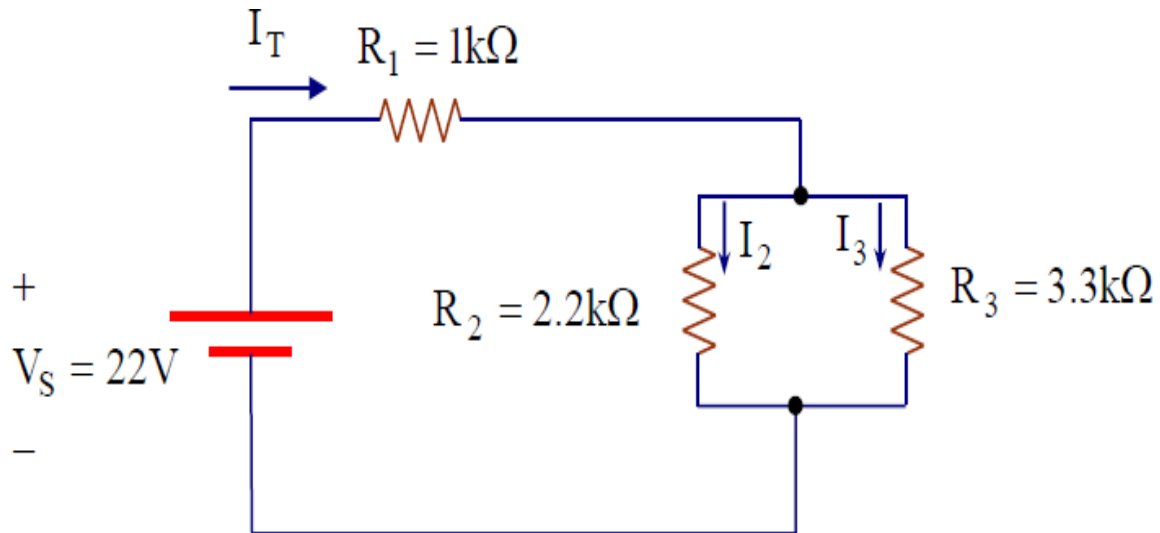
$$R_{T23} = 47 + 47 = 94\Omega$$

$$R_T = R_1 + (R_{T456} // R_{T23}) = 100 + 48.4 = 148.4\Omega$$



Example 11 :- for agivin circuit find the current passing in each resistors ?

Solution :-



$$R_T = R_1 + (R_2 // R_3)$$

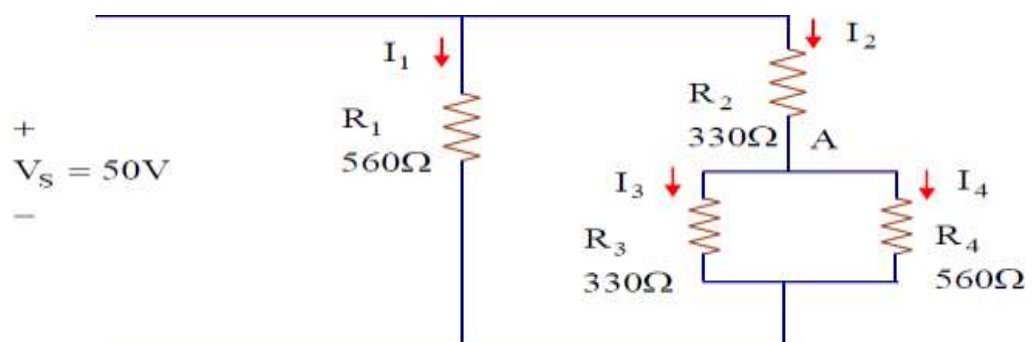
$$R_T = 1 + \left(\frac{2.2 \times 3.3}{2.2 + 3.3} \right) = 2.32 \text{ K}\Omega$$

$$I_T = \frac{V_S}{R_T} = \frac{22}{2.32 \times 1000} = 9.48 \text{ mA}$$

$$I_2 = \frac{I_T \times R_3}{R_2 + R_3} = \frac{9.48 \times 10^{-3} \times 3.3 \times 10^3}{(2.2 + 3.3) \times 1000} = 5.688 \text{ mA}$$

$$I_3 = \frac{9.48 \times 10^{-3} \times 2.2 \times 10^3}{(2.2 + 3.3) \times 1000} = 3.79 \text{ mA}$$

Example-12: Find the current through each resistor when the applied voltage is 50v



Solution:

$$R_{234} = R_2 + (R_3 // R_4)$$

$$R_{234} = 330 + \left(\frac{330 \times 560}{330 + 560} \right) = 537.64 \Omega$$

$$I_2 = \frac{V}{R_{234}} = \frac{50}{537.64} = 0.092 A$$

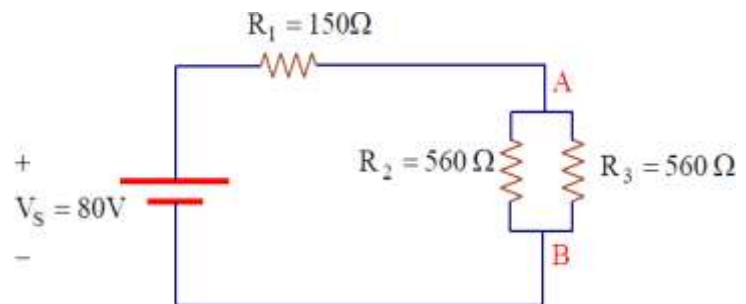
$$I_3 = \frac{I_2 \times R_4}{R_3 + R_4} = \frac{0.092 \times 560}{560 + 330} = 0.057 A$$

$$I_4 = I_2 - I_3 = 0.092 - 0.057 = 0.035 A$$

$$I_1 = \frac{V}{R_1} = \frac{50}{560} = 0.089 A$$

Example 13 : for agivin circuit ; find the voltage across the points A and B ?

Solution :



$$R_T = R_1 + (R_2 // R_3)$$

$$R_T = 150 + \left(\frac{560 \times 560}{560 + 560} \right) = 430 \Omega$$

$$I = \frac{V_S}{R_T} = \frac{80}{430} = 0.186 A$$

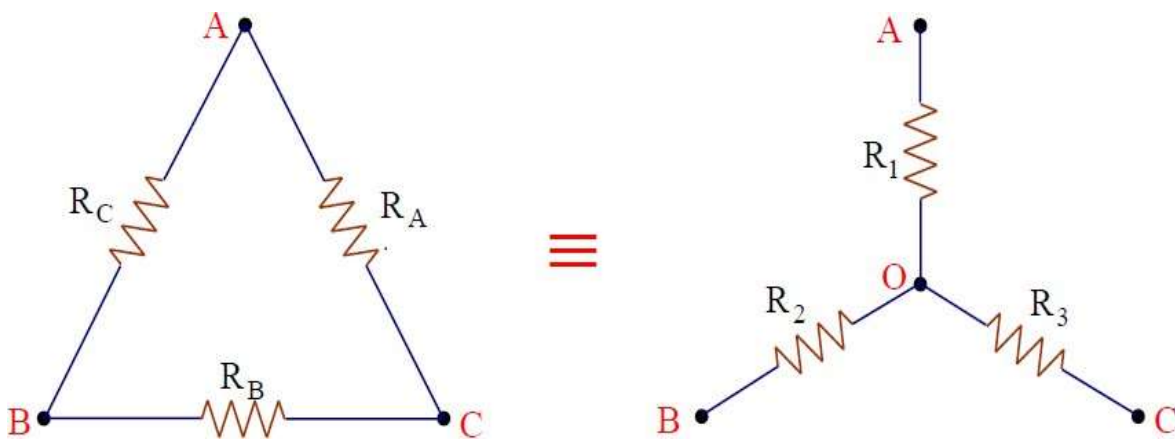
$$R_{AB} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{560 \times 560}{560 + 560} = 280 \Omega$$

Voltage drop across AB is: $V_{AB} = R_{AB} \times I = 280 \times 0.186 = 52.09 V$

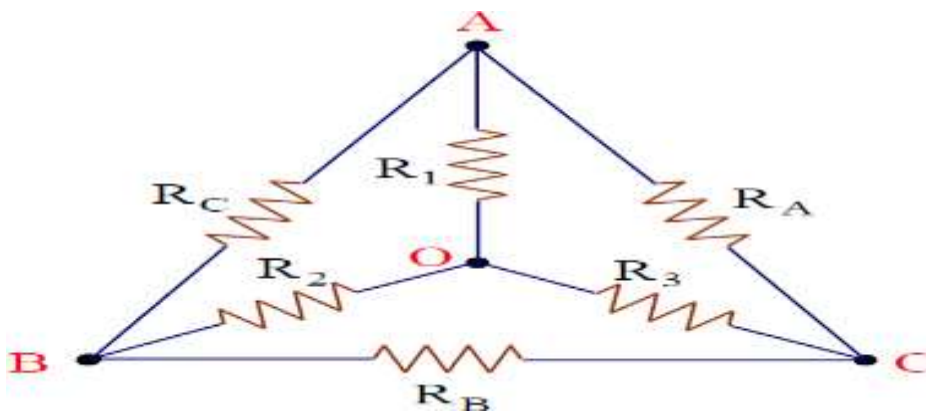
Star and delta connection:

In solving networks having considerable number of branches; one sometimes due to a large number of equations that have to be solved. This complicated network can be simplified by replacing delta meshes by equivalent star system and vice versa.

Suppose we are given three resistance R_A , R_B and R_C connected in delta fashion between terminal A; B and C as shown in figure. these resistances can be replaced by three resistance R_1 , R_2 and R_3 in star as shown in.



Delta to Star conversion:



$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_C R_A}{R_A + R_B + R_C}$$

Star to Delta conversion:

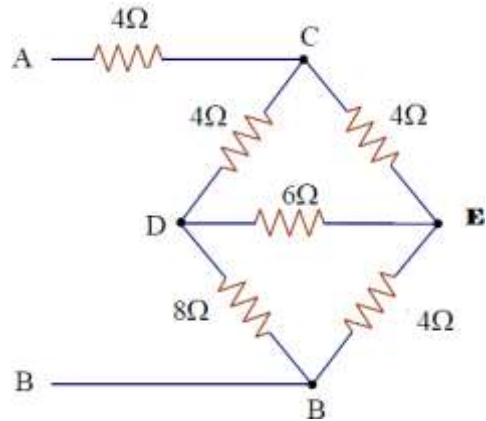
$$R_A = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_B = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Example-14: Find the total resistance between the points A and B of figure(a).

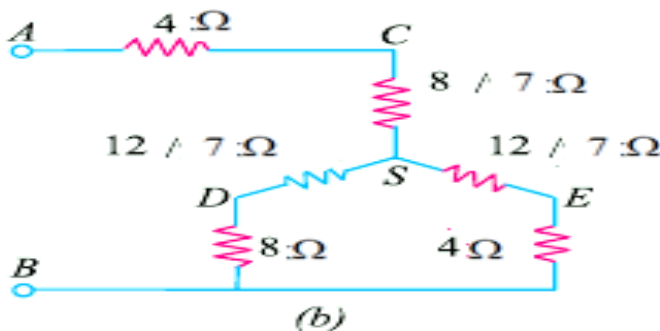
Solution:



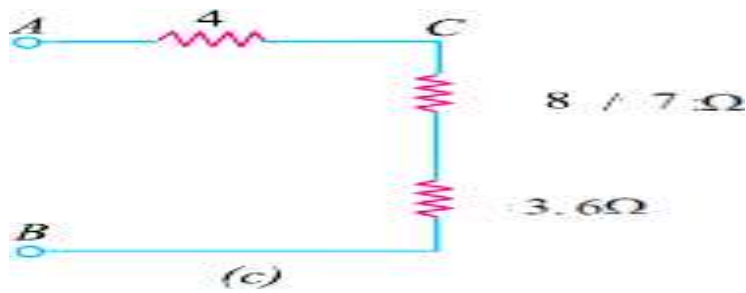
For finding R_{AB} we will convert the delta CDE of figure(a) into its equivalent star as shown in figure(b)

$$R_{CS} = 4 \times 4 / 14 = 16 / 14 = 8 / 7 \Omega \quad ; \quad R_{ES} = 4 \times 6 / 14 = 12 / 7 \Omega$$

$$R_{DS} = 6 \times 4 / 14 = 12 / 7 \Omega$$



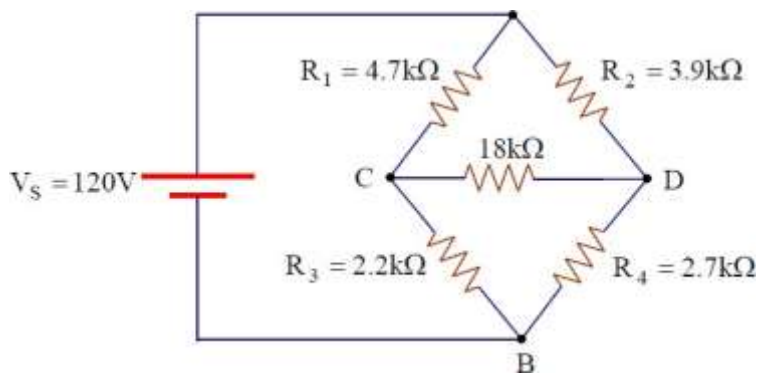
The two parallel resistances between S and B can be reduced to a single resistance of (3.6 Ω) as shown in figure (c).



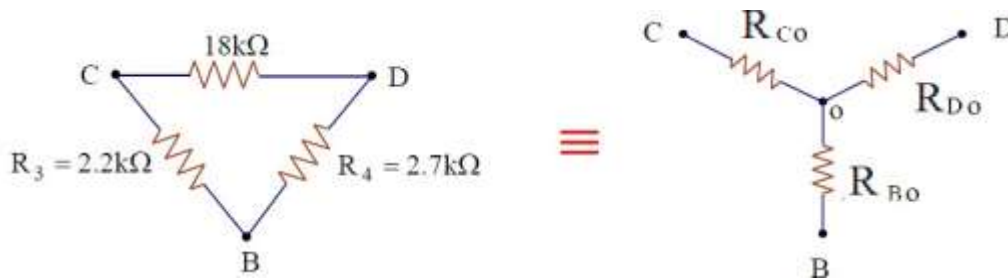
As seen from figure(c):

$$R_{AB} = 4 + (8/7) + (3.6) = 8.6\Omega$$

Example15 : for the circuit shown in figure ; find the current passing through ($R_L=18K\Omega$)



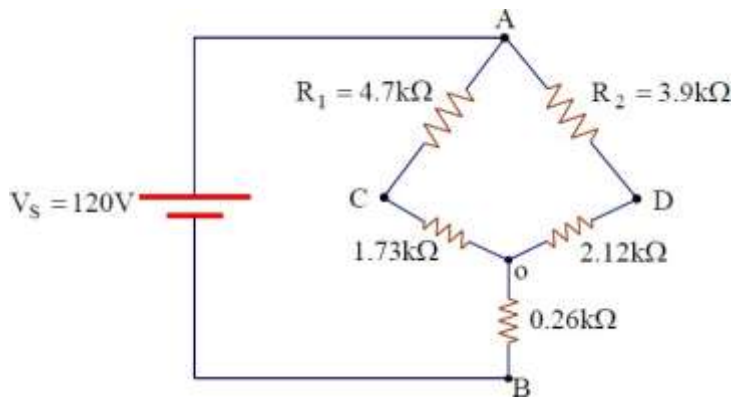
For finding R_T we will convert the delta CDB into its equivalent star as shown in figure



$$R_{Co} = \frac{18 \times 2.2}{18 + 2.2 + 2.7} = 1.73K\Omega$$

$$R_{Do} = \frac{18 \times 2.7}{18 + 2.2 + 2.7} = 2.12K\Omega$$

$$R_{Bo} = \frac{2.7 \times 2.2}{18 + 2.2 + 2.7} = 0.26K\Omega$$



$$R_T = \frac{(4.7+1.73) \times (3.9+2.12)}{(4.7+1.73) + (3.9+2.12)} + 0.26 \approx 3.37 K\Omega$$

$$I = \frac{V_S}{R_T} = \frac{120}{3.37 \times 1000} = 35.6 mA$$

By using current divider rule to finding I_1 and I_2

$$I_1 = \frac{I \times (3.9+2.12)}{(3.9+2.12) + (4.7+1.73)} = 17.2 mA$$

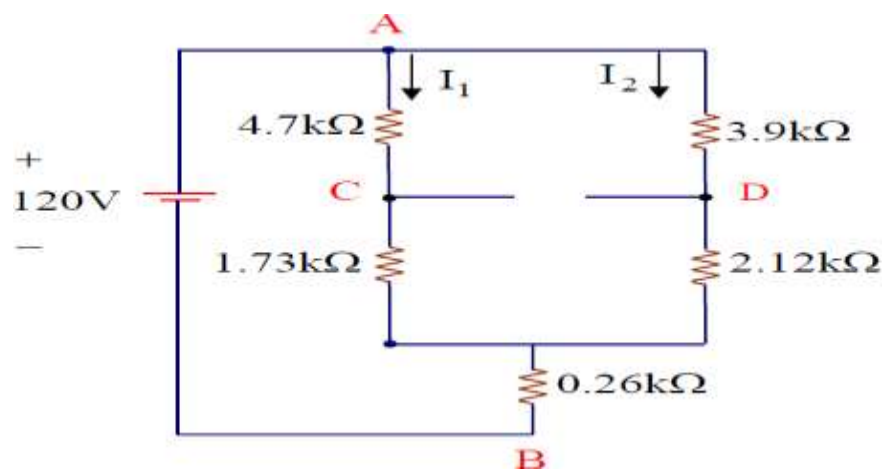
$$I_2 = \frac{I \times (4.7+1.73)}{(3.9+2.12) + (4.7+1.73)} = 18.38 mA$$

$$V_C = I_1 \times 1.73 \times 1000 = 17.2 \times 10^{-3} \times 1.73 \times 1000 = 39.18 V$$

$$V_D = I_2 \times 2.12 \times 1000 = 18.38 \times 10^{-3} \times 2.12 \times 1000 = 48.32 V$$

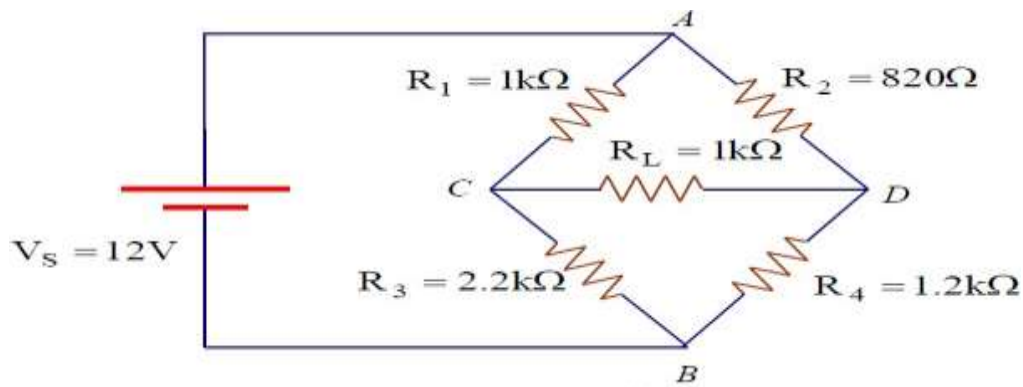
$$V_{CD} = V_C - V_D = 39.18 - 48.32 = (-9.14)$$

$$I_{18K\Omega} = \frac{V_{CD}}{18 \times 1000} = \frac{9.14}{18000} = 0.507 mA$$



Home work : for the circuit shown in figure find the current passing through

($R_L = 1\text{k}\Omega$).

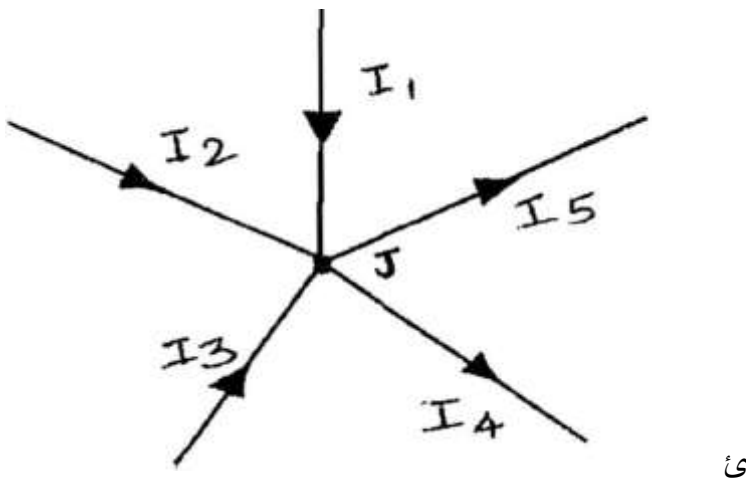


Kirchoff's Law

Kirchoff's law is used to find out the current flow in the network circuits easily where Ohm's law is not applicable. It is applicable both for DC and AC circuits.

Current law:

The sum of the current flowing towards a junction (node) is equal to the sum of the currents flowing away from it. This is called Kirchoff's current Law(K.C.L).



In the figure above J a junction (or node) formed by five conductors. The current in these conductors are I_1 , I_2 , I_3 , I_4 , and I_5 .

Some of these currents are flowing towards J others away from it.

According to Kirchoff's Law:

$$I_1 + I_2 + I_3 = I_4 + I_5$$

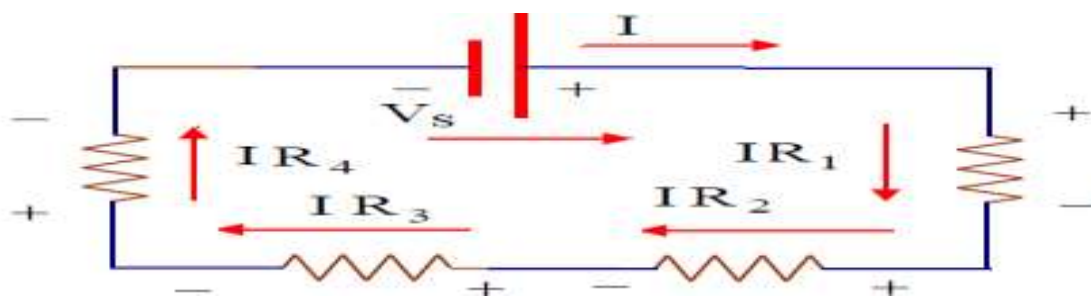
Flowing towards J = Flowing away from J

Otherwise

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0 \quad \text{this is known KCL equation}$$

Voltage Law:

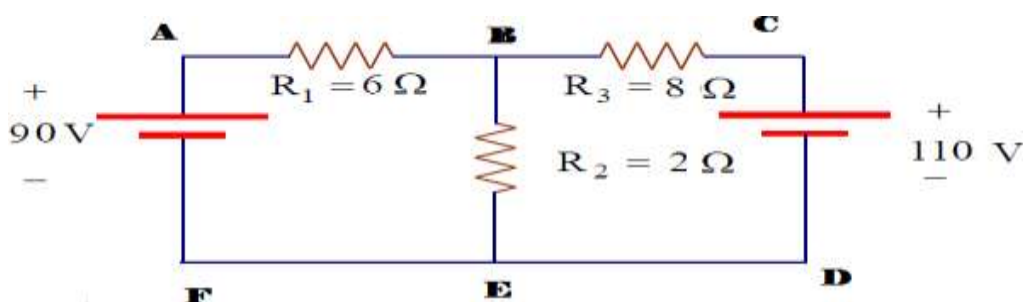
At any closed circuit the potential drop (IR) at each resistance is equal to the total voltage given to the circuit.



In a closed circuit, the sum of the potential drop is equal to the sum of the potential rises. This is called Kirchoff's Voltage Law.

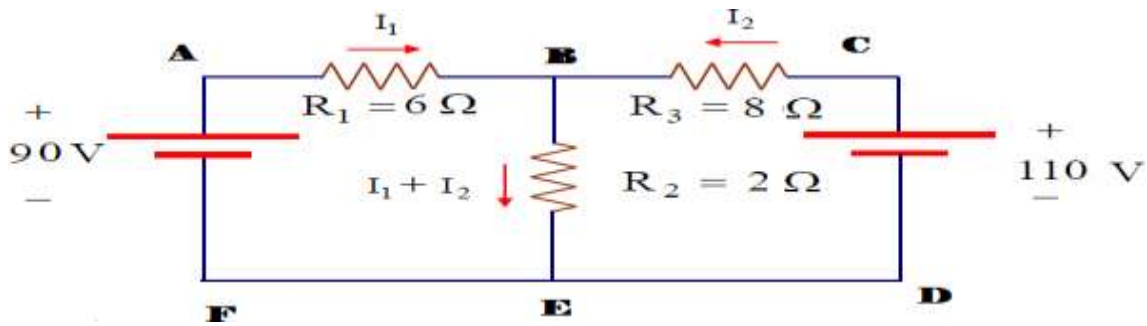
$$V_s = IR_1 + IR_2 + IR_3 + IR_4$$

Example-16: In the circuit of figure using Kirchoff's laws, find the current in the various elements.



Solution:

According to Kirchoff's first law mark the direction of current flow. According to second law, Write down the KVL equation in the closed circuits



ABEFA forms a closed circuit (LOOP 1)

$$6I_1 + 2(I_1 + I_2) = 90$$

$$6I_1 + 2I_1 + 2I_2 = 90$$

$$8I_1 + 2I_2 = 90 \quad \text{--- (1)}$$

CBEDC forms another closed circuit (LOOP2)

$$8I_2 + 2(I_1 + I_2) = 110$$

$$8I_2 + 2I_1 + 2I_2 = 110$$

$$2I_1 + 10I_2 = 110 \quad \text{--- (2)}$$

To solve, equation(2) is multiplied by 4.

$$8I_1 + 40I_2 = 440 \quad \text{--- (3)}$$

$$\text{subtracted eq (1) from eq (3)} \quad 8I_1 + 2I_2 = 90 \quad \text{--- (1)}$$

$$8I_1 + 40I_2 = 440$$

$$-38I_2 = -350$$

$$38I_2 = 350 \Rightarrow I_2 = \frac{350}{38} = 9.211A$$

Substitute the I_2 value in equation (1)

$$8I_1 + 2(9.211) = 90$$

$$8I_1 + 18.422 = 90$$

$$8I_1 = 90 - 18.422$$

$$I_1 = \frac{71.578}{8} = 8.947A$$

From this,

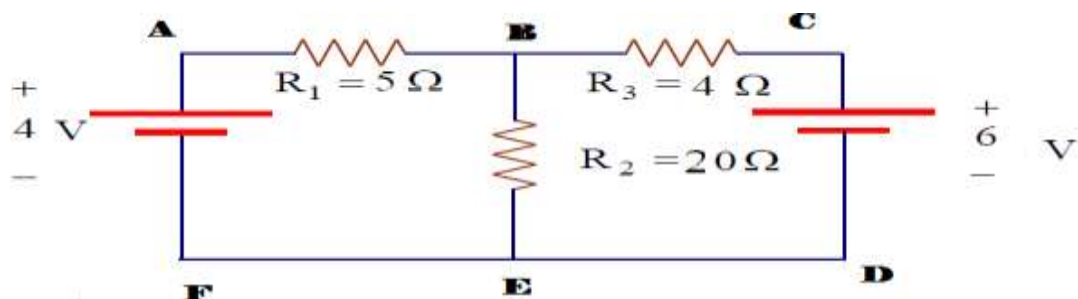
The current through 6Ω resistor is $= 8.947A$

The current through 8Ω resistor is $= 9.211A$

The current through 2Ω resistor is $= 8.947+9.211$

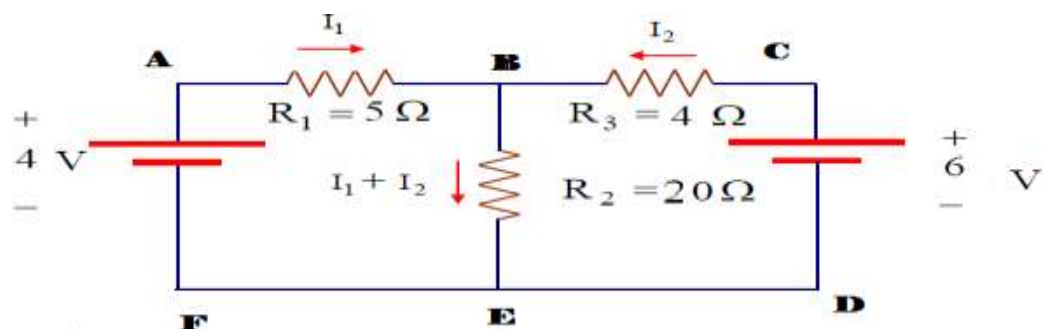
$= 18.158A$

Example-17: By using Kirchoff's law, calculate the current flowing through each resistor as shown in the figure.



In the LOOP (ABEFA)

$$5I_1 + 20(I_1 + I_2) = 4 \quad \text{--- (1)}$$



$$25I_1 + 20I_2 = 4 \quad \text{--- (1)}$$

In LOOP EDCBE

$$4I_2 + 20(I_1 + I_2) = 6 \quad \text{--- (2)}$$

$$20I_1 + 24I_2 = 6 \quad \text{--- (2)}$$

$$Eq(1) \times 4 \quad 100I_1 + 80I_2 = 16 \quad \text{--- (3)}$$

$$Eq(2) \times 5 \quad 100I_1 + 120I_2 = 30 \quad \text{--- (4)}$$

$$-40I_2 = -1$$

$$I_2 = 0.35A$$

Substitute $I_2 = 0.35A$ in Eq(1)

$$25I_1 + 20(0.35) = 4$$

$$I_1 = -0.12A$$

The path of the current flow in the 5Ω resistor is $-ve$ therefore we assume current direction is opposite to that as shown in figure.

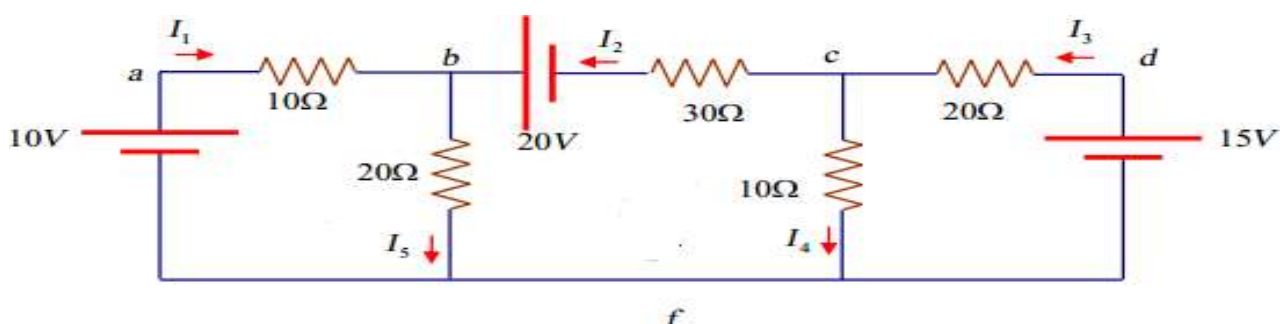
Current I_1 in 5Ω resistor = $0.12A$

Current I_2 in 4Ω resistor = $0.35A$

Current $I_1 + I_2$ in 20Ω resistor = $0.23A$

Example 18 : - by using Kirchoff's laws find the currents in each resistors for the circuit shown .

Solution :



We assume the currents in the each resistances and branches

applying K.C.L for nodes b and c

$$I_5 = I_1 + I_2 \quad \text{eq ----1}$$

$$I_4 = I_3 - I_2 \quad \text{eq ----2}$$

applying K.V.L for loop (1)

$$10 = 10I_1 + 20I_5$$

$$\therefore 10 = 10I_1 + 20I_1 + 20I_2$$

$$10 = 30I_1 - 20I_2 \quad \text{eq ----3}$$

applying K.V.L for loop(2)

$$20I_5 - 20 + 30I_2 - 10I_4 = 0$$

$$20 = 20(I_1 + I_2) + 30I_2 - 10(I_3 - I_2)$$

$$20 = 20I_1 + 60I_2 - 10I_3 \quad \text{eq ----4}$$

applying K.V.L for loop (3)

$$15 - 20I_3 - 10I_4 = 0$$

$$- 15 = -20I_3 + 10(I_3 - I_2)$$

$$- 15 = 10I_2 - 30I_3 \quad \text{eq ----5}$$

then solved eqs 3,4 and 5

$$10 = 30I_1 + 20I_2 - (0) I_3$$

$$20 = 20I_1 + 60I_2 - 10I_3$$

$$- 15 = (0) I_1 + 10I_2 - 30I_3$$

By using matrix method

$$\begin{bmatrix} 10 \\ 20 \\ -15 \end{bmatrix} = \begin{bmatrix} 30 & 20 & 0 \\ 20 & 60 & -10 \\ 0 & 10 & -30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$[V] = [R] \cdot [I]$$

$$\Delta = \begin{vmatrix} 30 & 20 & 0 \\ 20 & 60 & -10 \\ 0 & 10 & -30 \end{vmatrix}$$

$$|\Delta| = 30(60 * (-30) - (10) * (-10)) - (20) * (-20 * (-30) - 0) + 0(\dots\dots)$$

$$I_1 = \frac{1}{|\Delta|} \begin{vmatrix} 10 & 20 & 0 \\ 20 & 60 & -10 \\ -15 & 10 & -30 \end{vmatrix} = 0.05128A$$

$$I_2 = \frac{1}{|\Delta|} \begin{vmatrix} 30 & 10 & 0 \\ 20 & 20 & -10 \\ 0 & -15 & -30 \end{vmatrix} = 0.423A$$

$$I_3 = \frac{1}{|\Delta|} \begin{vmatrix} 30 & 20 & 10 \\ 20 & 60 & 20 \\ 0 & 10 & -15 \end{vmatrix} = 0.641A$$

$$I_4 = I_3 - I_2 = 0.218A$$

$$I_5 = I_1 + I_2 = 0.05128 + 0.42300 = 0.47428A$$

Maxwell's loop current method

This method is derived from (KVL) and can be applied by the following step:

1. Convert all current sources in the circuit to voltage sources.
2. Assign independent loop in the circuit to be analyzed (all the loop have the same direction).
3. Write the loop equations of the circuit as in the following formulas.

$$I_1 \sum R_{\text{loop1}} - I_2 \sum R_{\text{between loop1\&2}} - \dots - I_n \sum R_{\text{between loop1\&n}} = \sum V_{\text{loop1}} \dots (1)$$

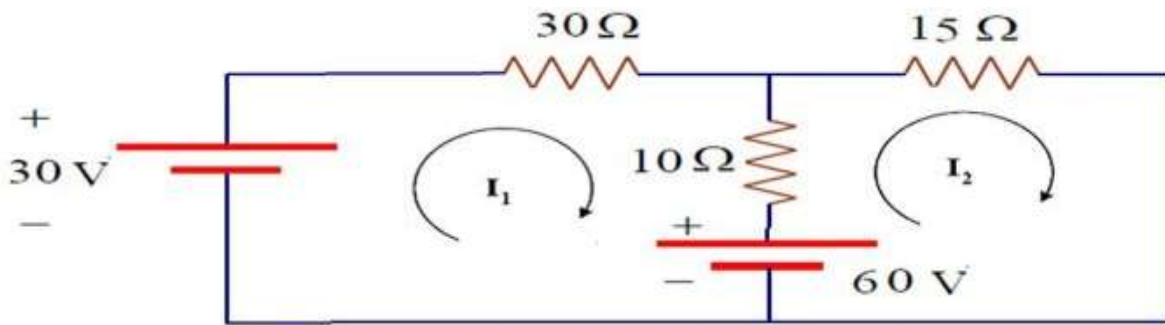
$$-I_1 \sum R_{\text{between loop1\&2}} + I_2 \sum R_{\text{loop2}} - \dots - I_n \sum R_{\text{between loop2\&n}} = \sum V_{\text{loop2}} \dots (2)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$-I_1 \sum R_{\text{between loop1\&n}} - I_2 \sum R_{\text{between loopn\&2}} - \dots + I_n \sum R_{\text{loopn}} = \sum V_{\text{loopn}} \dots (n)$$

Solve the above equation to find out the current value($I_1, I_2, \dots, I_n$)

Example-19: Find the current through the 10Ω resistor.



Solution:

Loop1 $30 - 30I_1 - (I_1 - I_2) - 60 = 0 \quad \text{--- eq(1)}$

Loop2 $60 - 10(I_2 - I_1) - 15I_2 = 0 \quad \text{--- eq(2)}$

Rewrite the above equation

$-40I_1 + 10I_2 = 30 \quad \text{--- eq(1)}$

$10I_1 - 25I_2 = -60 \quad \text{--- eq(2)}$

The current through 10 ohm resistor is $(I_1 - I_2)$ and can be determined using the determinates:

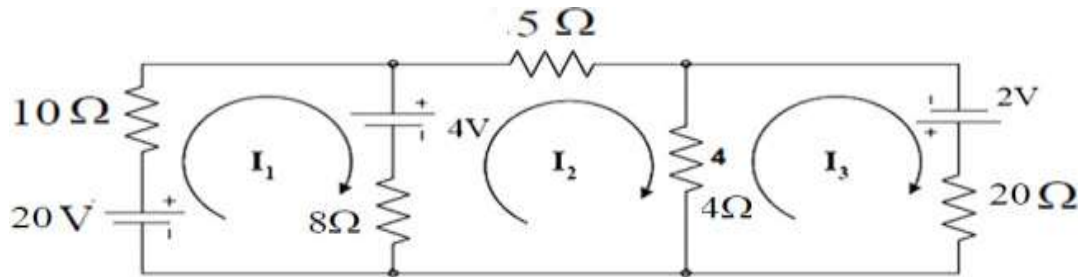
$$\begin{vmatrix} -40 & 10 \\ 10 & -25 \end{vmatrix} \times \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} 30 \\ -60 \end{vmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 30 & 10 \\ -60 & -25 \end{vmatrix}}{\begin{vmatrix} -40 & 10 \\ 10 & -25 \end{vmatrix}} = \frac{(30 \times (-25)) - ((-60) \times 10)}{((-40) \times (-25)) - (10 \times 10)} = \frac{-150}{900} = -0.166A$$

$$I_2 = \frac{\begin{vmatrix} -40 & 30 \\ 10 & -60 \end{vmatrix}}{\begin{vmatrix} -40 & 10 \\ 10 & -25 \end{vmatrix}} = \frac{((-40) \times (-60)) - (30 \times 10)}{((-40) \times (-25)) - (10 \times 10)} = \frac{-2100}{900} = -2.33A$$

$$I_{10\Omega} = I_2 - I_1 = -2.33 - (-0.166) = -2.164A$$

Example-20: Find the current through each branch .



$$I_1(10+8)-I_2(8)-I_3 \cdot 0 = 2-4 \dots\dots\dots 1$$

$$-I_1(8)+I_2(8+5+4)-I_3(4) = 4 \dots\dots\dots 2$$

$$I_1 \cdot 0 - I_2(4) + I_3(4+20) = 2 \dots\dots\dots 3$$

Rewrite the above equation:

$$18I_1 - 8I_2 = -2 \dots\dots\dots 1$$

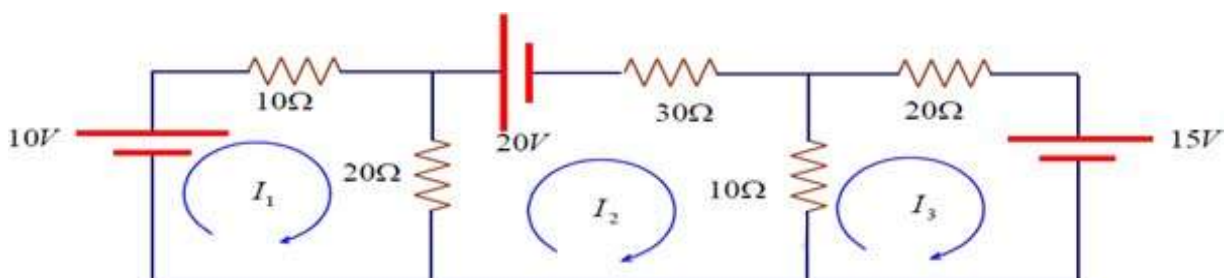
$$-8I_1 + 17I_2 - 4I_3 = 4 \dots\dots\dots 2$$

$$-4I_2 + 24I_3 = 2 \dots\dots\dots 3$$

Solve the above equation we obtain: $I_1 = -0.54\text{A}$, $I_2 = 0.92\text{A}$, $I_3 = 0.678\text{A}$

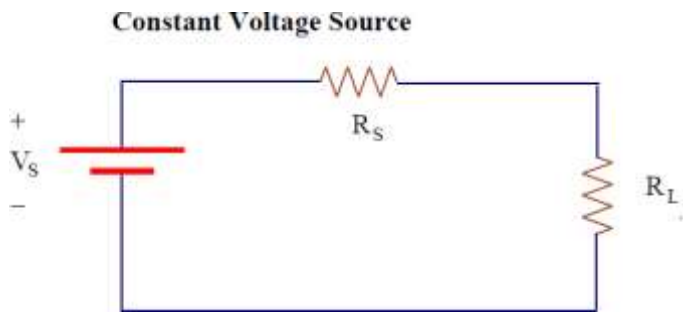
Home work

Example : find the current in each resistor by using Maxwell's loop current method



Type of D.C source

1-constant voltage source .

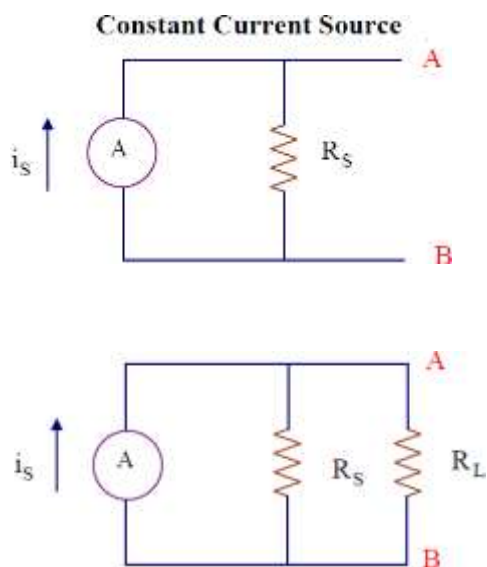


R_S = the internal resistance of source

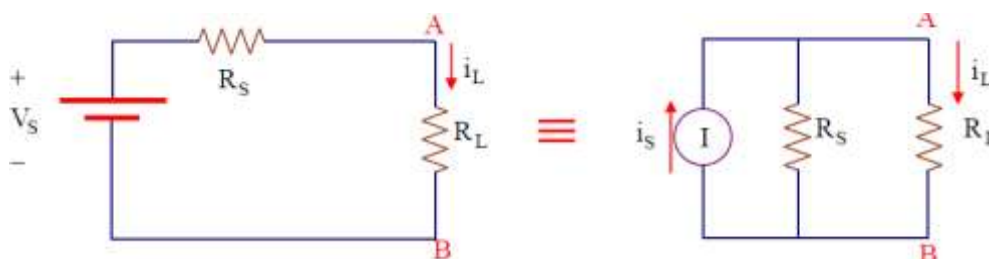
R_L = the load resistance

V_S = the voltage source or electromotive force (E .m .f)

2- constant current source



Convert voltage source to current source

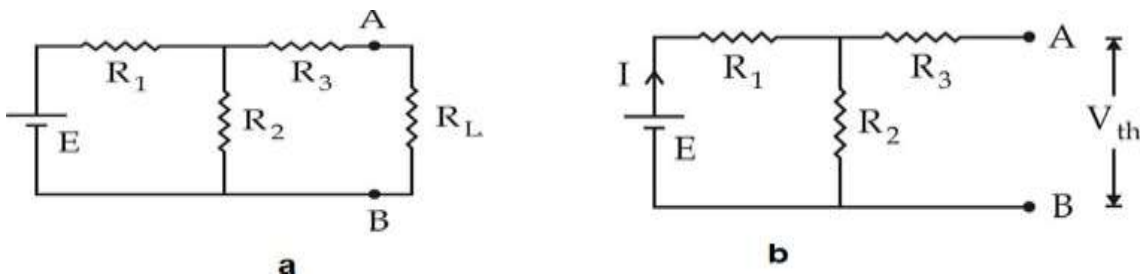


$$I_L = \frac{V_S}{R_S + R_L} \quad I_L = \left(\frac{R_S}{R_S + R_L} \right) i_S \quad V_S = R_S \cdot i_S$$

Thevenin's theorem

Statement: A linear network consisting of a number of voltage sources and resistances can be replaced by an equivalent network having a single voltage source called Thevenin's voltage (V_{Th}) and a single resistance called Thevenin's resistance (R_{Th}).

Explanation:



Consider a network or a circuit as shown figure (a) . Let E be the emf of the cell having its internal resistance $r = 0$. R_L load resistance

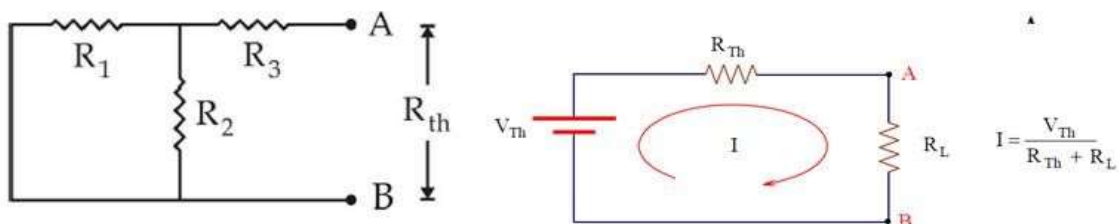
The load resistance (R_L) is removed figure (b).the current (I) in the circuit is

$$I = \frac{E}{R_1 + R_2}$$

The voltage across AB = Thevenin's voltage (V_{Th})

$$V_{Th} = V_{AB} = I \times R_2 = \frac{E \times R_2}{R_1 + R_2}$$

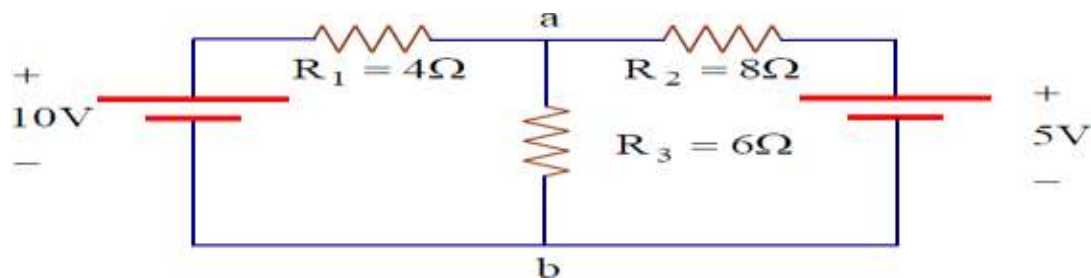
To find R_{Th} :



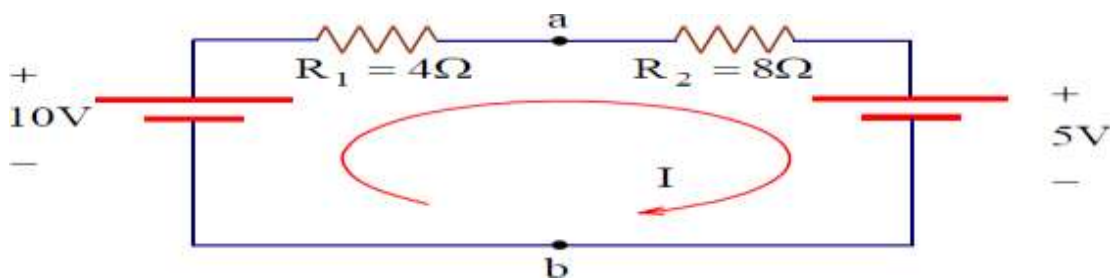
$$R_{Th} = R_3 + \frac{R_1 \times R_2}{R_1 + R_2}$$

Example-21: For the circuit in figure find the current in ($R=6\Omega$) by using Thevenin's theorem.

Solution:



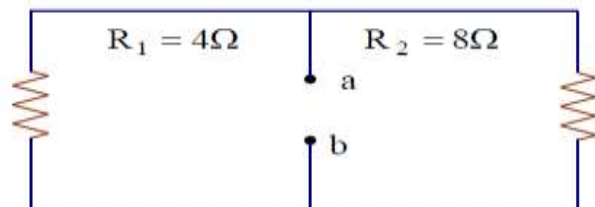
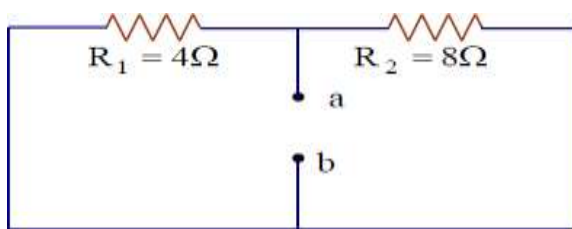
To find the (V_{Th})



$$10 - 5 = I(4 + 8) \Rightarrow I = \frac{5}{12} A$$

$$V_{Th} = 10 - 4 \times \frac{5}{12} = 8.33V \text{ or } V_{Th} = 5 + 8 \times \frac{5}{12}$$

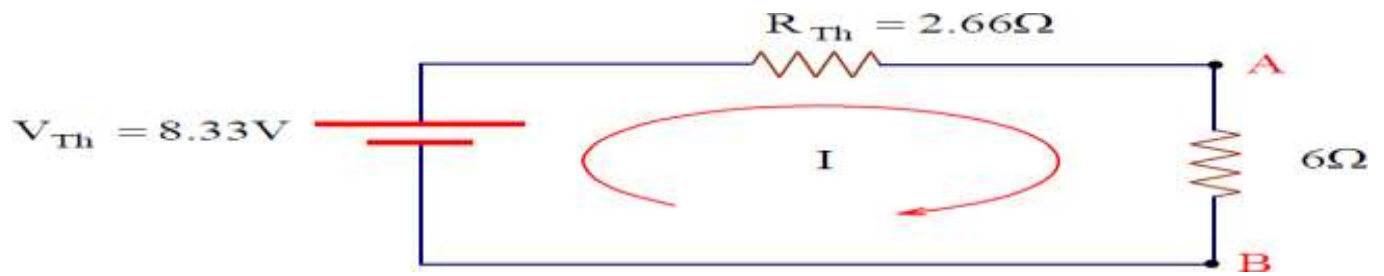
To find (R_{Th})



$$R_{Th} = \frac{4 \times 8}{4 + 8} = 2.66\Omega$$

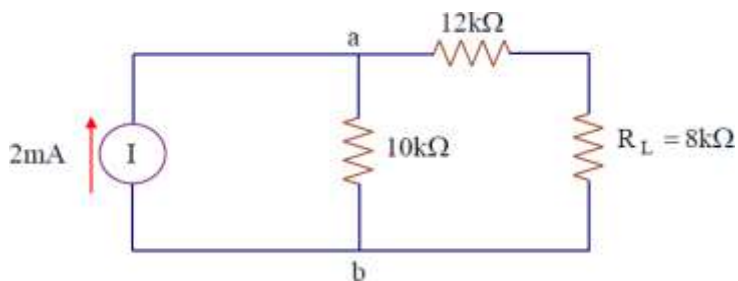
To find the current in ($R=6\Omega$) by using *Thevenin's* equivalent circuit

$$I_{6\Omega} = \frac{8.33}{2.66 + 6} = 0.96A$$



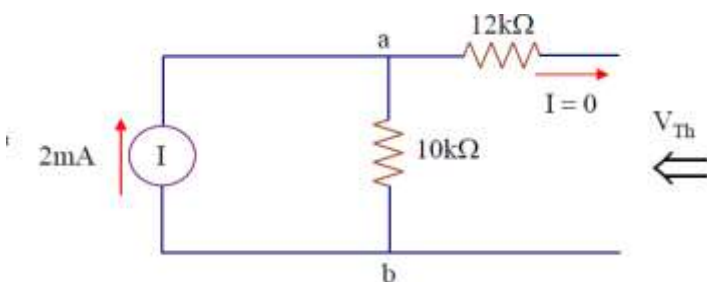
Example-22: For the circuit in figure find the current in ($R=8\Omega$) by using Thevenin's theorem.

Solution:

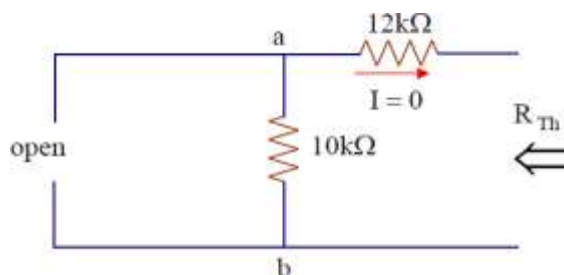


To find the (V_{Th})

$$V_{Th} = 2 \times 10^{-3} \times 10 \times 1000 = 20V$$

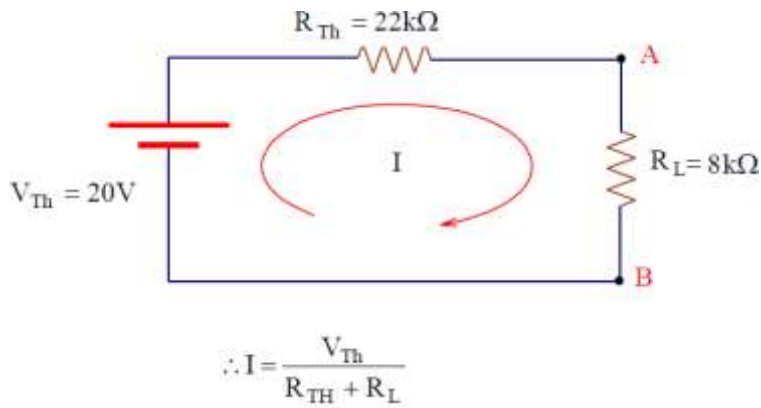


To find (R_{Th})



$$R_{Th} = 10 + 12 = 22\Omega$$

To find the current in ($R=8\Omega$) by using *Thevenin's* equivalent circuit



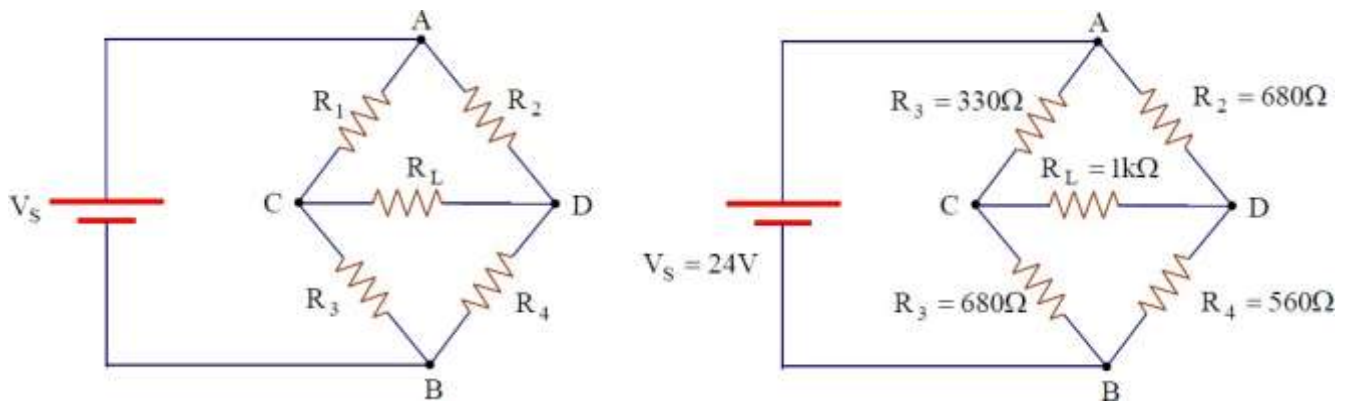
$$I = \frac{20}{(22+8) \times 1000} = \frac{20}{30000} = \frac{2}{3} \text{ mA}$$

Second method by using C.D.R s

$$I_{8K\Omega} = \frac{2 \times 10^{-3} \times 10 \times 1000}{(8+12+10) \times 1000} = \frac{2}{3} \text{ mA}$$

Example 23:- find the current in ($R_L= 1K\Omega$) by using Thevenin's theorem .

Solution :



To find the (V_{Th})

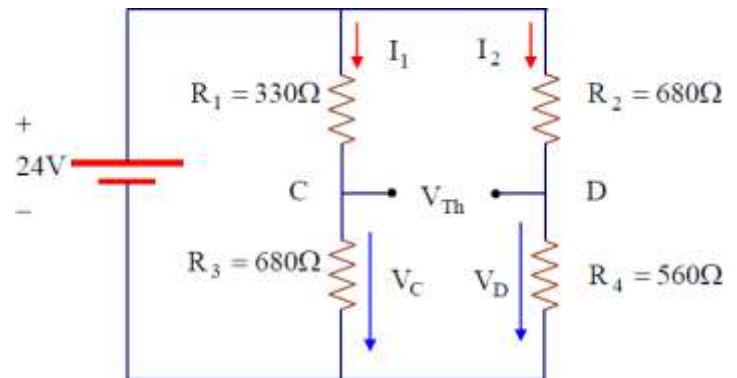
$$V_{Th} = V_C - V_D = \left(\frac{V_S \times R_3}{R_3 + R_1} \right) - \left(\frac{V_S \times R_4}{R_4 + R_2} \right) =$$

$$V_{Th} = \left(\frac{24 \times 680}{680 + 330} \right) - \left(\frac{24 \times 560}{560 + 680} \right) = 16.158 - 10.838 = 5.32V$$

Or by finding I_1 and I_2

$$I_1 = \frac{V_S}{R_1 + R_3} \quad \therefore V_C = I_1 \times R_3$$

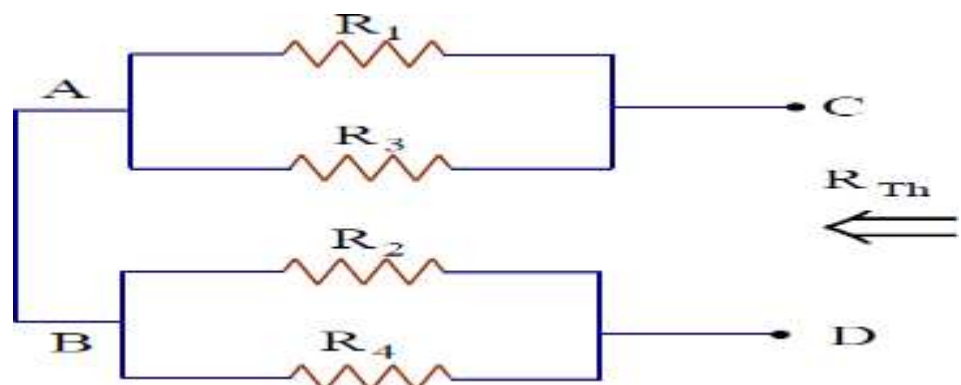
$$I_2 = \frac{V_S}{R_2 + R_4} \quad \therefore V_D = I_2 \times R_4$$

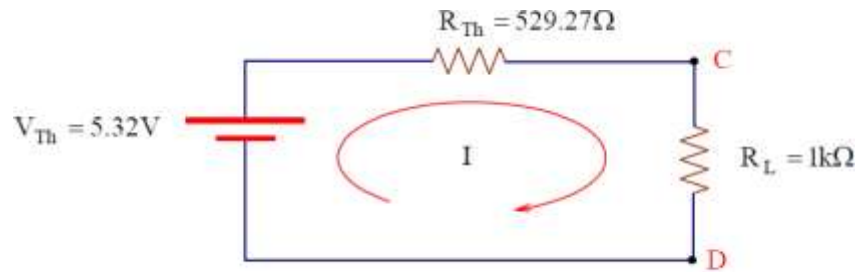


To find (R_{Th})

$$R_{Th} = \frac{R_1 \times R_3}{R_1 + R_3} + \frac{R_2 \times R_4}{R_2 + R_4} = \frac{330 \times 680}{330 + 680} + \frac{560 \times 680}{560 + 680} =$$

$$222.178 + 307.096 = 526.27\Omega$$





$$\therefore I_{CD} = \frac{V_{Th}}{R_{Th} + R_L}$$

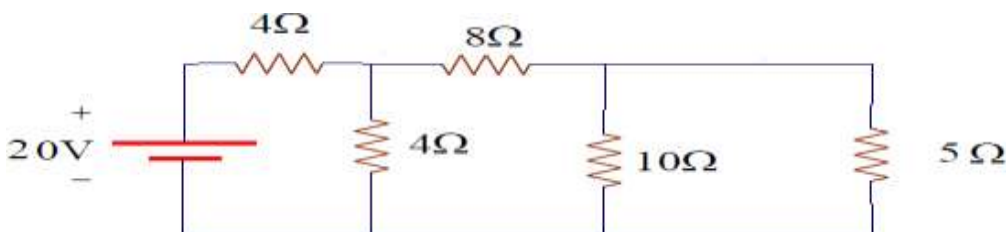
$$I_{CD} = \frac{5.32}{529.27 + 1000} = 3.5\text{mA}$$

Norton's Theorem:

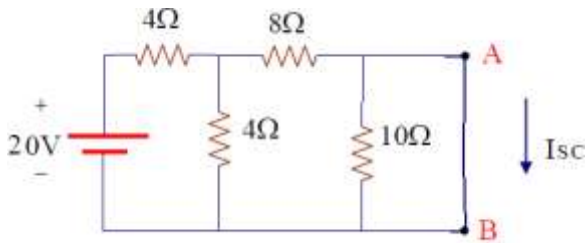
This theorem is an alternative to the Thevenin's theorem. In fact it is the dual of Thevenin's theorem. Whereas Thevenin's theorem reduces a two-terminal active network of linear resistances and generators to an equivalent constant-voltage source and series resistance. Norton's theorem replaces the network by an equivalent constant-current and a parallel resistance.

Example-24: Apply Norton's theorem to calculate current flowing through 5Ω resistor of figure (a) below.

Solution:



Remove 5Ω resistor and put a short circuit across terminals A and B as shown in figure (b).



Let us find I_{SC} . A combination of 4Ω and 8Ω in series with a 4Ω resistance.

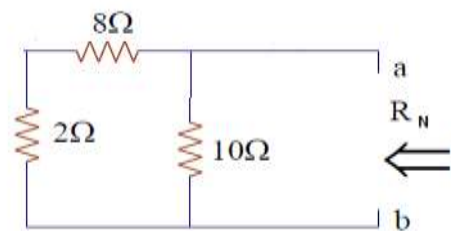
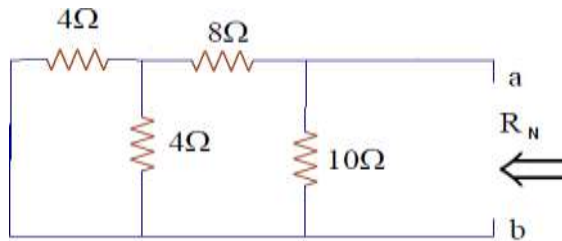
$$R_T = 4 + \frac{4 \times 8}{4 + 8} = \frac{20}{3} \Omega$$

$$\text{Hence } I = \frac{20}{R_T} = \frac{20}{(20/3)} = 3A$$

$$I_{SC} = 3 \times \frac{4}{4 + 8} = 1A$$

In figure (c) battery has been removed by short circuit and Norton's resistance is:

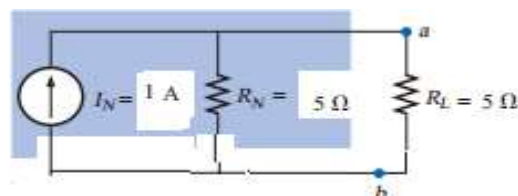
$$[(4 \parallel 4) + 8] \parallel 10 = 5\Omega$$



Hence; figure (e) gives Norton's equivalent circuit

Now join 5Ω resistance back across terminals A and B. the current flowing through it is:

$$I = \frac{R_N I}{R_L + R_N} = \frac{(5 \Omega)(1 \text{ A})}{5 \Omega + 5 \Omega} = 0.5 \text{ A}$$



Example-25: Apply Norton's theorem to calculate current flowing through 80Ω resistor of

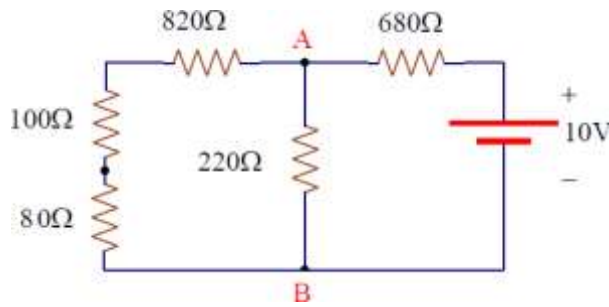


figure (a) below.

Electric Power in D.C circuit :

Power is defined as the product of voltage and current. Unit of power is watts and denoted by the letter "P".

$$P = V \times I \quad \text{watts} \quad P = I^2 \times R \quad P = \frac{V^2}{R} \text{ watt}$$

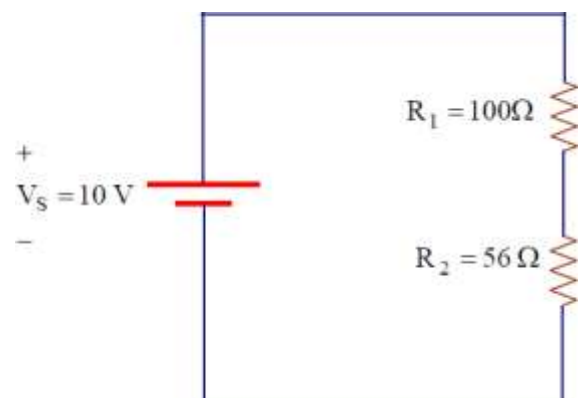
Example 26 :- for the circuit in figure shown find (P_1) and (P_2) .

Solutin :-

$$I = \frac{V_s}{R_1 + R_2} = \frac{10}{100 + 56} = 0.064A$$

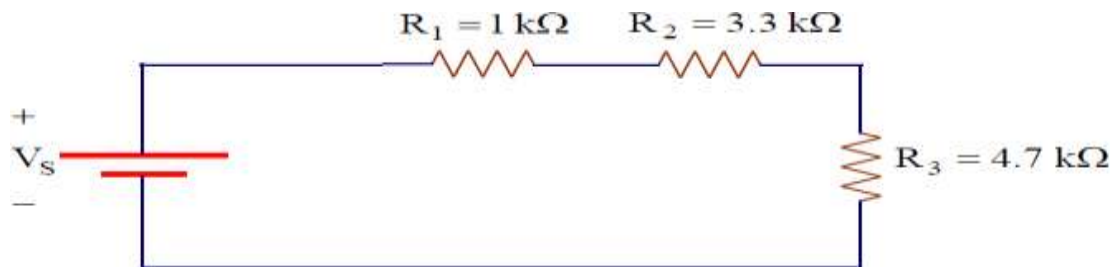
$$P_1 = V_1 \times I = (R_1 \times I) \times I = I^2 \times R_1 = 0.064^2 \times 100 = 0.4096 \text{ w}$$

$$P_2 = V_2 \times I = (R_2 \times I) \times I = I^2 \times R_2 = 0.064^2 \times 56 = 0.2293 \text{ w}$$



Example27 : for the circuit in figure find (V_s) if ($P_1 = 0.5 \text{ w}$) .

Solution :



$$P_1 = I^2 \times R_1 \Rightarrow I = \sqrt{\frac{P_1}{R_1}} = \sqrt{\frac{0.5}{1000}} = 0.0223A$$

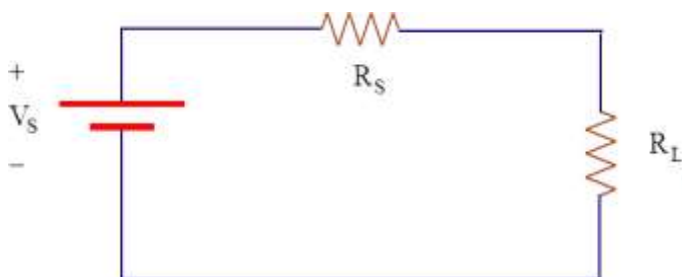
$$V_S = I \times (R_1 + R_2 + R_3) = 0.0223 \times 9000 = 200.1V$$

Maximum power transfer

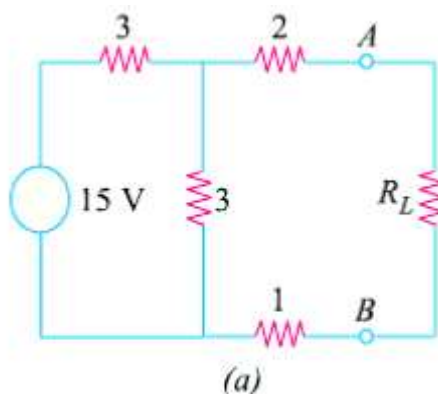
A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals.

maximum power(P_{max})when $R_L = R_S$

And using Thevenin's theorem then find $P_{max} = \frac{V_{Th}^2}{4R_{Th}}$



Example-28: for the circuit in figure(a); at maximum power transfer find the value of R_L and P_{max} (all resistance in ohm) .

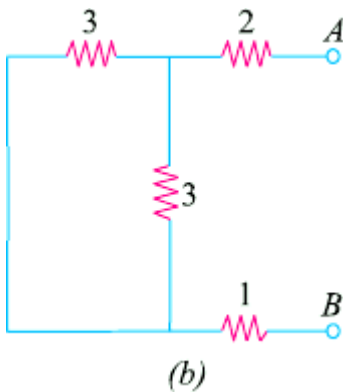


Solution:

We will remove R_L and find the equivalent Thevenin's source. As seen from figure(b) V_{Th} equals the drop across the vertical resistor of 3Ω because no current flows through 2Ω and 1Ω resistors.

$$V_{Th} = 15 \times \frac{3}{3+3} = 7.5v$$

Thevenin's resistance can be found by replacing 15v source with short circuit as shown in figure(b)

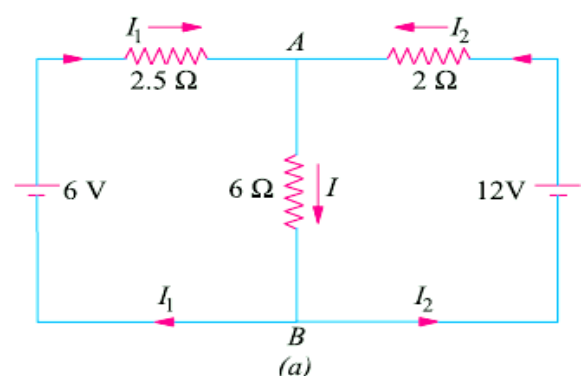


$$R_{Th} = 2 + 1 + \frac{3 \times 3}{3+3} = 4.5\Omega, \quad P_{max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{7.5^2}{4 \times 4.5} = 3.125 \text{ w}$$

Superposition theorem

In a linear network having number of voltage or current sources and resistances, the current through any branch of the network is the algebraic sum of the currents due to each of the sources when acting independently.

Example-29: In figure(a) below find the different currents flowing in the branches.



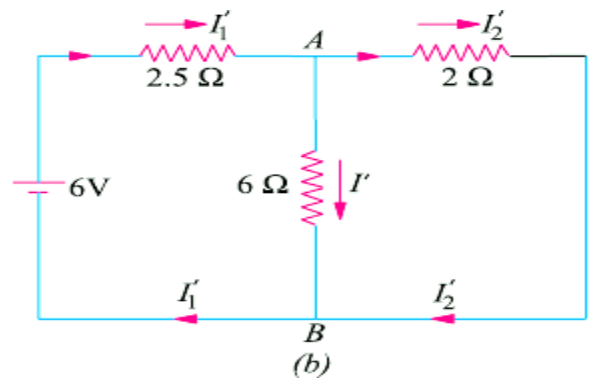
Solution:

In figure(b) the 12v battery has been removed. The various currents can be found by applying Ohm's law:

$$R_T = 2.5 + \frac{2 \times 6}{2+6} = 4\Omega$$

$$I'_1 = \frac{6}{R_T} = \frac{6}{4} = 1.5A; \quad I' = 1.5 \times \frac{2}{6+2} = 1.5 \times \frac{2}{8} = \frac{6}{8} = 0.375A$$

$$I'_2 = 1.5 \times \frac{6}{6+2} = 1.5 \times \frac{6}{8} = 1.125$$



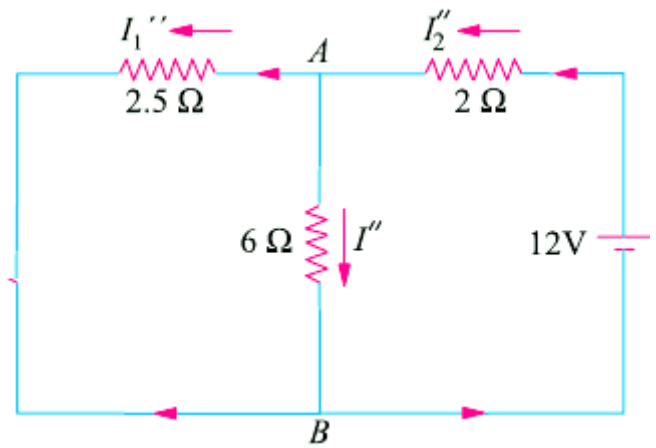
In figure(c) 6v battery has been removed. The various currents and their direction are as shown:

$$R_T = 2 + \frac{2.5 \times 6}{2.5+6} = 3.764\Omega$$

$$I'' = \frac{12}{R_T} = \frac{12}{3.764} = 3.18A; \quad I'' = 3.18 \times \frac{2.5}{6+2.5} = 3.18 \times \frac{2.5}{8.5} = \frac{6}{4} = 0.935A$$

$$I''_1 = 3.18 \times \frac{6}{6+2.5} = 3.18 \times \frac{6}{8.5} = 2.245A$$

It is
charging
current

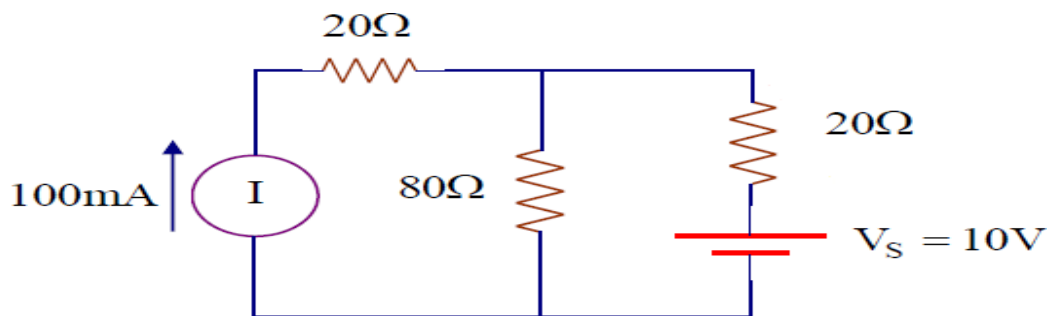


$$I_1 = I'_1 + I''_1 = 1.5 - 2.245 = -0.745A$$

$$I_2 = I'_2 + I''_2 = -1.125 + 3.18 = 2.055A$$

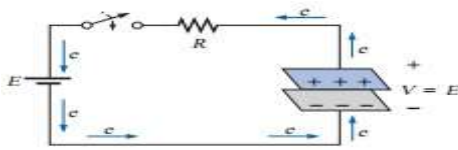
$$I = I' + I'' = 0.375 + 0.935 = 1.31A$$

Example-30: In figure(a) below find the different currents flowing in the branches.

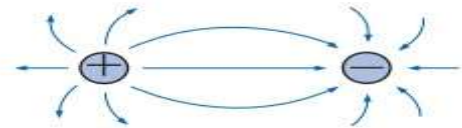


Capacitors

Electric flux lines always extend from a positively charged body to a negatively charged body, always extend or terminate perpendicular to the charged surfaces, and never intersect.



Fundamental charging network.



Electric flux distribution

CAPACITANCE

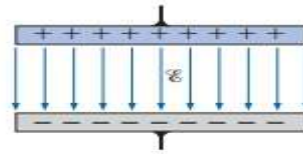
A capacitor has a capacitance of 1 farad if 1 coulomb of charge is deposited on the plates by a potential difference of 1 volt across the plates.

$$C = \frac{Q}{V}$$

C = farads (F)
 Q = coulombs (C)
 V = volts (V)

$$\mathcal{E} = \frac{V}{d}$$

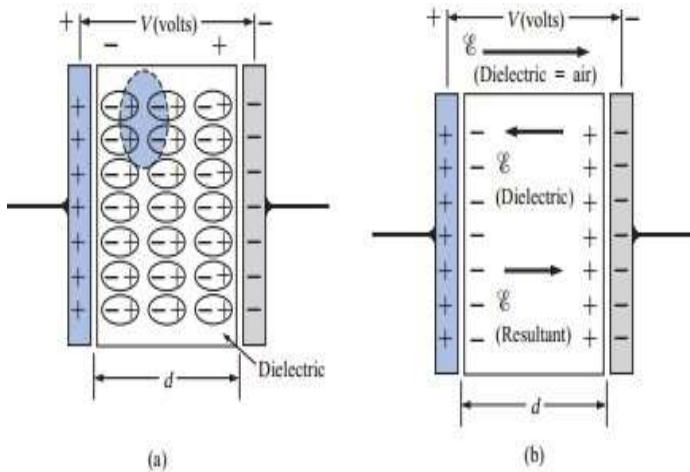
(volts/meter, V/m)



Electric flux distribution between the plates of a capacitor.

$$\mathcal{E} = \frac{V}{d}$$

(volts/meter, V/m)



Effect of a dielectric on the field distribution between the plates of a capacitor:

(a) alignment of dipoles in the dielectric;

(b) electric field components between the plates of a capacitor with a dielectric present.

The ratio of the flux density to the electric field intensity in the dielectric is called the **permittivity** of the dielectric:

$$\epsilon = \frac{D}{\mathcal{E}} \quad (\text{farads/meter, F/m})$$

For a vacuum, the value of ϵ (denoted by ϵ_o) is 8.85×10^{-12} F/m. The ratio of the permittivity of any dielectric to that of a vacuum is called the **relative permittivity**, ϵ_r . It simply compares the permittivity of the dielectric to that of air. In equation form,

$$\epsilon_r = \frac{\epsilon}{\epsilon_o}$$

$$\epsilon = \epsilon_r \epsilon_o$$

Note that ϵ_r is a dimensionless quantity. The relative permittivity, or **dielectric constant**, as it is often called, is provided in Table 10.1 for various dielectric materials.

$$\epsilon = \frac{D}{\mathcal{E}} = \frac{\psi/A}{V/d} = \frac{Q/A}{V/d} = \frac{Qd}{VA}$$

$$C = \frac{Q}{V}$$

$$\epsilon = \frac{Cd}{A}$$

TABLE 10.1

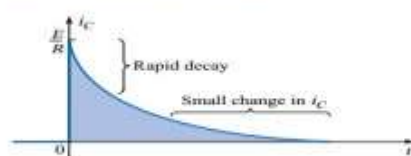
Relative permittivity (dielectric constant) of various dielectrics.

Dielectric	ϵ_r (Average Values)
Vacuum	1.0
Air	1.0006
Teflon	2.0
Paper, paraffined	2.5
Rubber	3.0
Transformer oil	4.0
Mica	5.0
Porcelain	6.0
Bakelite	7.0
Glass	7.5
Distilled water	80.0
Barium-strontium titanite (ceramic)	7500.0

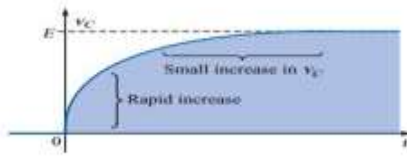
$$C = \epsilon \frac{A}{d} \quad (\text{F})$$

$$C = \epsilon_o \epsilon_r \frac{A}{d} = 8.85 \times 10^{-12} \epsilon_r \frac{A}{d} \quad (\text{F})$$

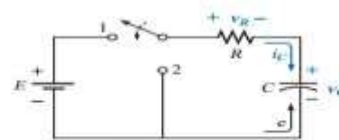
TRANSIENTS IN CAPACITIVE NETWORKS: CHARGING PHASE



i_C during the charging phase.



v_C during the charging phase.

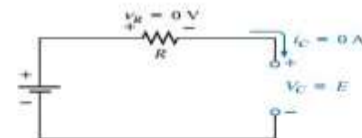


Basic charging network.

When the switch is closed at $t = 0$ s, the current

jumps to a value limited only by the resistance of the network and then decays to zero as the plates are charged.

is the source voltage since $i = i_C = i_R = 0$ A and $v_R = i_R R = (0)R = 0$ V. For all future analysis:



Open-circuit equivalent for a capacitor following the charging phase.

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.

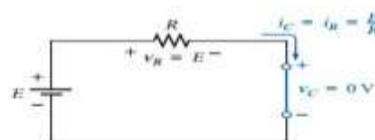
Looking back at the instant the switch is closed, we can also surmise that a capacitor behaves as a short circuit the moment the switch is closed in a dc charging network, as shown in Fig. The current $i = i_C = i_R = E/R$, and the voltage $v_C = E - v_R = E - i_R R = E - (E/R)R = E - E = 0$ V at $t = 0$ s.

Through the use of calculus, the following mathematical equation for the charging current i_C can be obtained:

$$i_C = \frac{E}{R} e^{-t/RC}$$

$$\tau = RC \quad (\text{seconds, s})$$

time constant $= \tau = RC$



Short-circuit equivalent for a capacitor (switch closed, $t = 0$).

The current i_C of a capacitive network is essentially zero after five time constants of the charging phase have passed in a dc network.

$$v_C = E(1 - e^{-t/RC})$$

EXAMPLE

- Find the mathematical expressions for the transient behavior of v_C , i_C , and v_R for the circuit of Fig. 10.35 when the switch is moved to position 1. Plot the curves of v_C , i_C , and v_R .
- How much time must pass before it can be assumed, for all practical purposes, that $i_C \cong 0$ A and $v_C \cong E$ volts?

Solutions:

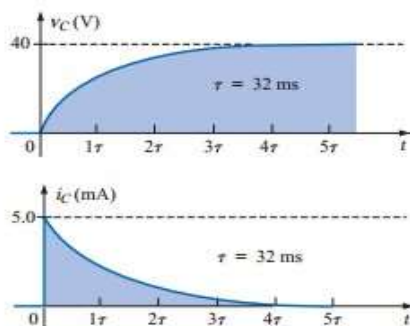
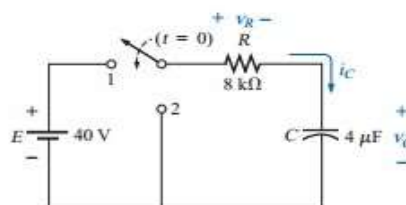
- $\tau = RC = (8 \times 10^3 \Omega)(4 \times 10^{-6} \text{ F}) = 32 \times 10^{-3} \text{ s} = 32 \text{ ms}$
By Eq. (10.15),

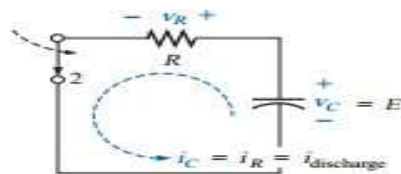
$$v_C = E(1 - e^{-t/\tau}) = 40(1 - e^{-t/(32 \times 10^{-3})})$$

$$i_C = \frac{E}{R} e^{-t/\tau} = \frac{40 \text{ V}}{8 \text{ k}\Omega} e^{-t/(32 \times 10^{-3})} = (5 \times 10^{-3}) e^{-t/(32 \times 10^{-3})}$$

$$v_R = E e^{-t/\tau} = 40 e^{-t/(32 \times 10^{-3})}$$

- $5\tau = 5(32 \text{ ms}) = 160 \text{ ms}$





$$v_C = Ee^{-t/RC}$$

discharging

$$i_C = \frac{E}{R}e^{-t/RC}$$

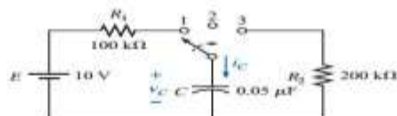
discharging

Demonstrating the discharge behavior of a capacitive network.

The voltage $v_R = v_C$ and

$$v_R = Ee^{-t/RC}$$

discharging



EXAMPLE

- Find the mathematical expression for the transient behavior of the voltage across the capacitor of Fig. if the switch is thrown into position 1 at $t = 0$ s.
- Repeat part (a) for i_C .
- Find the mathematical expressions for the response of v_C and i_C if the switch is thrown into position 2 at 30 ms (assuming that the leakage resistance of the capacitor is infinite ohms).
- Find the mathematical expressions for the voltage v_C and current i_C if the switch is thrown into position 3 at $t = 48$ ms.
- Plot the waveforms obtained in parts (a) through (d) on the same time axis for the voltage v_C and the current i_C .

Solutions:

- a. Charging phase:

$$\begin{aligned} v_C &= E(1 - e^{-t/\tau}) \\ \tau &= R_1 C = (100 \times 10^3 \Omega)(0.05 \times 10^{-6} \text{ F}) = 5 \times 10^{-3} \text{ s} \\ &= 5 \text{ ms} \\ v_C &= 10(1 - e^{-t/(5 \times 10^{-3})}) \end{aligned}$$

$$\begin{aligned} \text{b. } i_C &= \frac{E}{R_1} e^{-t/\tau} \\ &= \frac{10 \text{ V}}{100 \times 10^3 \Omega} e^{-t/(5 \times 10^{-3})} \\ i_C &= (0.1 \times 10^{-3}) e^{-t/(5 \times 10^{-3})} \end{aligned}$$

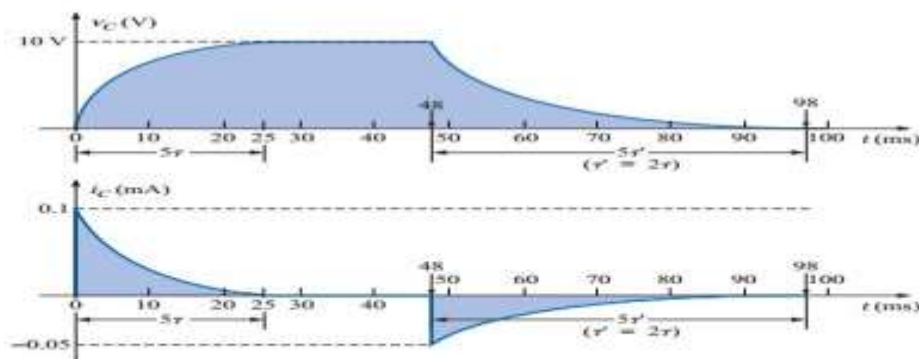
- c. Storage phase:

$$\begin{aligned} v_C &= E = 10 \text{ V} \\ i_C &= 0 \text{ A} \end{aligned}$$

- d. Discharge phase (starting at 48 ms with $t = 0$ s for the following equations):

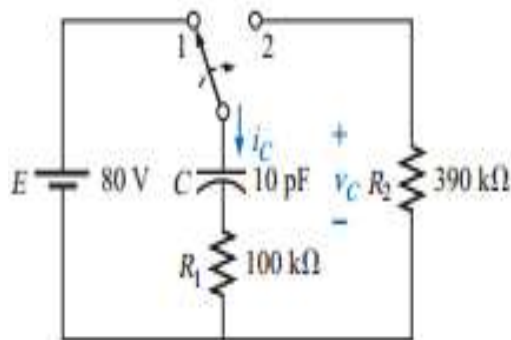
$$\begin{aligned} v_C &= Ee^{-t'/\tau'} \\ \tau' &= R_2 C = (200 \times 10^3 \Omega)(0.05 \times 10^{-6} \text{ F}) = 10 \times 10^{-3} \text{ s} \\ &= 10 \text{ ms} \\ v_C &= 10e^{-t'/(10 \times 10^{-3})} \\ i_C &= -\frac{E}{R_2} e^{-t'/\tau'} \\ &= -\frac{10 \text{ V}}{200 \times 10^3 \Omega} e^{-t'/(10 \times 10^{-3})} \\ i_C &= -(0.05 \times 10^{-3}) e^{-t'/(10 \times 10^{-3})} \end{aligned}$$

e.



The waveforms for the network of Fig.

Homework



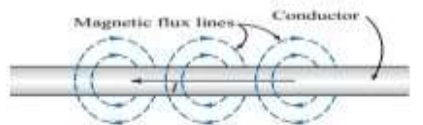
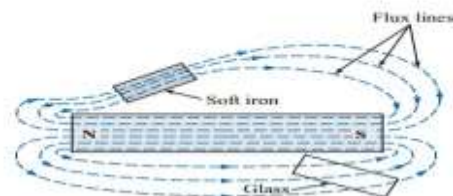
For the network of Fig.

- Find the mathematical expression for the voltage across the capacitor after the switch is thrown into position 1.
- Repeat part (a) for the current i_C .
- Find the mathematical expressions for the voltage v_C and current i_C if the switch is thrown into position 2 at a time equal to five time constants of the charging circuit.
- Plot the waveforms of v_C and i_C for a period of time extending from $t = 0$ to $t = 30 \mu\text{s}$.

Magnetic Circuits

MAGNETIC FIELDS

In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops, as in Fig. The symbol for magnetic flux is the Greek letter Φ



Magnetic flux lines around a current-carrying conductor.



FLUX DENSITY

The number of flux lines per unit area is called the **flux density**, is denoted by the capital letter B .

$$B = \frac{\Phi}{A}$$

B = teslas (T)
 Φ = webers (Wb)
 A = square meters (m^2)

PERMEABILITY

The permeability of free space μ_0 (vacuum) is

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{Wb}}{\text{A} \cdot \text{m}}$$

The ratio of the permeability of a material to that of free space is called its **relative permeability**; that is,

$$\mu_r = \frac{\mu}{\mu_0}$$

In general, for ferromagnetic materials, $\mu_r \geq 100$, and for nonmagnetic materials, $\mu_r = 1$.

Since μ_r is a variable, dependent on other quantities of the magnetic circuit, values of μ_r are not tabulated. Methods of calculating μ_r from the data supplied by manufacturers will be considered in a later section.

RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} \quad (\text{ohms, } \Omega)$$

The **reluctance** of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathcal{R} = \frac{l}{\mu A} \quad (\text{rels, or At/Wb})$$

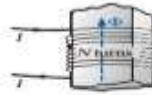
OHM'S LAW FOR MAGNETIC CIRCUITS

Effect = $\frac{\text{cause}}{\text{opposition}}$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

The magnetomotive force $\mathcal{F}$ is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig.). In equation form,

$$\mathcal{F} = NI \quad (\text{ampere-turns, At})$$



MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** (H). In equation form,

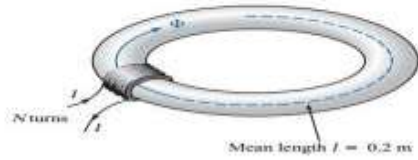
$$H = \frac{\mathcal{F}}{l} \quad (\text{At/m})$$

Substituting for the magnetomotive force will result in

$$H = \frac{NI}{l} \quad (\text{At/m})$$

For the magnetic circuit of Fig. , if $NI = 40 \text{ At}$ and $l = 0.2 \text{ m}$, then

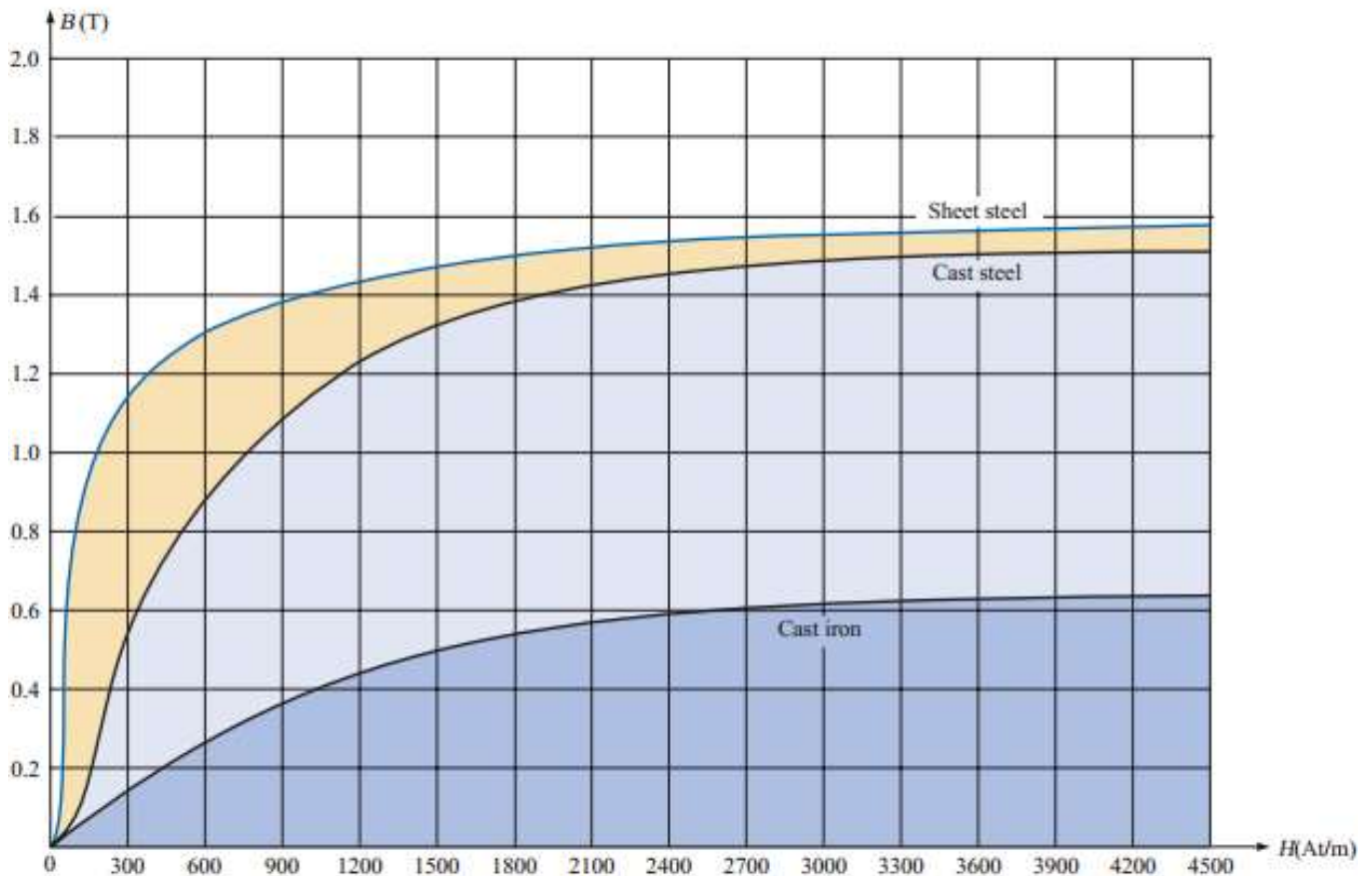
$$H = \frac{NI}{l} = \frac{40 \text{ At}}{0.2 \text{ m}} = 200 \text{ At/m}$$



Defining the magnetizing force of a magnetic circuit.

The flux density and the magnetizing force are related by the following equation:

$$B = \mu H$$



Normal magnetization curve for three ferromagnetic materials.

AMPÈRE'S CIRCUITAL LAW

	Electric Circuits	Magnetic Circuits
Cause	E	$\mathcal{F}$
Effect	I	Φ
Opposition	R	$\mathcal{R}$

If we apply the "cause" analogy to Kirchhoff's voltage law ($\sum_{\text{C}} V = 0$), we obtain the following:

$$\sum_{\text{C}} \mathcal{F} = 0 \quad (\text{for magnetic circuits})$$

$$\mathcal{F} = NI$$

resulting in the following for magnetic circuits:

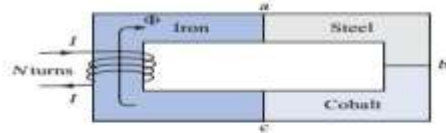
$$\mathcal{F} = \Phi \mathcal{R} \quad (\text{At})$$

$$\mathcal{F} = HI \quad (\text{At})$$

Applying Ampère's circuital law, we have $\sum_{\text{C}} \mathcal{F} = 0$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\text{impressed mmf}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

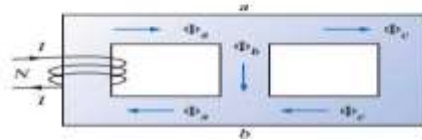


Series magnetic circuit of three different materials.

THE FLUX Φ

$$\Phi_a = \Phi_b + \Phi_c \quad (\text{at junction } a)$$

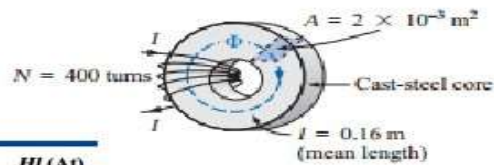
$$\Phi_b + \Phi_c = \Phi_a \quad (\text{at junction } b)$$



EXAMPLE For the series magnetic circuit of Fig.

- Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.
- Determine μ and μ_r for the material under these conditions.

Solutions:



Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
4×10^{-4}	2×10^{-3}			0.16	

- The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4} \text{ Wb}}{2 \times 10^{-3} \text{ m}^2} = 2 \times 10^{-1} \text{ T} = 0.2 \text{ T}$$

Using the B - H curves of Fig. 11.24, we can determine the magnetizing force H :

$$H (\text{cast steel}) = 170 \text{ At/m}$$

Applying Ampère's circuital law yields

$$NI = HI$$

$$\text{and} \quad I = \frac{HI}{N} = \frac{(170 \text{ At/m})(0.16 \text{ m})}{400 \text{ t}} = 68 \text{ mA}$$

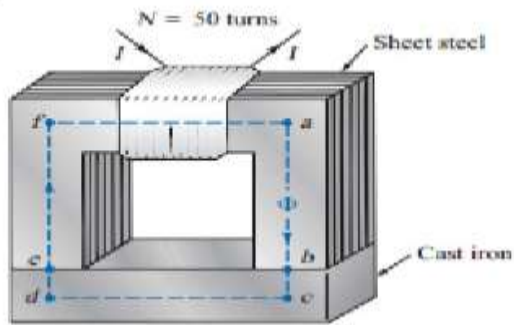
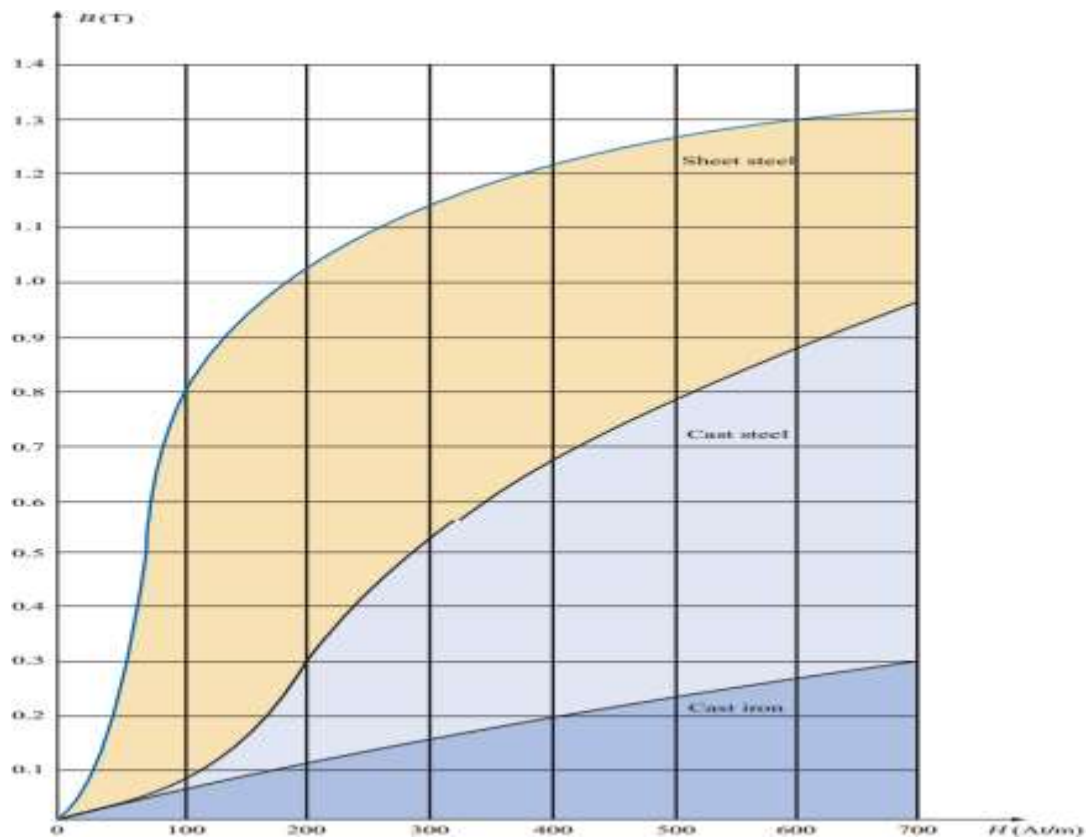
(Recall that t represents turns.)

- The permeability of the material can be found using Eq. (11.8):

$$\mu = \frac{B}{H} = \frac{0.2 \text{ T}}{170 \text{ At/m}} = 1.176 \times 10^{-3} \text{ Wb/A} \cdot \text{m}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = 935.83$$



$$l_{efab} = 304.8 \times 10^{-3} \text{ m}$$

$$l_{bede} = 127 \times 10^{-3} \text{ m}$$

$$\Phi = 3.5 \times 10^{-4} \text{ Wb}$$

$$\text{Area (throughout)} = 6.452 \times 10^{-4} \text{ m}^2$$

EXAMPLE 11.4 The electromagnet of Fig. has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core.

Solution:

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
<i>efab</i>	3.5×10^{-4}	6.452×10^{-4}			304.8×10^{-3}	
<i>bede</i>	3.5×10^{-4}	6.452×10^{-4}			127×10^{-3}	

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{3.5 \times 10^{-4} \text{ Wb}}{6.452 \times 10^{-4} \text{ m}^2} = 0.542 \text{ T}$$

and the magnetizing force is

$$H(\text{sheet steel, Fig. 11.15}) \approx 70 \text{ At/m}$$

$$H(\text{cast iron, Fig. 11.15}) \approx 1600 \text{ At/m}$$

$$H_{efab}l_{efab} = (70 \text{ At/m})(304.8 \times 10^{-3} \text{ m}) = 21.34 \text{ At}$$

$$H_{bcde}l_{bcde} = (1600 \text{ At/m})(127 \times 10^{-3} \text{ m}) = 203.2 \text{ At}$$

Section	Φ (Wb)	A (m ²)	B (T)	H (At/m)	l (m)	HI (At)
efab	3.5×10^{-4}	6.452×10^{-4}	0.542	70	304.8×10^{-3}	21.34
bcde	3.5×10^{-4}	6.452×10^{-4}	0.542	1600	127×10^{-3}	203.2

Applying Ampère's circuital law,

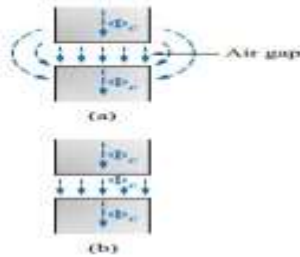
$$NI = H_{efab}l_{efab} + H_{bcde}l_{bcde}$$

$$= 21.34 \text{ At} + 203.2 \text{ At} = 224.54 \text{ At}$$

$$(50 \text{ t})I = 224.54 \text{ At}$$

$$I = \frac{224.54 \text{ At}}{50 \text{ t}} = 4.49 \text{ A}$$

AIR GAPS



Air gaps: (a) with fringing; (b) ideal.

The flux density of the air gap in Fig. 11.16 is given by

$$B_g = \frac{\Phi_g}{A_g}$$

where, for our purposes,

$$\Phi_g = \Phi_{\text{core}}$$

and

$$A_g = A_{\text{core}}$$

$$H_g = \frac{B_g}{\mu_0}$$

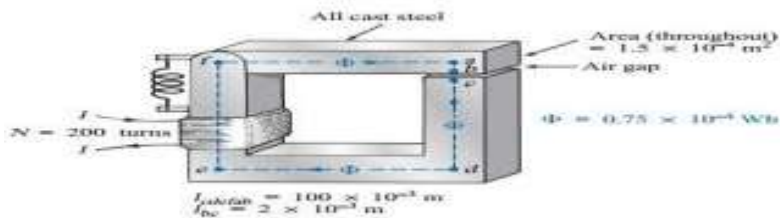
and the mmf drop across the air gap is equal to $H_g l_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_0} = \frac{B_g}{4\pi \times 10^{-7}}$$

and

$$H_g = (7.96 \times 10^5) B_g \quad (\text{At/m})$$

EXAMPLE 11.6 Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4} \text{ Wb}$ in the series magnetic circuit of Fig. 11.17.



Solution:

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4} \text{ Wb}}{1.5 \times 10^{-4} \text{ m}^2} = 0.5 \text{ T}$$

From the B - H curves of Fig. 11.15,

$$H(\text{cast steel}) \approx 280 \text{ At/m}$$

Applying Eq. (11.15),

$$H_g = (7.96 \times 10^5) B_g = (7.96 \times 10^5)(0.5 \text{ T}) = 3.98 \times 10^5 \text{ At/m}$$

The mmf drops are

$$H_{\text{core}}l_{\text{core}} = (280 \text{ At/m})(100 \times 10^{-3} \text{ m}) = 28 \text{ At}$$

$$H_g l_g = (3.98 \times 10^5 \text{ At/m})(2 \times 10^{-3} \text{ m}) = 796 \text{ At}$$

Applying Ampère's circuital law,

$$NI = H_{\text{core}}l_{\text{core}} + H_g l_g$$

$$= 28 \text{ At} + 796 \text{ At}$$

$$(200 \text{ t})I = 824 \text{ At}$$

$$I = 4.12 \text{ A}$$

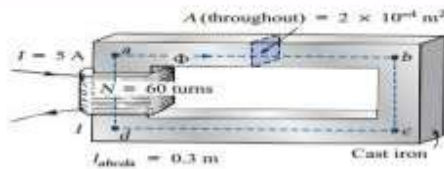
Quantity

Symbol

Unit

Abbreviation

Magnetic flux	Φ	weber	wb
Flux density	B	Weber / sequare meter	Tesla(T)
Permibility	μ	Weber /Amper. turn	Wb/ At
Reluctance	R	Amper.turn/ weber	At/wb
Magnetizing force	H	Amper. .turn/ meter	At /m
Magnetomotive force	m.m.f	Amper.turn/ meter	At/m



EXAMPLE
circuit of Fig.

Calculate the magnetic flux Φ for the magnetic circuit of Fig.

Solution:

By Ampère's circuital law,

$$NI = H_{abceda} l_{abceda}$$

$$H_{abceda} = \frac{NI}{l_{abceda}} = \frac{(60 \text{ t})(5 \text{ A})}{0.3 \text{ m}}$$

$$= \frac{300 \text{ At}}{0.3 \text{ m}} = 1000 \text{ At/m}$$

or

$$B_{abceda} \text{ (from Fig.)} \approx 0.39 \text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39 \text{ T})(2 \times 10^{-4} \text{ m}^2) = 0.78 \times 10^{-4} \text{ Wb}$$

EXAMPLE

Find the magnetic flux Φ for the series magnetic circuit of Fig. for the specified impressed mmf.

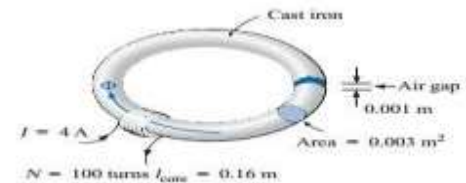
Solution:

$$NI = H_g l_g$$

$$H_g = \frac{NI}{l_g} = \frac{400 \text{ At}}{0.001 \text{ m}} = 4 \times 10^5 \text{ At/m}$$

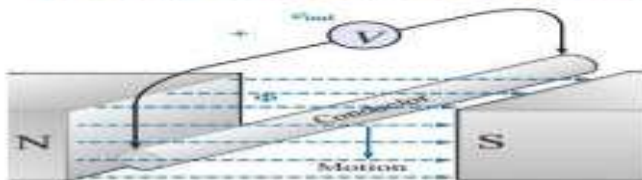
$$B_g = \mu_0 H_g = (4\pi \times 10^{-7})(4 \times 10^5 \text{ At/m})$$

$$= 0.503 \text{ T}$$



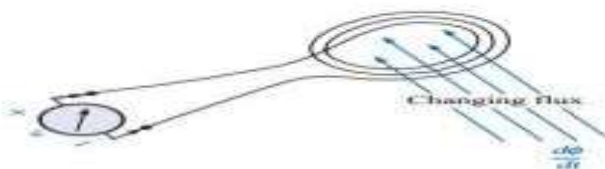
Inductors

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION



Generating an induced voltage by moving a conductor through a magnetic field.

$$e = N \frac{d\Phi}{dt} \quad (\text{volts, V})$$



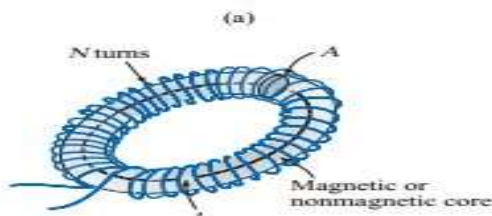
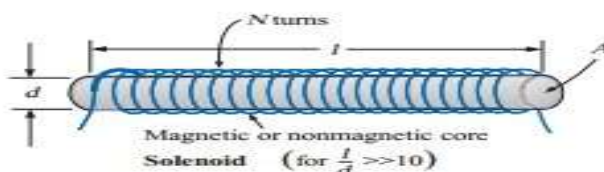
SELF-INDUCTANCE

The ability of a coil to oppose any change in current is a measure of the **self-inductance** L of the coil. For brevity, the prefix *self* is usually dropped. Inductance is measured in henries (H), after the American physicist Joseph Henry

$$L = \frac{N^2 \mu A}{l} \quad (\text{henries, H})$$

$$L = \frac{N^2 \mu_r \mu_o A}{l} = \mu_r \frac{N^2 \mu_o A}{l}$$

$$L = \mu_r L_o$$



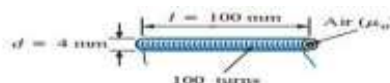
EXAMPLE Find the inductance of the air-core coil of Fig. 12.6.

Solution:

$$\mu = \mu_r \mu_o = (1)(\mu_o) = \mu_o$$

$$A = \frac{\pi d^2}{4} = \frac{(\pi)(4 \times 10^{-3} \text{ m})^2}{4} = 12.57 \times 10^{-6} \text{ m}^2$$

$$L_o = \frac{N^2 \mu_o A}{l} = \frac{(100)^2 (4\pi \times 10^{-7} \text{ Wb/A}\cdot\text{m})(12.57 \times 10^{-6} \text{ m}^2)}{0.1 \text{ m}} = 1.58 \mu\text{H}$$



EXAMPLE Repeat Example but with an iron core and conditions such that $\mu_r = 2000$.

Solution: By Eq. (12.3),

$$L = \mu_r L_o = (2000)(1.58 \times 10^{-6} \text{ H}) = 3.16 \text{ mH}$$

TYPES OF INDUCTORS



Inductor symbols.

INDUCED VOLTAGE

The inductance of a coil is also a measure of the change in flux linking a coil due to a change in current through the coil; that is,

$$L = N \frac{d\phi}{di} \quad (\text{H})$$

where N is the number of turns, ϕ is the flux in webers, and i is the current through the coil. If a change in current through the coil fails to result in a significant change in the flux linking the coil through its center, the resulting inductance level will be relatively small.

$$e_L = N \frac{d\phi}{dt} = \left(N \frac{d\phi}{di} \right) \left(\frac{di}{dt} \right)$$

$$e_L = L \frac{di}{dt} \quad (\text{V})$$

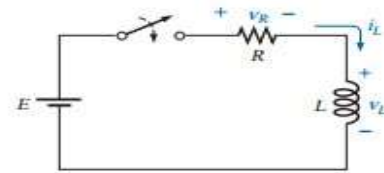
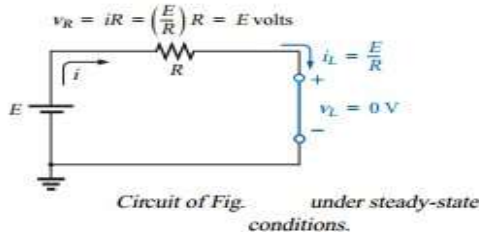
$$v_L = L \frac{di}{dt}$$

$$v_L = L \frac{di}{dt} = L(0) = 0 \text{ V}$$

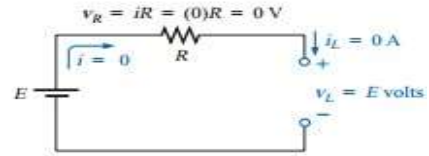
R-L TRANSIENTS: STORAGE CYCLE

The changing voltages and current that result during the storing of energy in the form of a magnetic field by an inductor in a dc circuit can best be described using the circuit of Fig. At the instant the switch is closed, the inductance of the coil will prevent an instantaneous change in current through the coil. The potential drop across the coil, v_L , will equal the impressed voltage E as determined by Kirchhoff's voltage law since $v_R = iR = (0)R = 0$ V. The current i_L will then build up from zero, establishing a voltage drop across the resistor and a corresponding drop in v_L . The current will continue to increase until the voltage across the inductor drops to zero volts and the full impressed voltage appears across the resistor. Initially, the current i_L increases quite rapidly, followed by a continually decreasing rate until it reaches its maximum value of E/R .

An ideal inductor ($R = 0 \Omega$) assumes a short-circuit equivalent in a dc network once steady-state conditions have been established.



Basic R-L transient network.

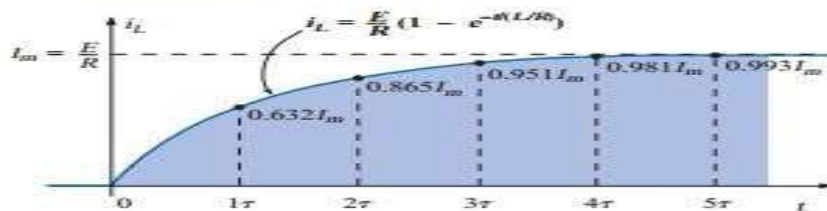


Circuit of Fig. the instant the switch is closed.

The equation for the current i_L during the storage phase is the following:

$$i_L = I_m(1 - e^{-t/\tau}) = \frac{E}{R}(1 - e^{-t/(L/R)})$$

$$\tau = \frac{L}{R} \quad (\text{seconds, s})$$



Plotting the waveform for i_L during the storage cycle.

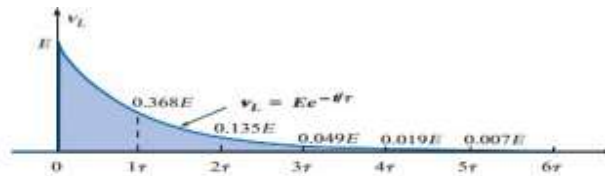
The fact that τ has the units of time can be verified by taking the equation for the induced voltage

$$v_L = L \frac{di}{dt}$$

the storage phase has passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.

the storage phase can be described mathematically by the following equation:

$$v_L = Ee^{-t/\tau}$$



Plotting the voltage v_R versus time for the network of Fig.

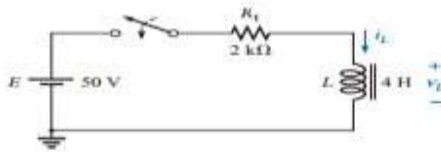
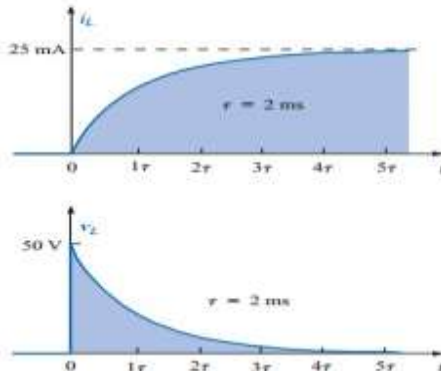


FIG.
Example 1



In five time constants, $i_L = E/R$, $v_L = 0$ V, and the inductor can be replaced by its short-circuit equivalent.

Since

$$v_R = i_R R = i_L R$$

then

$$v_R = \left[\frac{E}{R} (1 - e^{-t/\tau}) \right] R$$

and

$$v_R = E(1 - e^{-t/\tau})$$

and the curve for v_R will have the same shape as obtained for i_L .

EXAMPLE Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit of Fig. after the closing of the switch. Sketch the resulting curves.

Solution:

$$\tau = \frac{L}{R_1} = \frac{4 \text{ H}}{2 \text{ k}\Omega} = 2 \text{ ms}$$

$$I_m = \frac{E}{R_1} = \frac{50}{2 \text{ k}\Omega} = 25 \times 10^{-3} \text{ A} = 25 \text{ mA}$$

$$i_L = (25 \times 10^{-3})(1 - e^{-t/(2 \times 10^{-3})})$$

$$v_L = 50e^{-t/(2 \times 10^{-3})}$$

Sinusoidal Alternating Waveforms

SINUSOIDAL ac VOLTAGE CHARACTERISTICS AND DEFINITIONS

Generation

Waveform: The path traced by a quantity, such as the voltage in Fig. plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_i , e_o).

Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters (such as E_m for sources of voltage and V_m for the voltage drop across a load). For the waveform of Fig. the average value is zero volts, and E_m is as defined by the figure.

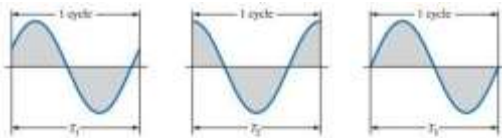
Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of Fig. the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by E_{pp} or V_{pp} , the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. is a periodic waveform.

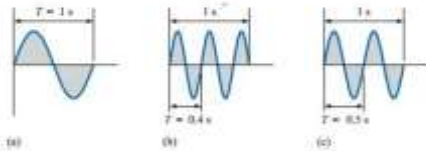
Period (T): The time interval between successive repetitions of a periodic waveform (the period $T_1 = T_2 = T_3$ in Fig.), as long as successive similar points of the periodic waveform are used in determining T .

Cycle: The portion of a waveform contained in one period of time. The cycles within T_1 , T_2 , and T_3 of Fig. may appear different in Fig. 13.4, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.



Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform of Fig. (a) is 1 cycle per second, and for Fig. (b), 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. (c)], the frequency would be 2 cycles per second.



Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

$$f = \frac{1}{T}$$

$$f = \text{Hz}$$

$$T = \text{seconds (s)}$$

$$T = \frac{1}{f}$$

EXAMPLE Find the period of a periodic waveform with a frequency of
a. 60 Hz.
b. 1000 Hz.

Solutions:

a. $T = \frac{1}{f} = \frac{1}{60 \text{ Hz}} = 0.01667 \text{ s}$ or **16.67 ms**

(a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000 \text{ Hz}} = 10^{-3} \text{ s} = \mathbf{1 \text{ ms}}$

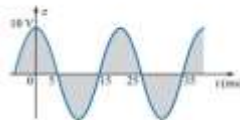


FIG. Example

EXAMPLE Determine the frequency of the waveform of Fig.

Solution: From the figure, $T = (25 \text{ ms} - 5 \text{ ms}) = 20 \text{ ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3} \text{ s}} = \mathbf{50 \text{ Hz}}$$

THE SINE WAVE

$$\text{Radians} = \left(\frac{\pi}{180^\circ} \right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180^\circ}{\pi} \right) \times (\text{radians})$$

Applying these equations, we find

$$90^\circ: \text{ Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{2} \text{ rad}$$

$$30^\circ: \text{ Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{6} \text{ rad}$$

$$\frac{\pi}{3} \text{ rad}: \text{ Degrees} = \frac{180^\circ}{\pi} \left(\frac{\pi}{3} \right) = 60^\circ$$

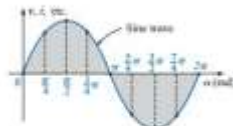
$$\frac{3\pi}{2} \text{ rad}: \text{ Degrees} = \frac{180^\circ}{\pi} \left(\frac{3\pi}{2} \right) = 270^\circ$$

The velocity with which the radius vector rotates about the center, called the **angular velocity**, can be determined from the following equation:

$$\text{Angular velocity} = \frac{\text{distance (degrees or radians)}}{\text{time (seconds)}}$$

EXAMPLE Determine the angular velocity of a sine wave having a frequency of 60 Hz.

Solution: $\omega = 2\pi f = (2\pi)(60 \text{ Hz}) = \mathbf{377 \text{ rad/s}}$



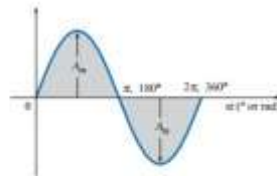
Plotting a sine wave versus radians.

GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

the sinusoidal waveform is

$$A_m \sin \alpha$$

$$A_m \sin \omega t$$



Basic sinusoidal function.

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

EXAMPLE

Given $e = 5 \sin \alpha$, determine e at $\alpha = 40^\circ$ and $\alpha = 0.8\pi$.

Solution: For $\alpha = 40^\circ$,

$$e = 5 \sin 40^\circ = 5(0.6428) = \mathbf{3.214 \text{ V}}$$

For $\alpha = 0.8\pi$,

$$\alpha (^\circ) = \frac{180^\circ}{\pi} (0.8\pi) = 144^\circ$$

$$\text{and } e = 5 \sin 144^\circ = 5(0.5878) = \mathbf{2.939 \text{ V}}$$

PHASE RELATIONS

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive-going (increasing with time) slope before 0° , as shown in Fig. the expression is

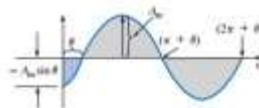
$$A_m \sin(\omega t + \theta)$$

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

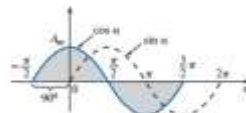
$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\begin{aligned} \cos \alpha &= \sin(\alpha + 90^\circ) \\ \sin \alpha &= \cos(\alpha - 90^\circ) \\ -\sin \alpha &= \sin(\alpha \pm 180^\circ) \\ -\cos \alpha &= \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ) \\ &\text{etc.} \end{aligned}$$

$$\begin{aligned} \sin(-\alpha) &= -\sin \alpha \\ \cos(-\alpha) &= \cos \alpha \end{aligned}$$



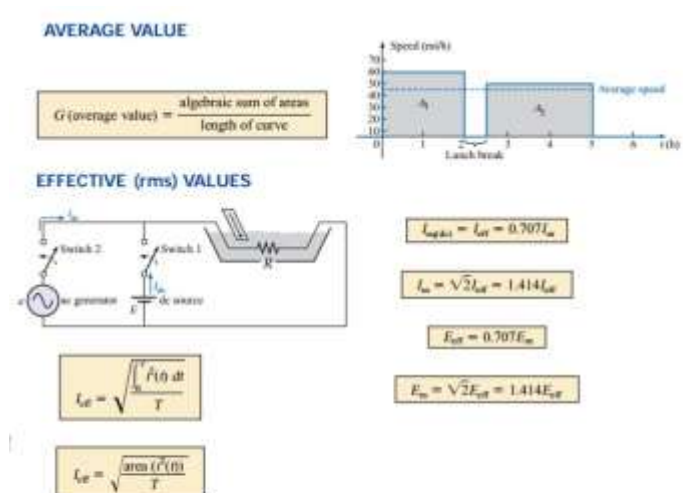
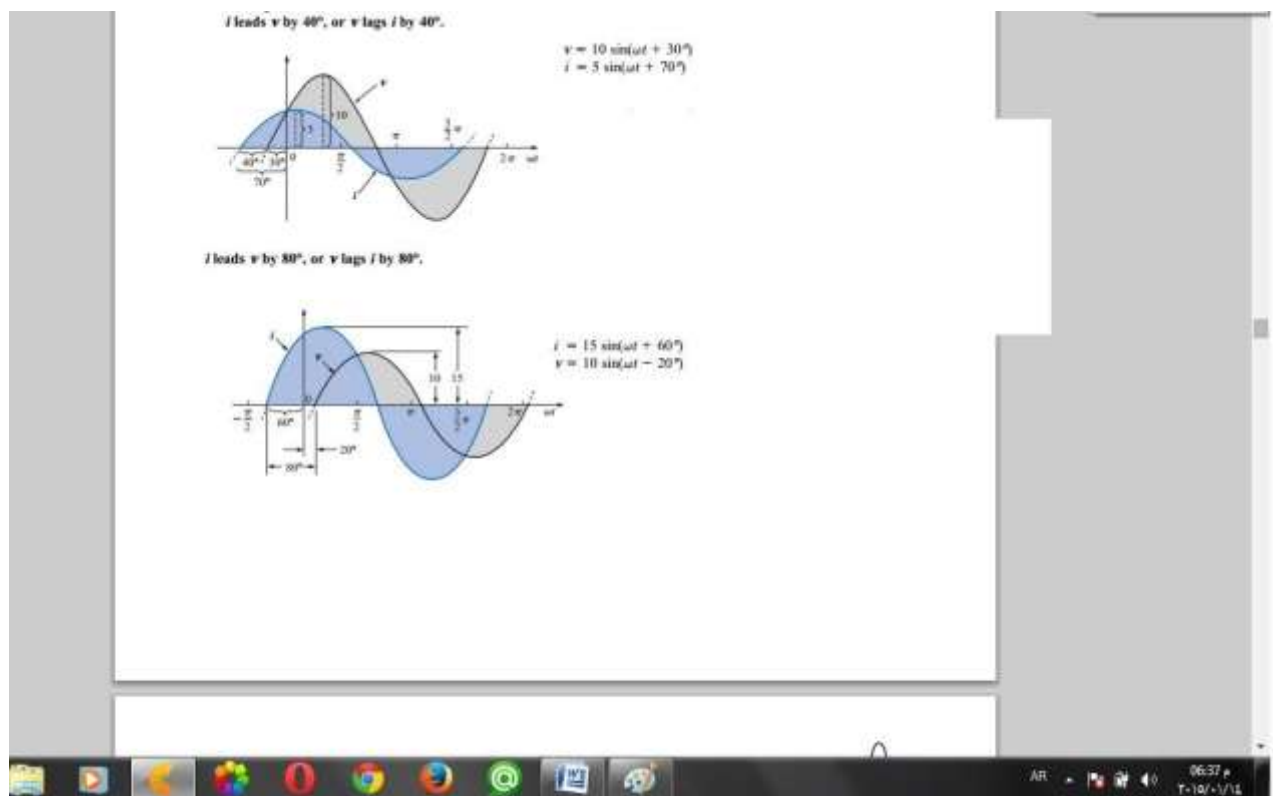
Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0° .



Phase relationship between a sine wave and a cosine wave.



Graphic tool for finding the relationship between specific sine and cosine functions.



RESPONSE OF BASIC R, L, AND C ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

Resistor

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

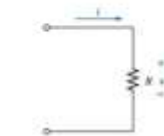
$$I_m = \frac{V_m}{R}$$

In addition, for a given i

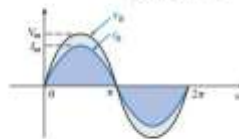
$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

$$V_m = I_m R$$

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.



Determining the sinusoidal response for a resistive element.



The voltage and current of a resistive element are in phase.

$$\frac{d}{dt} \cos(\theta) = -\sin(\theta) = -\cos(\theta + 90^\circ)$$

Inductor

$$v_L = L \frac{di_L}{dt}$$

and, applying differentiation,

$$\frac{di_L}{dt} = \frac{d}{dt} (I_m \sin \omega t) = \omega I_m \cos \omega t$$

$$\text{Therefore, } v_L = L \frac{di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \cos \omega t$$

$$v_L = V_m \sin(\omega t + 90^\circ)$$

$$V_m = \omega L I_m$$

$$i_L = I_m \sin(\omega t \pm \theta)$$

$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90^\circ)$$

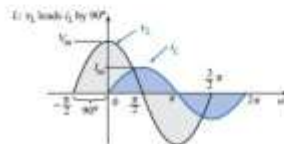
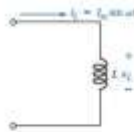
for an inductor, v_L leads i_L by 90° , or i_L lags v_L by 90° .

The quantity ωL , called the **reactance** (from the word reaction) of an inductor, is symbolically represented by X_L and is measured in ohms; that is,

$$X_L = \omega L \quad (\text{ohms, } \Omega)$$

In an Ohm's law format, its magnitude can be determined from

$$X_L = \frac{V_m}{I_m} \quad (\text{ohms, } \Omega)$$

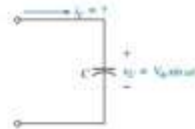


For a pure inductor, the voltage across the coil leads the current through the coil by 90° .

Capacitor

$$i_C = C \frac{dv_C}{dt}$$

$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$



Investigating the sinusoidal response of a capacitive element

Therefore,

$$i_C = C \frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

$$i_C = I_m \sin(\omega t + 90^\circ)$$

$$I_m = \omega C V_m$$

for a capacitor, i_C leads v_C by 90° , or v_C lags i_C by 90° .

$$v_C = V_m \sin(\omega t + \theta)$$

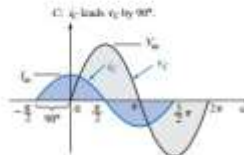
$$i_C = \omega C V_m \sin(\omega t + \theta + 90^\circ)$$

The quantity $1/\omega C$, called the **reactance** of a capacitor, is symbolically represented by X_C and is measured in ohms; that is,

$$X_C = \frac{1}{\omega C} \quad (\text{ohms}, \Omega)$$

In an Ohm's law format, its magnitude can be determined from

$$X_C = \frac{V_m}{I_m} \quad (\text{ohms}, \Omega)$$



The current of a purely capacitive element leads the voltage across the element by 90° .

EXAMPLE The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10Ω . Sketch the curves for v and i .

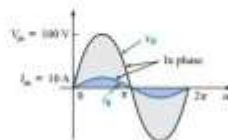
a. $v = 100 \sin 377t$

b. $v = 25 \sin(377t + 60^\circ)$

Solutions:

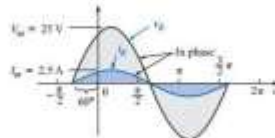
a. $I_m = \frac{V_m}{R} = \frac{100 \text{ V}}{10 \Omega} = 10 \text{ A}$

$$i = 10 \sin 377t$$



b. $I_m = \frac{V_m}{R} = \frac{25 \text{ V}}{10 \Omega} = 2.5 \text{ A}$

$$i = 2.5 \sin(377t + 60^\circ)$$



EXAMPLE The current through a 0.1-H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the v and i curves.

a. $i = 10 \sin 377t$

b. $i = 7 \sin(377t - 70^\circ)$

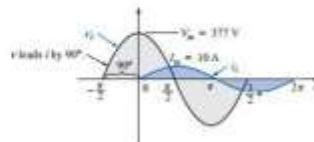
Solutions:

a. $X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7 \Omega$

$$V_m = I_m X_L = (10 \text{ A})(37.7 \Omega) = 377 \text{ V}$$

and we know that for a coil v leads i by 90° . Therefore,

$$v = 377 \sin(377t + 90^\circ)$$



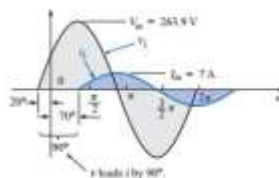
b. X_L remains at 37.7Ω .

$$V_m = I_m X_L = (7 \text{ A})(37.7 \Omega) = 263.9 \text{ V}$$

and we know that for a coil v leads i by 90° . Therefore,

$$v = 263.9 \sin(377t - 70^\circ + 90^\circ)$$

$$v = 263.9 \sin(377t + 20^\circ)$$



EXAMPLE 14.5 The voltage across a $1\text{-}\mu\text{F}$ capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

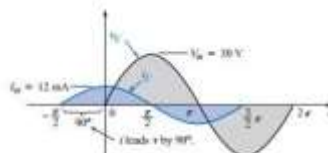
Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{(400 \text{ rad/s})(1 \times 10^{-6} \text{ F})} = \frac{10^6}{400} = 2500 \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30 \text{ V}}{2500 \Omega} = 0.0120 \text{ A} = 12 \text{ mA}$$

and we know that for a capacitor i leads v by 90° . Therefore,

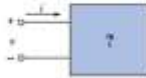
$$i = 12 \times 10^{-3} \sin(400t + 90^\circ)$$



The curves are sketched in Fig.

EXAMPLE For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor, and determine the value of C , L , or R if sufficient data are provided (Fig. 1.11).

- a. $v = 100 \sin(\omega t + 40^\circ)$
 $i = 20 \sin(\omega t + 40^\circ)$
 b. $v = 1000 \sin(377t + 10^\circ)$
 $i = 5 \sin(377t - 80^\circ)$
 c. $v = 500 \sin(157t + 30^\circ)$
 $i = 1 \sin(157t + 120^\circ)$
 d. $v = 50 \cos(\omega t + 20^\circ)$
 $i = 5 \sin(\omega t + 110^\circ)$



Solutions:

a. Since v and i are in phase, the element is a resistor;

$$R = \frac{V_m}{I_m} = \frac{100 \text{ V}}{20 \text{ A}} = 5 \Omega$$

b. Since v leads i by 90° , the element is an inductor; and

$$X_L = \frac{V_m}{I_m} = \frac{1000 \text{ V}}{5 \text{ A}} = 200 \Omega$$

so that $X_L = \omega L = 200 \Omega$ or

$$L = \frac{200 \Omega}{\omega} = \frac{200 \Omega}{377 \text{ rad/s}} = 0.531 \text{ H}$$

c. Since i leads v by 90° , the element is a capacitor;

$$X_C = \frac{V_m}{I_m} = \frac{500 \text{ V}}{1 \text{ A}} = 500 \Omega$$

$$X_C = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega X_C} = \frac{1}{(157 \text{ rad/s})(500 \Omega)} = 12.74 \mu\text{F}$$

$$\begin{aligned} \text{d. } v &= 50 \cos(\omega t + 20^\circ) = 50 \sin(\omega t + 20^\circ + 90^\circ) \\ &= 50 \sin(\omega t + 110^\circ) \end{aligned}$$

Since v and i are in phase, the element is a resistor; and

$$R = \frac{V_m}{I_m} = \frac{50 \text{ V}}{5 \text{ A}} = 10 \Omega$$

current easy to develop high voltage.

AC equipment is low cost.

Possible to convert to DC.

Easy to step down of setup the voltage by transformer.

AC motors are cheapest.

Disadvantage:

Can not able to store in battery.

Because of high starting current in AC the voltage drop is occurred.

The speed of the AC motors is depending up on the frequency.

According to the induction load; power factor gets low.

If coil rotate in magnetic field or magnetic field rotate inside the coil there is an alternating e. m. f. generate in the coil. The generated emf is proportional to the number of turns of coil, magnetic field strength, and the angle between the coil and magnetic field.

$$e = BLv \sin \theta$$

From this:

L = Length of the conductor.

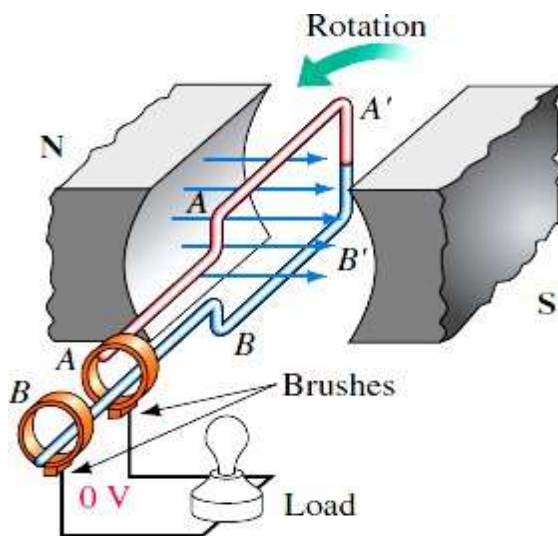
$v = \text{Velocity of conductor.}$

$B = \text{Flux density.}$

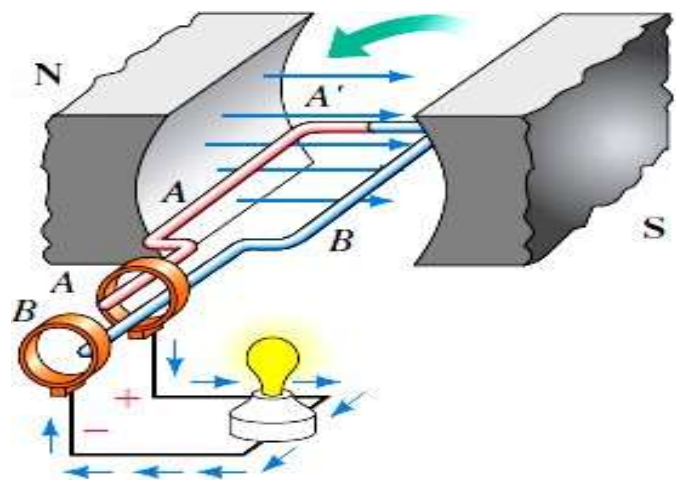
$\theta = \text{angle between field to conductor.}$

$e = \text{generated AC emf}$

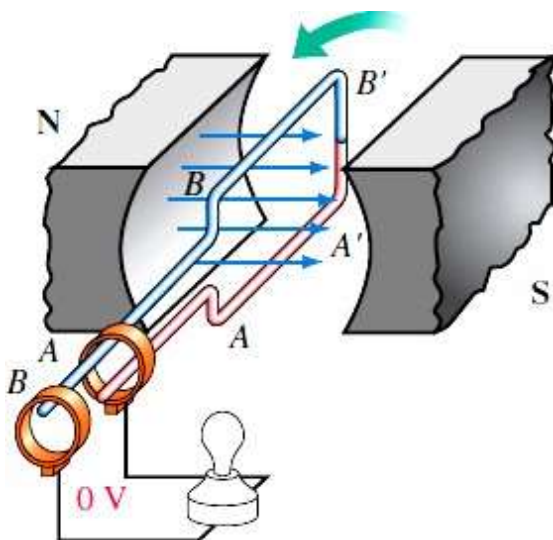
The generated AC emf value is depending upon the sine value of the angle between the magnetic field and conductor.



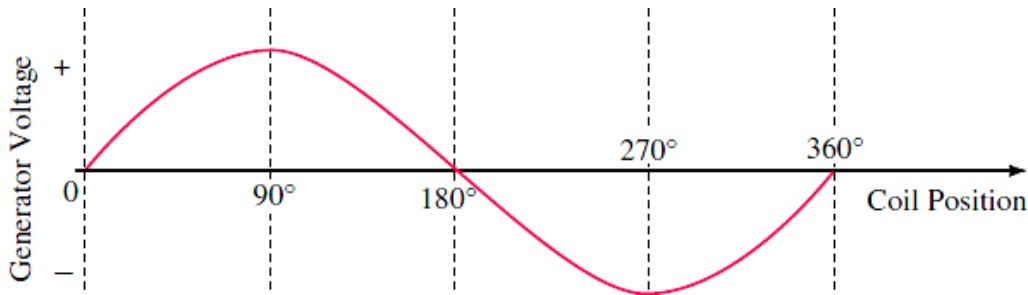
(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.



(b) 90° Position: Coil end A is positive with respect to B. Current direction is out of slip ring A.



(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.



Cycle:

An alternating current complete set of one positive half cycle and one negative half cycle is called one cycle.

Time period:

The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by the letter "T".

Frequency:

The number of cycle per second is called the frequency of the alternating quantity. The unit is hertz (Hz).

Instantaneous value:

The alternating quantity changes at every time.

$$v = V_{max} \sin \omega t \quad \text{or} \quad i = I_{max} \sin \omega t$$

Maximum value:

The maximum value positive or negative of an alternating quantity is known as its maximum value. Denoted by " I_{max} or V_{max} ".

Effective value and RMS value:

The effective value of an alternating current is given by that DC current which when flowing through a given circuit for a given time produces the same heat as produced by

the alternating current when flowing through the same circuit for the same time also is called root mean square value RMS. The voltmeter and ammeter are read the effective value only.

$$RMS\ value = \frac{I_{max}}{\sqrt{2}} \quad or \quad \frac{V_{max}}{\sqrt{2}}$$

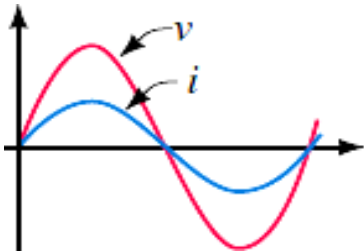
Average value:

The average value is calculated by the averages of the maximum value of alternating quantity at different instances.

$$Average\ value = \frac{2I_{max}}{\pi} \quad or \quad \frac{2V_{max}}{\pi}$$

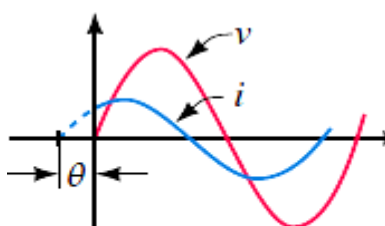
In phase:

If waveform of two AC quantities (voltage or current) get the maximum and zero at same time then they are said to be in phase.

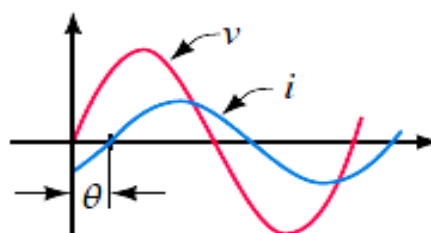


Out of phase:

If in AC circuit two quantities namely voltage or current waves get the maximum and zero at different value then they are said to be out of phase.



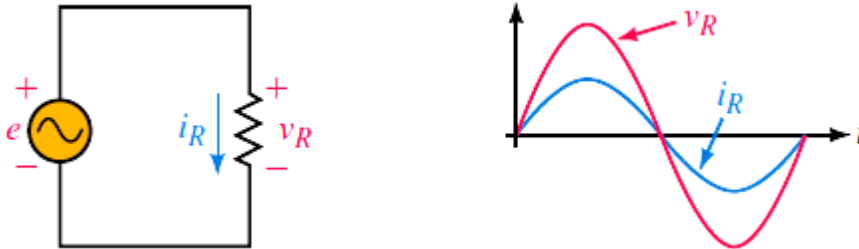
Current leads



Current lags

AC circuit with pure resistance:

A circuit without inductance and capacitance is called pure resistance circuit as shown in figure.



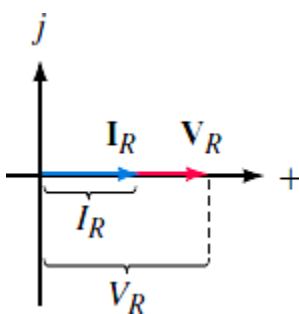
By applying Ohm's law:

$$i_R = \frac{v_R}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

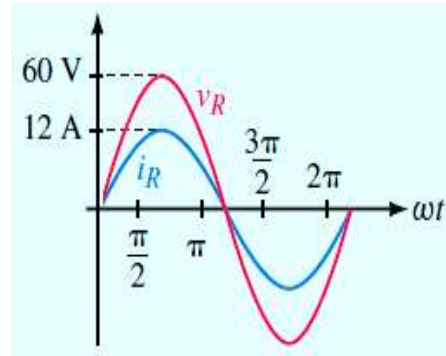
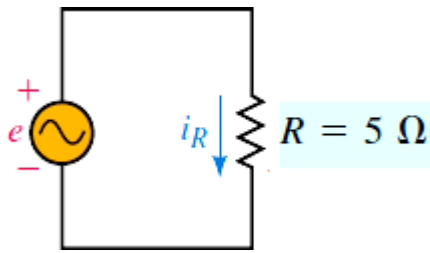
where $I_m = V_m/R$

Transposing, $V_m = I_m R$

Note that current and voltage are in phase:



Example-21: For the circuit shown in figure find the value of i_R , if $v(t)=60\sin\omega t$.

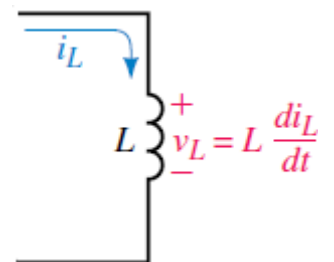


Solution:

$$i_R = \frac{v}{R} = \frac{60 \sin(\omega t)}{5} = 12 \sin(\omega t) \quad A$$

AC circuit with Purely Inductive Load:

Consider a purely inductive circuit with an inductor connected to an AC generator, as shown in Figure.



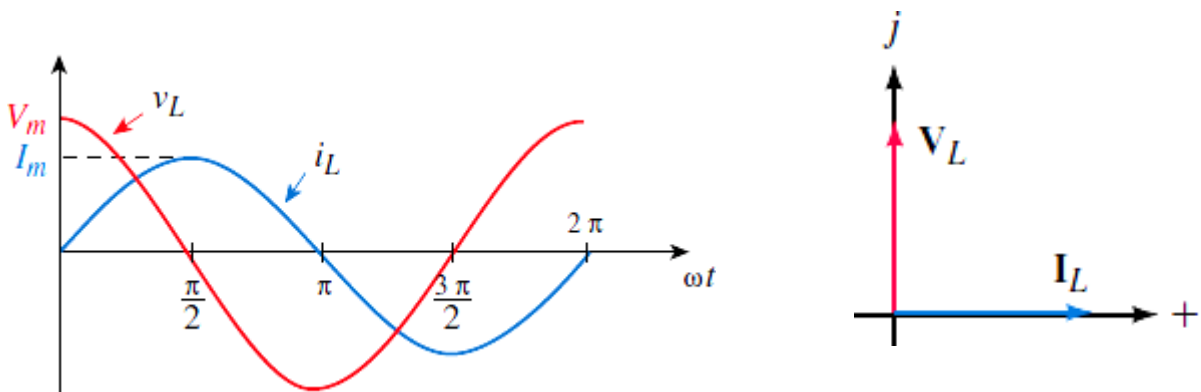
$$v_L = L \frac{di_L}{dt} = L \frac{d}{dt} (I_m \sin \omega t) = \omega L I_m \cos \omega t = V_m \cos \omega t$$

Utilizing the trigonometric identity $\cos \omega t = \sin(\omega t + 90^\circ)$ you can write this as:

$$v_L = V_m \sin(\omega t + 90^\circ)$$

$$V_m = \omega L I_m$$

Voltage and current waveforms are shown in Figure (a), and phasors in Figure (b). As you can see, for a purely inductive circuit, current lags voltage by 90°



From Equation above, we see that the ratio V_m to I_m is:

$$\frac{V_m}{I_m} = \omega L$$

This ratio is defined as inductive reactance and is given the symbol X_L . Since the ratio of volts to amps is ohms, reactance has units of ohms.

$$X_L = \frac{V_m}{I_m} \quad (\Omega)$$

Combining Equations above yields:

$$X_L = \omega L \quad (\Omega)$$

Where ω is radians per second $\omega = 2\pi f$

Example-22: The voltage across a 0.2H inductance is $v_L = 100 \sin(400t - 70^\circ)$ V.

Determine i_L and sketch it

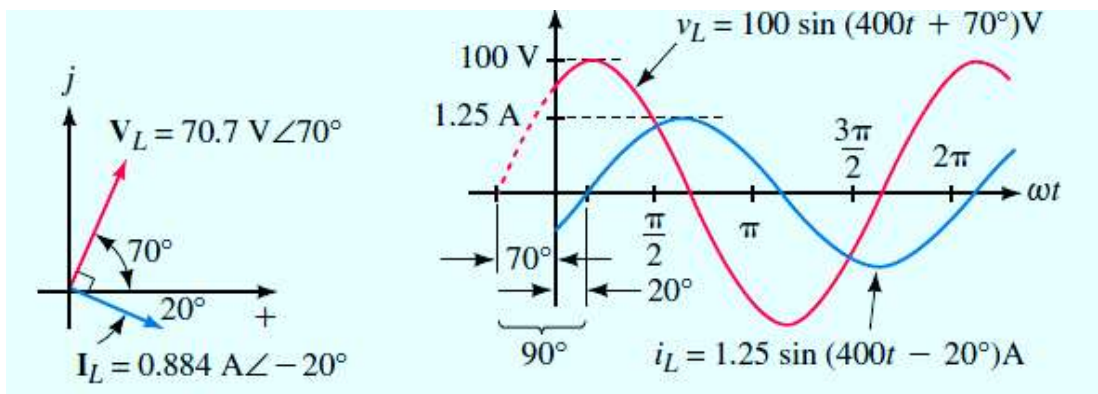
Solution:

$$\omega = 400 \text{ rad/s} \quad \text{therefore } x_L = \omega L = 400 \times 0.2 = 80 \Omega$$

$$I_m = \frac{V_m}{x_L} = \frac{100}{80} = 1.25 \text{ A}$$

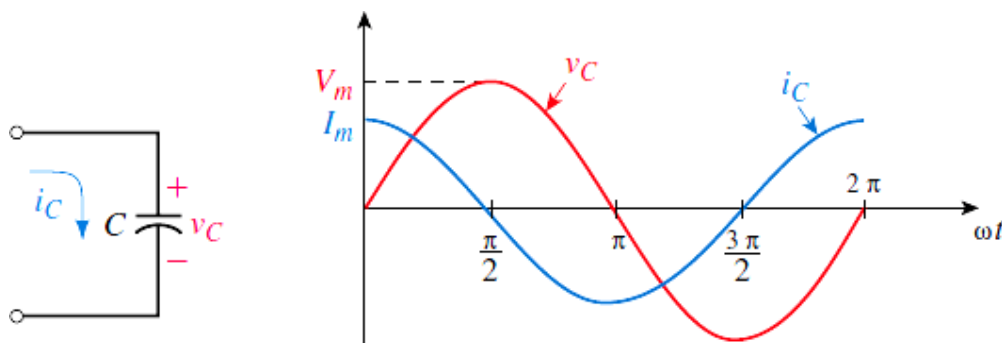
The current lags the voltage by 90° therefore:

$$i_L = 1.25 \sin(400t - 20^\circ) \text{ A as indicated in figure below}$$



AC circuit with Purely Capacitive Load:

In the purely capacitive case, both resistance R and inductance L are zero. The circuit diagram is shown in Figure



For capacitance, current is proportional to the rate of change of voltage:

$$i_C = C \frac{dv_C}{dt} = C \frac{d}{dt}(V_m \sin \omega t) = \omega C V_m \cos \omega t = I_m \cos \omega t$$

Using the appropriate trigonometric identity, this can be written as:

$$i_C = I_m \sin(\omega t + 90^\circ)$$

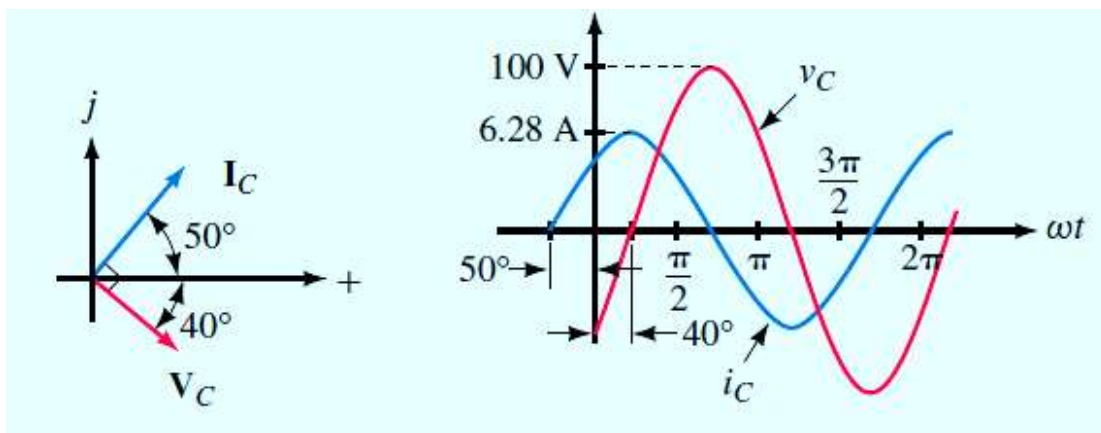
Where $I_m = \omega C V_m = \frac{V_m}{X_C}$

X_C is called the capacitance reactance

Example-23: The voltage across a 10-mF capacitance is $v_C = 100 \sin(\omega t - 40^\circ)$ V and $f = 1000$ Hz. Determine i_C and sketch its waveform.

Solution:

Since current leads voltage by 90° , $i_C = 6.28 \sin(6283t - 50^\circ)$ A as indicated in Figure below.



Impedance:

The opposition that a circuit element presents to current in the phasor domain is defined as its impedance.

$$Z = \frac{V}{I} \text{ (ohms)}$$

For resistor:

$$Z_R = R$$

For inductance:

$$Z_L = j\omega L = jX_L$$

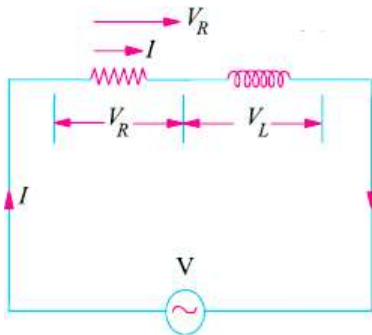
For capacitance:

$$Z_C = -j\frac{1}{\omega C} = -jX_C \quad (\text{ohms})$$

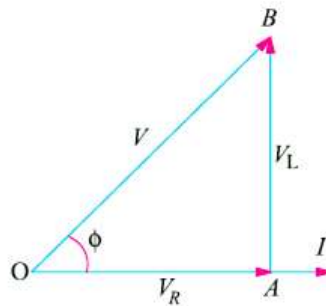
Series AC circuit:

AC through Resistance and Inductance:

A pure resistance R and inductance L are shown in figure(a) is connected in series.



Figure(a)



Figure(b)

Let V=r.m.s. value of the applied voltage; I=r.m.s. value of the resultant current.

$$V_R = IR \quad \text{voltage drop across } R(\text{in phase with } I)$$

$$V_L = I \times X_L \quad \text{voltage drop across inductance (lag of } I \text{ by } 90)$$

The applied voltage V is:

$$V = \sqrt{V_R^2 + V_L^2} \quad \text{or } V = I \times Z$$

where Z is the impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2}$$

From figure(b) it is clear that the applied voltage V leads the current I by an angle ϕ such that:

$$\tan \phi = \frac{V_L}{V_R} = \frac{X_L}{R}$$

Vector

$$|A| = \sqrt{a_1^2 + a_2^2}$$

$$\theta = \tan^{-1} \left(\frac{a_2}{a_1} \right)$$

$$a_1 = A \cos \theta$$

$$a_2 = A \sin \theta$$

rectangular form

$$\bar{A} = a_1 + ja_2$$

$$\bar{A} = A \cos \theta + jA \sin \theta = A(\cos \theta + j \sin \theta)$$

polar form

$$\bar{A} = |A| \angle \theta$$

Basic Calculations of Vectors

$$\bar{A} = a_1 + ja_2$$

$$\bar{B} = b_1 + jb_2$$

$$\bar{C} = c_1 + jc_2$$

$$\bar{D} = d_1 + jd_2$$

$$\bar{C} = \bar{A} + \bar{B}$$

$$\bar{D} = \bar{A} - \bar{B}$$

$$c_1 + jc_2 = a_1 + ja_2 + b_1 + jb_2 = (a_1 + b_1) + j(a_2 + b_2)$$

$$d_1 + jd_2 = (a_1 + ja_2) - (b_1 + jb_2) = (a_1 - b_1) + j(a_2 - b_2)$$

$$c_1 = a_1 + b_1$$

$$c_2 = a_2 + b_2$$

$$d_1 = a_1 - b_1$$

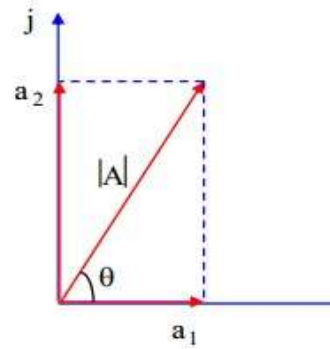
$$d_2 = a_2 - b_2$$

$$|C| = \sqrt{[(a_1 + b_1)^2 + (a_2 + b_2)^2]}$$

$$\theta_C = \tan^{-1} \frac{(a_2 + b_2)}{(a_1 + b_1)}$$

$$|D| = \sqrt{[(a_1 - b_1)^2 + (a_2 - b_2)^2]}$$

$$\theta_d = \tan^{-1} \frac{(a_2 - b_2)}{(a_1 - b_1)}$$



$$C = A \cdot B = (a_1 + ja_2) \cdot (b_1 + jb_2) \\ = (a_1b_1 - a_2b_2) + j(a_1b_2 + a_2b_1)$$

$$|C| = \sqrt{[(a_1b_1 - a_2b_2)^2 + (a_1b_2 + a_2b_1)^2]} \\ \theta_C = \tan^{-1} \frac{(a_1b_2 + a_2b_1)}{(a_1b_1 - a_2b_2)}$$

$$\overline{C} = \overline{A} \cdot \overline{B} = |A| \angle \theta_A \cdot |B| \angle \theta_B = |C| \angle \theta_C$$

$$|C| = |A| \cdot |B|$$

$$\angle \theta_C = \angle \theta_a + \angle \theta_b$$

$$\overline{C} = |A|e^{j\theta_a} \cdot |B|e^{j\theta_b} = |A| \cdot |B|e^{j(\theta_a + \theta_b)} = |C|e^{j\theta_c}$$

$$\overline{D} = \frac{\overline{A}}{\overline{B}}$$

$$|D| = \frac{|A|}{|B|}$$

$$\angle \theta_d = \angle \theta_a - \angle \theta_b$$

Example-24: A 60Hz sinusoidal voltage $V = 141\sin\omega t$ is applied to a series R-L circuit. The value of the resistance and the inductance are 3Ω and $0.0106H$ respectively.

Write the expression for the instantaneous current in the circuit.

Compute the r.m.s. value and the phase of the voltage across the resistance and the inductance.

Solution:

$$V_m = 141 \text{ V}; V = 141/\sqrt{2} = 100 \text{ V} \quad \therefore V = 100 + j0$$

$$X_L = 2\pi \times 60 \times 0.0106 = 4 \Omega. Z = 3 + j4 = 5 \angle 53.1^\circ$$

$$(i) I = V/Z = 100 \angle 0^\circ / 5 \angle 53.1^\circ = 20 \angle -53.1^\circ$$

Since angle is minus, the current lags behind the voltage by 53.1°

$$(ii) I_m = \sqrt{2} \times 20 = 28.28; \therefore i = 28.28 \sin(\omega t - 53.1^\circ)$$

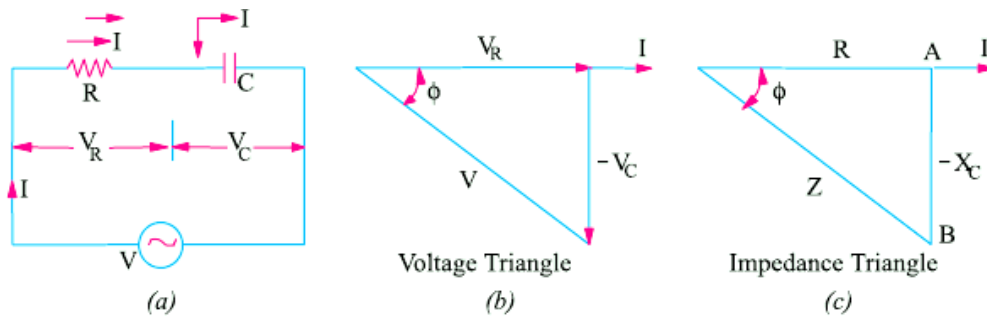
$$(iii) V_R = IR = 20 \angle -53.1^\circ \times 3 = 60 \angle -53.1^\circ \text{ volt.}$$

$$V_L = jIX_L = 1 \angle 90^\circ \times 20 \angle -53.1^\circ \times 4 = 80 \angle 36.9^\circ$$

AC through Resistance and Capacitance:

The circuit is shown in figure(a). Here $V_R = I.R$ = drop across R in phase with I. $V_C = I.X_C$ = drop across capacitor lagging I by 90° .

Ac capacitive reactance X_C is taken negative; V_C is shown along negative direction of Y-axis in the voltage triangle figure(b).



$$V = \sqrt{V_R^2 + (-V_C)^2} \quad \text{or } V = I \times Z$$

where Z is the impedance of the circuit

$$Z = \sqrt{R^2 + (-X_C)^2}$$

From figure(b) it is clear that the current I leads the applied voltage V by an angle ϕ such that:

$$\tan \phi = \frac{-V_C}{V_R} = \frac{-X_C}{R}$$

Example-25: A pure resistance of 50Ω is in series with a pure capacitance of $100\mu\text{f}$. The series combination is connected across 100V ; 50Hz supply. Find:

The impedance.

Current.

Phase angle.

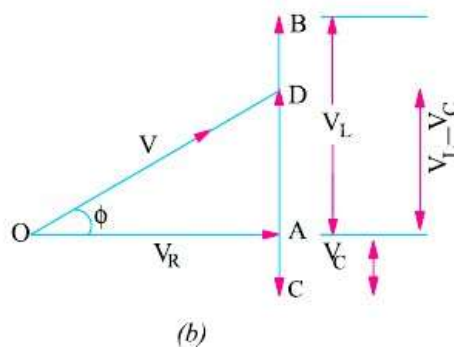
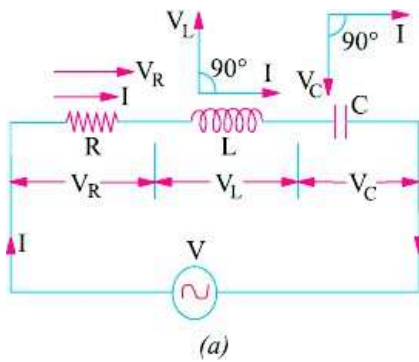
Voltage across resistor and capacitor.

Solution:

$$\begin{aligned} (a) \quad Z &= \sqrt{50^2 + 32^2} = 59.4 \, \Omega & (b) \quad I &= V/Z = 100/59.4 = 1.684 \, \text{A} \\ (c) \quad \text{p.f.} &= R/Z = 50/59.4 = 0.842 \, (\text{lead}) & (d) \quad \phi &= \cos^{-1}(0.842) = 32^\circ 36' \\ (e) \quad V_R &= IR = 50 \times 1.684 = 84.2 \, \text{V} & V_C &= IX_C = 32 \times 1.684 = 53.9 \, \text{V} \end{aligned}$$

Resistance; Inductance and capacitance in series:

The three are shown in figure(b) joined in series across an AC supply of r.m.s. voltage V



Let $V_R = I.R = \text{voltage drop across } R$

In phase with I

$V_L = I.X_L = \text{voltage drop across } L$

Leading I by 90°

$V_C = I.X_C = \text{voltage drop across } C$

Lagging by 90°

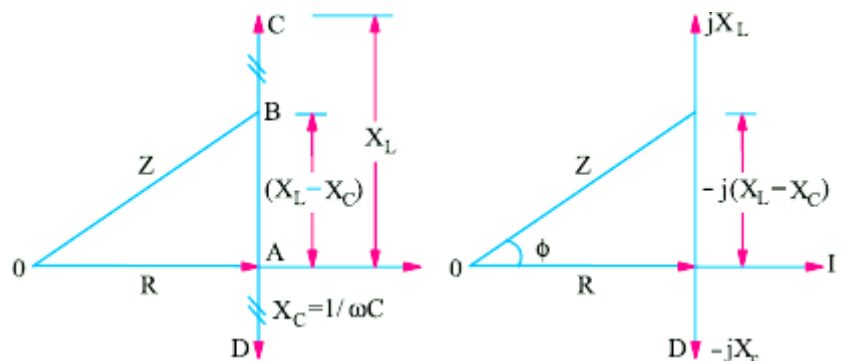
In the voltage triangle of figure(b):

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad ; \quad V = Z \times I$$

Where Z is the impedance of the circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase angle ϕ is given by:

$$\tan \phi = \frac{(X_L - X_C)}{R}$$



Example-26: A resistance of 20Ω an inductance of 0.2H and a capacitance of $100\mu\text{f}$ are connected in series across 220V ; 50Hz mains. Detemine:

Impedance.

Current.

Voltage across R; L; and C.

Solution:

Solution. $X_C = 0.2 \times 314 = 63 \Omega$, $C = 10 \mu F = 100 \times 10^{-6} = 10^{-4}$ farad

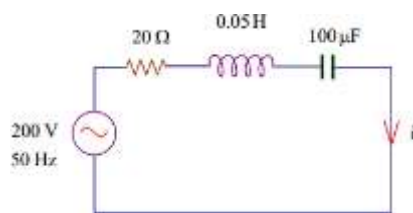
$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 10^{-4}} = 32 \Omega \quad X = 63 - 32 = 31 \Omega \text{ (inductive)}$$

$$(a) \quad Z = \sqrt{(20^2 + 31)^2} = 37 \Omega \quad (b) \quad I = 220/37 = 6 \text{ A (approx)}$$

$$(c) \quad V_R = I \times R = 6 \times 20 = 120 \text{ V}; \quad V_L = 6 \times 63 = 278 \text{ V}, \quad V_C = 6 \times 32 = 192 \text{ V}$$

Example-27: A resistance of 20Ω an inductance of 0.05H and a capacitance of $100\mu\text{f}$ are connected in series across 200V ; 50Hz mains. Detemine:

Impedance ,Current ,Voltage across R; L; and C ,and draw phasor daigram .



$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314 \text{ rad/sec}$$

$$X_L = \omega L = 314 \times 0.05 = 15.7 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(314 \times 100 \times 10^{-6})} = 31.84 \Omega$$

$$\bar{Z} = 20 + j(15.7 - 31.84) = 20 - j16.14$$

$$|Z| = \sqrt{(20)^2 + (16.14)^2} = 25.7 \Omega$$

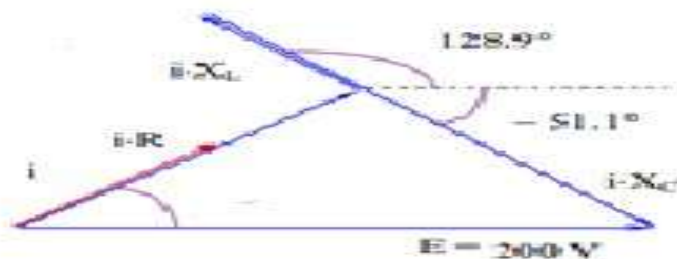
$$\theta_z = \tan^{-1}\left(\frac{-16.14}{20}\right) = -38.9^\circ$$

$$\bar{i} = \frac{\bar{E}}{\bar{Z}} = \frac{200 \text{ V} \angle 0^\circ}{25.7 \Omega \angle -38.9^\circ} = 7.78 \angle 38.9^\circ \text{ Amp}$$

$$\bar{E}_R = \bar{i} \cdot R = 7.78 \text{ A} \angle 38.9^\circ \cdot 20 \Omega = 155.6 \text{ V} \angle 38.9^\circ$$

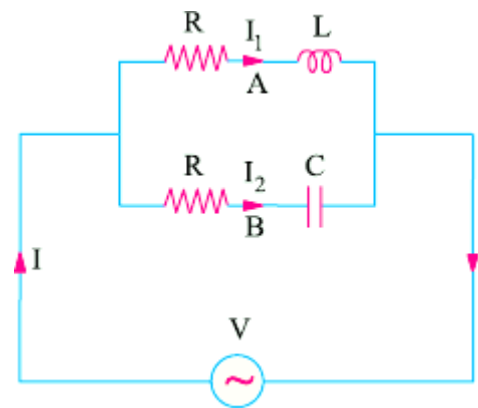
$$\bar{E}_L = \bar{i} \cdot X_L = 7.78 \text{ A} \angle 38.9^\circ \cdot 15.7 \Omega \angle 90^\circ = 122.146 \text{ V} \angle 128.9^\circ$$

$$\bar{E}_C = \bar{i} \cdot X_C = 7.78 \text{ A} \angle 38.9^\circ \cdot 31.84 \Omega \angle -90^\circ = 247.72 \text{ V} \angle -51.1^\circ$$



AC parallel circuit:

Consider the circuits shown in figure below. Here two branch A and B have been joined in parallel across an r.m.s supply of V volts. The voltage across two parallel branches A and B is the same. But currents through them are different.



For branch A

$$Z_1 = \sqrt{R_1^2 + X_L^2} : I_1 = \frac{V}{Z_1}$$

Current I_1 lags behind the applied voltage by ϕ_1

For branch B

$$Z_2 = \sqrt{\frac{R^2}{2} + \frac{X^2}{c}} : I_2 = \frac{V}{Z_2}$$

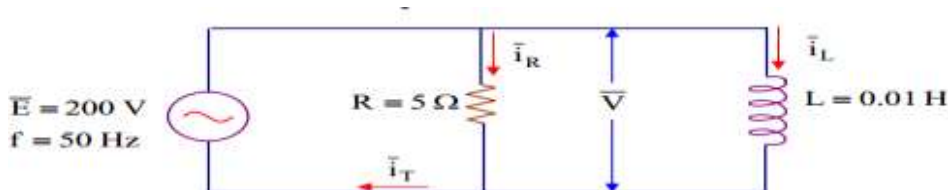
The total current I is:

$$I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) = V(Y_1 + Y_2) = VY$$

where $Y = \text{total admittance} = Y_1 + Y_2$

It should be noted that admittance are added for parallel branches; whereas for branches in series it is the impedances which are added.

Example-28: for the circuit shown find (I_L and I_R) .

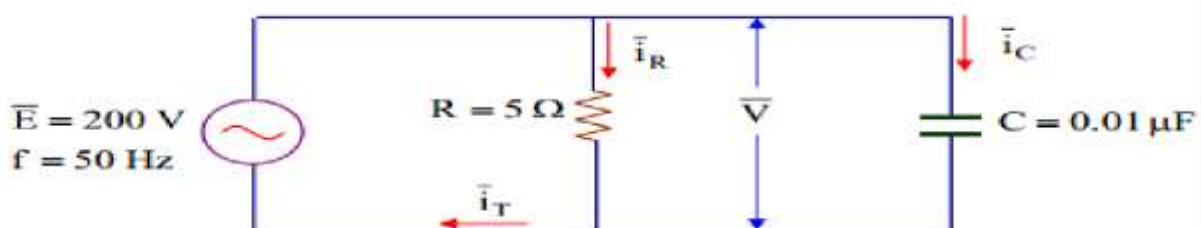


$$\bar{I}_R = \frac{E \angle 0^\circ}{R \angle 0^\circ} = \frac{E}{R} \angle 0^\circ = \frac{200 \text{ V} \angle 0^\circ}{5 \Omega \angle 0^\circ} = 40 \text{ A} \angle 0^\circ$$

$$\bar{I}_L = \frac{E}{\omega L} \angle -90^\circ = \frac{E}{2 \pi f L} \angle -90^\circ$$

$$\bar{I}_L = \frac{200 \text{ V}}{2 \pi \cdot 50 \cdot 1 \times 10^{-2} \Omega} \angle -90^\circ = 63.7 \text{ A} \angle -90^\circ$$

Example-29: for the circuit shown find (I_C , I_R and I_T) .



Solution:

$$I_R = \frac{E}{R} = \frac{200 \angle 0^\circ}{5 \angle 0^\circ} = 40 \angle 0^\circ \text{ A}$$

$$I_R = \frac{E}{X_C} = \omega EC \angle 90^\circ = 200 \times 2 \times 50 \times 3.14 \times 0.01 \times 10^{-6} = 62.8 \times 10^{-3} \angle 90^\circ \text{ A}$$

$$I_T = I_R + I_C = 40 \angle 0^\circ + 62.8 \times 10^{-3} \angle 90^\circ =$$

Example-27: Two circuit the impedance of which are given by $Z_1 = 10 + j15\Omega$ and $Z_2 = 6 - j8\Omega$ are connected in parallel. If the total current applied is 15A. what are the current taken by each branch.

Solution. Let $I = 15 \angle 0^\circ$; $Z_1 = 10 + j15 = 18 \angle 57^\circ$

$$Z_2 = 6 - j8 = 10 \angle -53.1^\circ$$

$$\text{Total impedance, } Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10 + j15)(6 - j8)}{16 + j7}$$

$$= 9.67 - j3.6 = 10.3 \angle -20.4^\circ$$

Applied voltage is given by

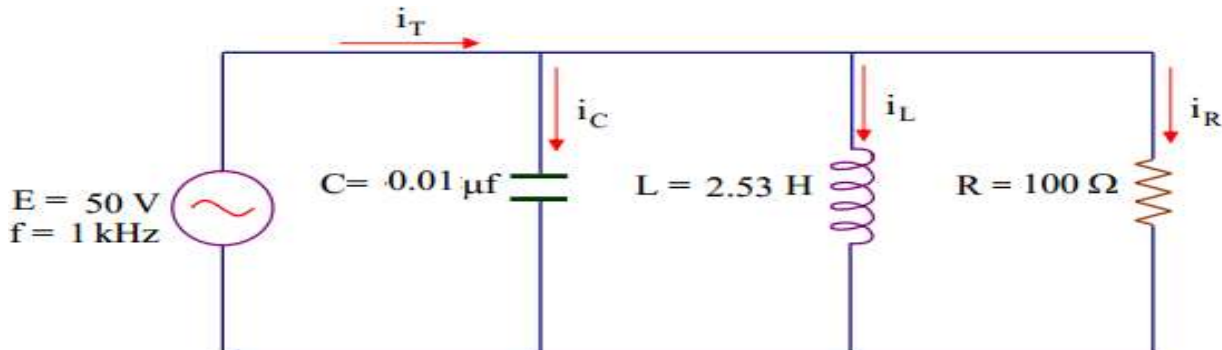
$$V = IZ = 15 \angle 0^\circ \times 10.3 \angle -20.4^\circ = 154.5 \angle -20.4^\circ$$

$$I_1 = V/Z_1 = 154.5 \angle -20.4^\circ / 18 \angle 57^\circ = 8.58 \angle -77.4^\circ$$

$$I_2 = V/Z_2 = 154.5 \angle -20.4^\circ / 10 \angle -53.1^\circ$$

$$= 15.45 \angle 32.7^\circ$$

Example-29: for the circuit shown find (I_C , I_R , I_L and I_T).



$$i_R = \frac{50 \text{ V } \angle 0^\circ}{100 \Omega \angle 0^\circ} = 0.5 \text{ A } \angle 0^\circ$$

$$i_L = \frac{50 \text{ V } \angle 0^\circ}{(2 \times \pi \times 1000 \times 2.53) \Omega \angle 90^\circ} = 3.145 \text{ mA } \angle -90^\circ$$

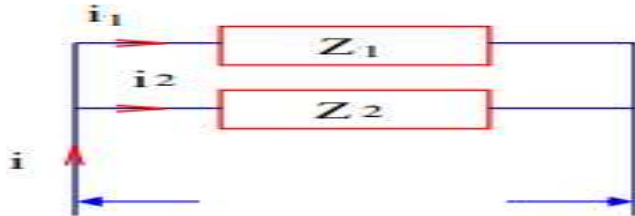
$$i_C = \frac{50 \text{ V } \angle 0^\circ}{(1/2 \times \pi \times 1000 \times 0.01 \times 10^{-6}) \Omega \angle -90^\circ} = 3.145 \text{ mA } \angle 90^\circ$$

$$i_T = i_R + i_L + i_C = 0.5 \text{ A } \angle 0^\circ + 3.145 \text{ mA } \angle -90^\circ + 3.145 \text{ mA } \angle 90^\circ$$

$$i_T = 0.5 \text{ A } \angle 0^\circ$$

Example-27: for the circuit the shown if $Z_1 = 6 + j8\Omega$ and $Z_2 = 8 - j6\Omega$ are connected in parallel. If the applied voltage is (100V , 60Hz) ,what are the current taken by each branch.

Solution :



$$Z_1 = 6 + j8 = 10 \Omega \angle 51.3^\circ$$

$$Z_2 = 8 - j6 = 10 \Omega \angle -36.9^\circ$$

$$E = 100 \text{ V} \angle 0^\circ$$

$$i_1 = \frac{100 \angle 0^\circ}{10 \angle 51.3^\circ} = 10 \text{ A} \angle -51.3^\circ$$

$$i_2 = \frac{100 \angle 0^\circ}{10 \angle -36.9^\circ} = 10 \text{ A} \angle 36.9^\circ$$

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(6 + j8)(8 - j6)}{(6 + j8)(8 - j6)}$$

$$Z_t = \frac{96 + j28}{14 + j2} = \frac{100 \angle 16.26^\circ}{14.14 \angle 8.13^\circ} = 7.1 \Omega \angle 8.13^\circ$$

$$i = \frac{E}{Z_t} = \frac{100 \angle 0^\circ}{7.1 \angle 8.13^\circ} = 14.1 \text{ A} \angle -8.13^\circ$$

Power in the AC circuit

Apparent power (S): It is given by product of r.m.s. values of applied voltage and circuit current.

$$S = V.I = I^2.Z \quad (\text{VA})$$

Active power (P): It is the power which is actually dissipated in the circuit resistance.

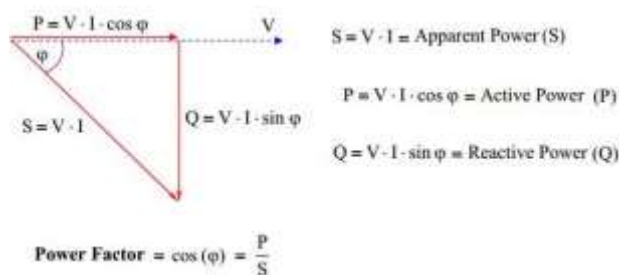
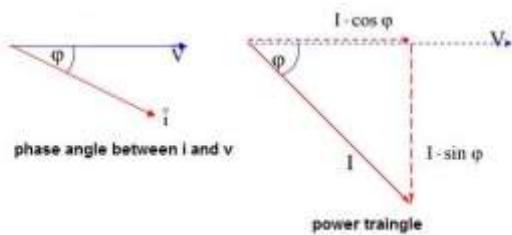
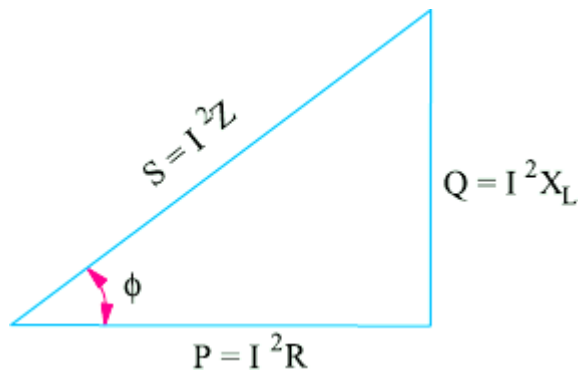
$$P = V.I \cos \phi = I^2.R \quad (\text{watt})$$

Reactance power (Q): it is the power developed in the reactance of the circuit.

$$Q = I.X \sin \phi = I^2.X$$

These three powers are shown in the power triangle of figure below.

$$S = \sqrt{P^2 + Q^2}$$



Power factor

$$\cos \phi = \frac{R}{Z} = \frac{P}{S}$$

Example-28: A 60Hz sinusoidal voltage $v = 141 \sin \omega t$ is applied to a series R-L circuit. The values of the resistance and the inductance are 3Ω and 0.0106H respectively.

Find the average power dissipated by the circuit.

Calculate the power factor of the circuit.

Solution. $V_m = 141 \text{ V}$; $V = 141/\sqrt{2} = 100 \text{ V} \quad \therefore V = 100 + j0$

$$X_L = 2\pi \times 60 \times 0.0106 = 4 \Omega. \quad Z = 3 + j4 = 5 \angle 53.1^\circ$$

$$I = V/Z = 100 \angle 0^\circ / 5 \angle 53.1^\circ = 20 \angle -53.1^\circ$$

Since angle is minus, the current lags behind the voltage by 53.1°

$$I_m = \sqrt{2} \times 20 = 28.28; \quad \therefore i = 28.28 \sin(\omega t - 53.1^\circ)$$

$$(i) \quad P = VI \cos \phi = 100 \times 20 \times \cos 53.1^\circ = \mathbf{1200 \text{ W.}}$$

$$(ii) \quad \text{p.f.} = \cos \phi = \cos 53.1^\circ = \mathbf{0.6.}$$

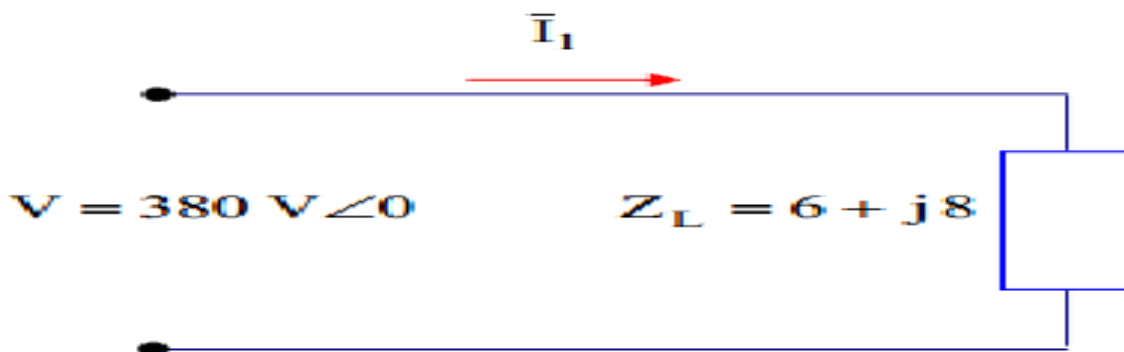
Example-29: In an alternating circuit; the impressed voltage is given by $V = (100 - j50)$ volts and the current is $I = (3 - j4)\text{A}$. Determine the real and reactive power in the circuit.

Solution. Power will be found by the conjugate method. Using current conjugate, we have

$$P_{IA} = (100 - j50)(3 + j4) = 300 + j400 - j150 + 200 = 500 + j250$$

Hence, real power is **500 W** and reactive power of VAR is **250**.

Example-29: for the circuit shown find (C) to make power factor of the circuit is unity.



Solution :

$$Z_L = \sqrt{(6)^2 + (8)^2} = 10 \Omega$$

$$\theta_{ZL} = \tan^{-1} \frac{8}{6} = 51.3^\circ$$

$$I_1 = \frac{380 \angle 0^\circ}{10 \angle 51.3^\circ} = 38 \text{ A} \angle -51.3^\circ$$

connect the capacitor (C) in figure

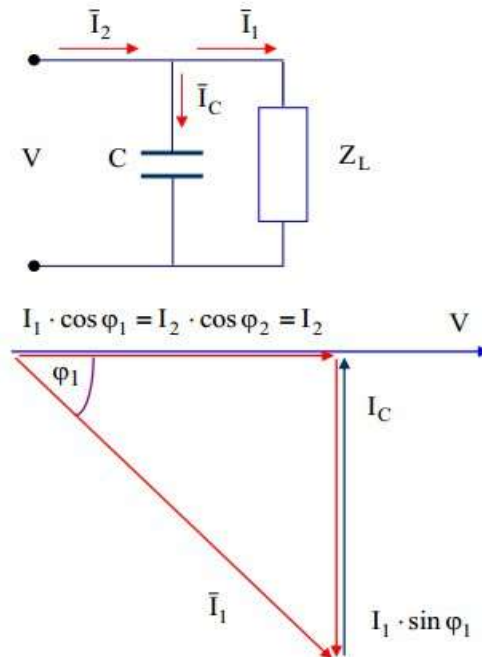
$$I_C = I_1 \cdot \sin \phi_1 = 38 \cdot \sin (-51.3^\circ) = -29.66 \text{ A}$$

$$\theta_C = 90^\circ$$

$$\therefore I_C = -29.66 \text{ A} \angle 90^\circ$$

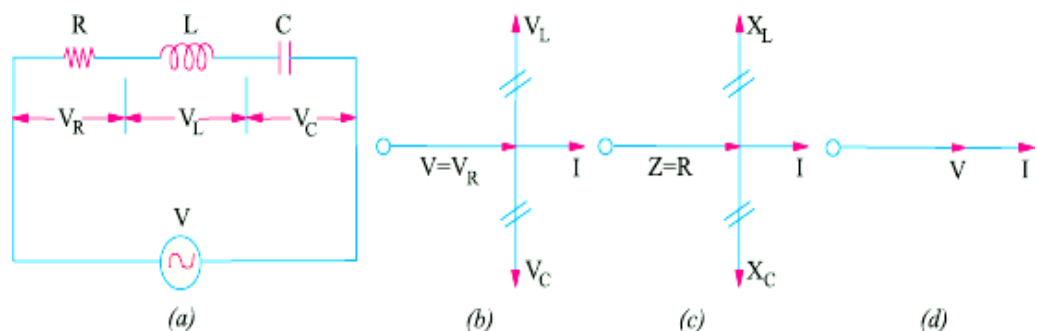
$$I_C = \frac{V \angle 0}{-j(1/\omega C)} = \frac{V \angle 0}{(1/\omega C) \angle -90} = V\omega C \angle 90$$

$$C = \frac{-29.66}{380 \times 2\pi \times 50} = -248.42 \mu\text{F}$$



Resonance in the R-L-C series circuit:

Let the R-L-C circuit shown in figure (a) is connected across an AC source of constant voltage V but of frequency varying from zero to infinity. There would be a certain frequency of the applied voltage which would make X_L equal to X_C in magnitude. In that case; $X=0$ and $Z=R$ as shown in figure (c). Under this condition the circuit is said to be in electrical resonance.



The frequency at which the net reactance of the series circuit is zero is called the resonant frequency f_0 . Its value can be found under:

$$X_L = X_C$$

Therefore
$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

If L in henry and C in farad then f_o is given in Hz.

When a series R-L-C circuit is in resonance it possesses minimum impedance ($Z=R$).

Hence circuit current is maximum. The current $I_o = V/R$ and is in phase with V.

Summary

When an R-L-C circuit is in resonance

1. net reactance of the circuit is zero i.e. $(X_L - X_C) = 0$ or $X = 0$.
2. circuit impedance is minimum i.e. $Z = R$. Consequently, circuit admittance is maximum.
3. circuit current is maximum and is given by $I_o = V/Z_o = V/R$.
4. power dissipated is maximum i.e. $P_o = I_o^2 R = V^2/R$.
5. circuit power factor angle $\theta = 0$. Hence, power factor $\cos \theta = 1$.

Q-factor of a resonant Series Circuit:

The Q-factor of an R-L-C series circuit can be defined by:

$$Q_o = \frac{2\pi f_o L}{R} \text{ or } Q_o = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Example-30: A coil having an inductance of 50mH and resistance 10Ω is connected in series with a 25 μ F capacitor across a 200V ac supply. Calculate: (a) resonance frequency of the circuit. (b) current flowing at resonance and (c) value of Q_o .

Solution. (a)
$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50 \times 10^{-3} \times 25 \times 10^{-6}}} = 142.3 \text{ Hz}$$

(b)
$$I_o = V/R = 200/10 = 20 \text{ A}$$

(c)
$$Q_o = \frac{\omega_o L}{R} = \frac{2\pi \times 142.3 \times 50 \times 10^{-3}}{10} = 4.47$$

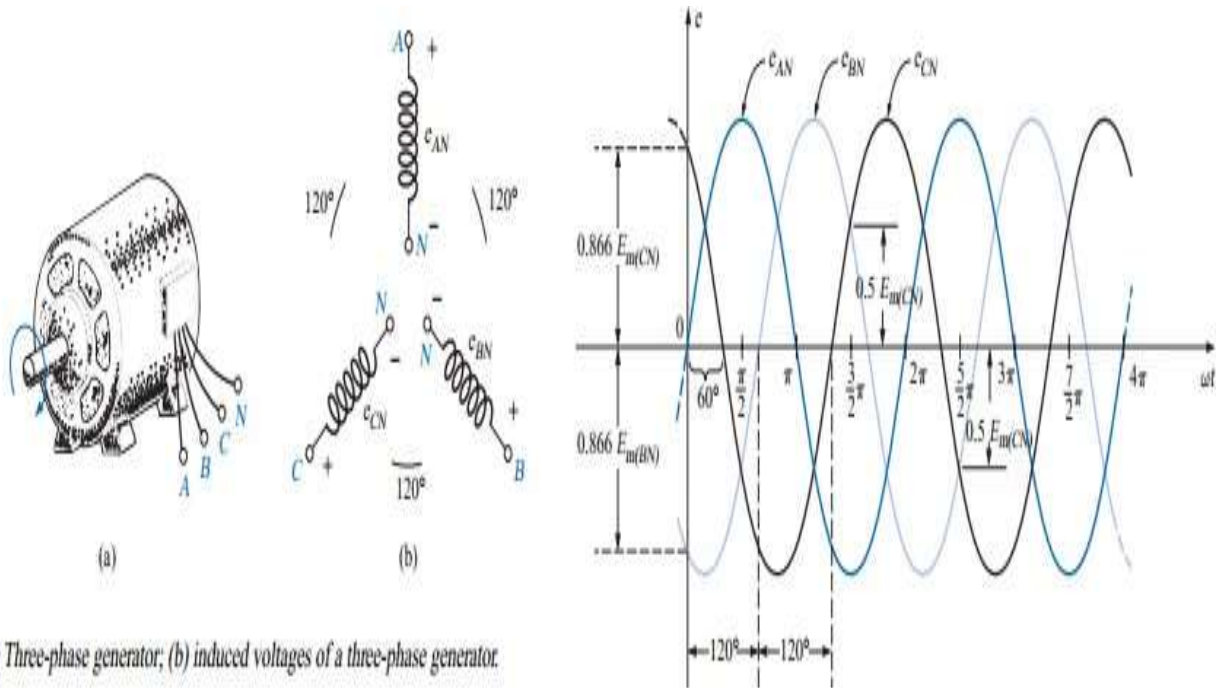
$$Q_o = \frac{1}{\omega_o CR} = \frac{1}{2\pi \times 142.3 \times 25 \times 10^{-6} \times 10} = 4.47$$

$$Q_o = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{50 \times 10^{-3}}{25 \times 10^{-6}}} = 4.47$$

Three phase system

Three phase generator:

Large scale generation of power is achieved by generating three phase e.m.f. using three separate windings insulated from each other. They are placed on the rotor of the alternator. The windings are displaced at angle of 120° with each other as shown in figure. When the rotor is rotated e.m.f. will be induced in the three coils (phases).



The instantaneous values of the three e.m.f. will be given by curves of figure

$$e_{AN} = E_{m(AN)} \sin \omega t$$

$$e_{BN} = E_{m(BN)} \sin(\omega t - 120^\circ)$$

$$e_{CN} = E_{m(CN)} \sin(\omega t - 240^\circ) = E_{m(CN)} \sin(\omega t + 120^\circ)$$

$$E_{AN} = 0.707 E_{m(AN)}$$

$$E_{BN} = 0.707 E_{m(BN)}$$

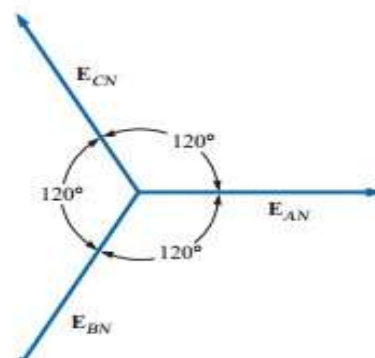
$$E_{CN} = 0.707 E_{m(CN)}$$

$$\mathbf{E}_{AN} = E_{AN} \angle 0^\circ$$

$$\mathbf{E}_{BN} = E_{BN} \angle -120^\circ$$

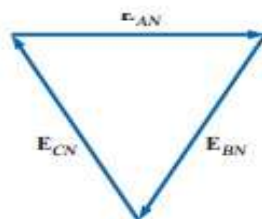
$$\mathbf{E}_{CN} = E_{CN} \angle +120^\circ$$

Phase voltages of a

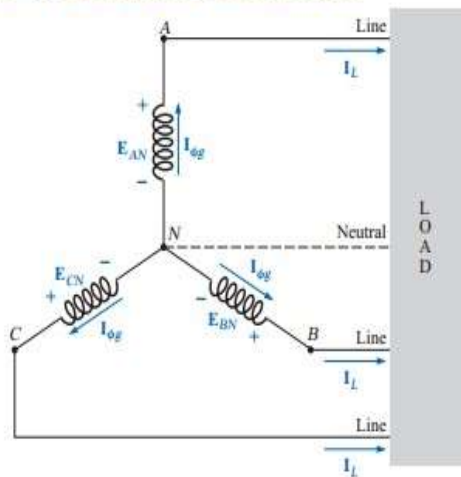


Phasor diagram for the phase voltages of a three-phase generator.

$$\mathbf{E}_{AN} + \mathbf{E}_{BN} + \mathbf{E}_{CN} = 0$$

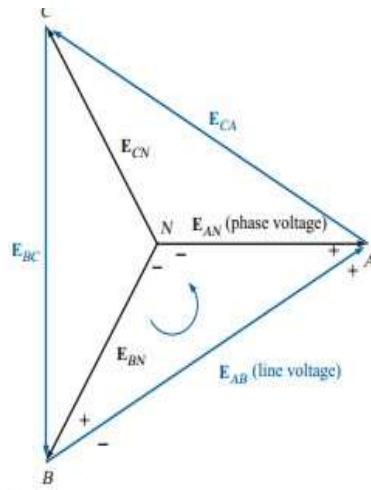


THE Y-CONNECTED GENERATOR



line current = phase current

$$I_L = I_{\phi}$$



Line and phase voltages of the Y-connected three-phase generator.

$$E_{AB} = \sqrt{3}E_{AN}$$

$$E_{AB} = E_{AB} \angle 30^\circ = \sqrt{3}E_{AN} \angle 30^\circ$$

$$E_{CA} = \sqrt{3}E_{CN} \angle 150^\circ$$

$$E_{BC} = \sqrt{3}E_{BN} \angle 270^\circ$$

$$\text{phase voltage} = E_{\phi}$$

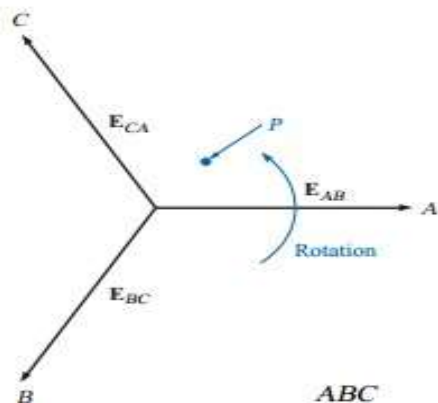
$$E_{AB} = E_{CA} = E_{BC} = \text{Line voltage} = E_L$$

$$E_L = \sqrt{3}E_{\phi}$$

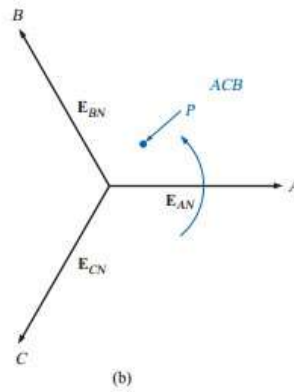
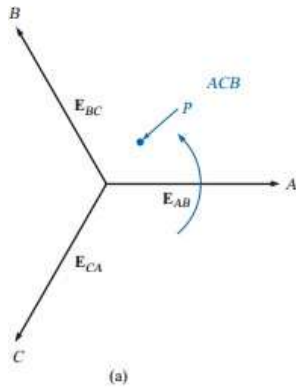
Numbering of phase

The three phases may be numbered 1; 2; 3 or a; b; c or as is customary they may be given three colour. The colours used commercially are red; yellow and blue. In this case the sequence is RYB.

PHASE SEQUENCE (Y-CONNECTED GENERATOR)



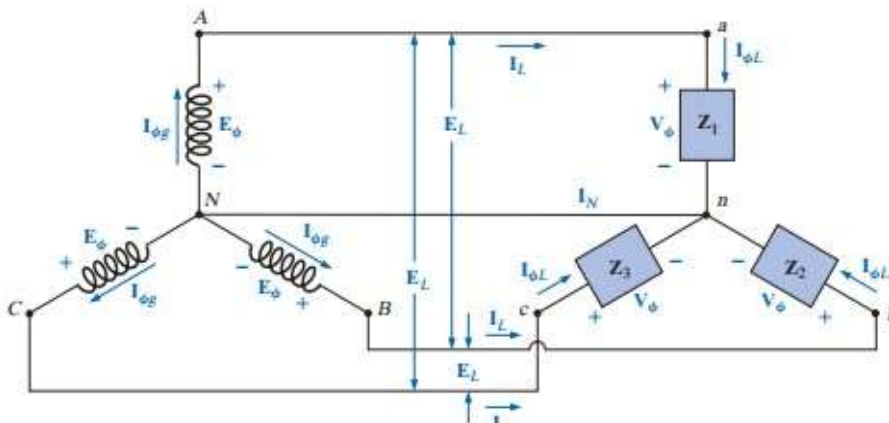
$$\text{Line voltages} \begin{cases} E_{AB} = E_{AB} \angle 0^\circ & (\text{reference}) \\ E_{CA} = E_{CA} \angle -120^\circ \\ E_{BC} = E_{BC} \angle +120^\circ \end{cases}$$



$$\text{Phase voltages} \begin{cases} E_{AN} = E_{AN} \angle 0^\circ & (\text{reference}) \\ E_{CN} = E_{CN} \angle -120^\circ \\ E_{BN} = E_{BN} \angle +120^\circ \end{cases}$$

Drawing the phasor diagram from the phase sequence.

THE Y-CONNECTED GENERATOR WITH A Y-CONNECTED LOAD



$$Z_1 = Z_2 = Z_3$$

$$I_{\phi g} = I_L = I_{\phi L}$$

$$V_\phi = E_\phi$$

$$E_L = \sqrt{3} V_\phi$$

Y-connected generator with a Y-connected load.

If the load is balanced, the **neutral connection** can be removed without affecting the circuit

EXAMPLE

The phase sequence of the Y-connected generator is ABC .

- Find the phase angles θ_2 and θ_3 .
- Find the magnitude of the line voltages.
- Find the line currents.
- Verify that, since the load is balanced, $I_N = 0$.

Solutions:

- For an ABC phase sequence,

$$\theta_2 = -120^\circ \quad \text{and} \quad \theta_3 = +120^\circ$$

- $E_L = \sqrt{3}E_\phi = (1.73)(120 \text{ V}) = 208 \text{ V}$. Therefore,

$$E_{AB} = E_{BC} = E_{CA} = 208 \text{ V}$$

- $V_\phi = E_\phi$. Therefore,

$$\begin{aligned} V_{an} &= E_{AN} & V_{bn} &= E_{BN} & V_{cn} &= E_{CN} \\ I_{\phi L} = I_{an} &= \frac{V_{an}}{Z_{an}} = \frac{120 \text{ V} \angle 0^\circ}{3 \Omega + j4 \Omega} = \frac{120 \text{ V} \angle 0^\circ}{5 \Omega \angle 53.13^\circ} \\ &= 24 \text{ A} \angle -53.13^\circ \end{aligned}$$

$$I_{bn} = \frac{V_{bn}}{Z_{bn}} = \frac{120 \text{ V} \angle -120^\circ}{5 \Omega \angle 53.13^\circ} = 24 \text{ A} \angle -173.13^\circ$$

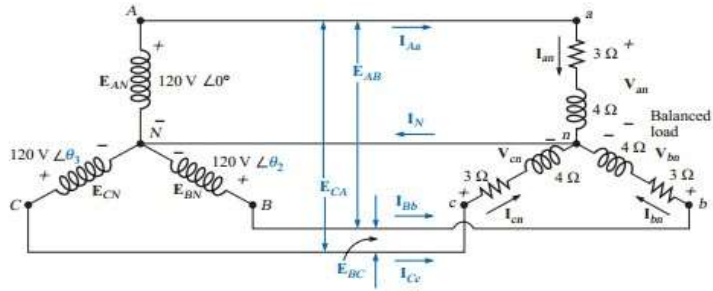
$$I_{cn} = \frac{V_{cn}}{Z_{cn}} = \frac{120 \text{ V} \angle +120^\circ}{5 \Omega \angle 53.13^\circ} = 24 \text{ A} \angle 66.87^\circ$$

and, since $I_L = I_{\phi L}$,

$$I_{Aa} = I_{an} = 24 \text{ A} \angle -53.13^\circ$$

$$I_{Bb} = I_{bn} = 24 \text{ A} \angle -173.13^\circ$$

$$I_{Cc} = I_{cn} = 24 \text{ A} \angle 66.87^\circ$$



- Applying Kirchhoff's current law, we have

$$I_N = I_{Aa} + I_{Bb} + I_{Cc}$$

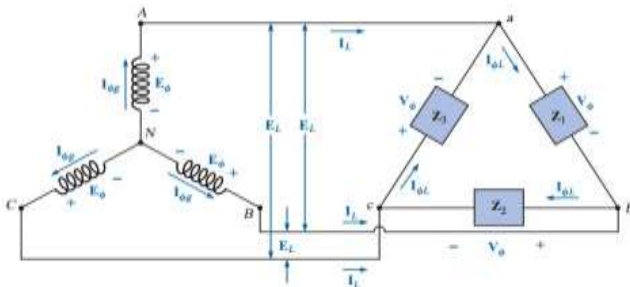
In rectangular form,

$$\begin{aligned} I_{Aa} &= 24 \text{ A} \angle -53.13^\circ = 14.40 \text{ A} - j19.20 \text{ A} \\ I_{Bb} &= 24 \text{ A} \angle -173.13^\circ = -22.83 \text{ A} - j2.87 \text{ A} \\ I_{Cc} &= 24 \text{ A} \angle 66.87^\circ = 9.43 \text{ A} + j22.07 \text{ A} \\ \Sigma(I_{Aa} + I_{Bb} + I_{Cc}) &= 0 + j0 \end{aligned}$$

and I_N is in fact equal to **zero**, as required for a balanced load.

THE Y-Δ SYSTEM

There is no neutral connection for the Y-Δ system variation in the impedance of a phase that produces an unbalanced system will simply vary the line and phase currents of the system.



Y-connected generator with a Δ-connected load.

For a balanced load,

$$Z_1 = Z_2 = Z_3$$

The voltage across each phase of the load is equal to the line voltage of the generator for a balanced or an unbalanced load:

$$V_\phi = E_L$$

$$I_L = \sqrt{3}I_\phi$$

EXAMPLE

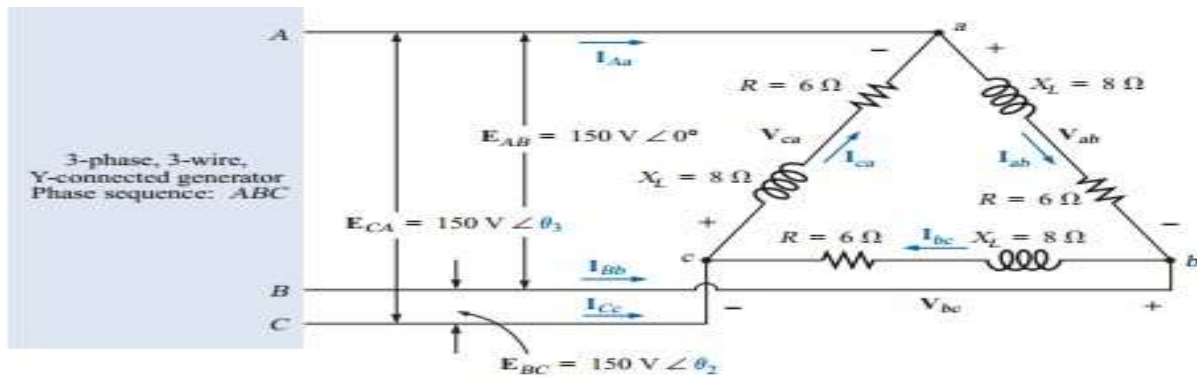
For the three-phase system of Fig.

- Find the phase angles θ_2 and θ_3 .
- Find the current in each phase of the load.
- Find the magnitude of the line currents.

Solutions:

- For an ABC sequence,

$$\theta_2 = -120^\circ \quad \text{and} \quad \theta_3 = +120^\circ$$



b. $V_\phi = E_L$. Therefore,

$$V_{ab} = E_{AB} \quad V_{ca} = E_{CA} \quad V_{bc} = E_{BC}$$

The phase currents are

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{150 \text{ V} \angle 0^\circ}{6 \Omega + j8 \Omega} = \frac{150 \text{ V} \angle 0^\circ}{10 \Omega \angle 53.13^\circ} = 15 \text{ A} \angle -53.13^\circ$$

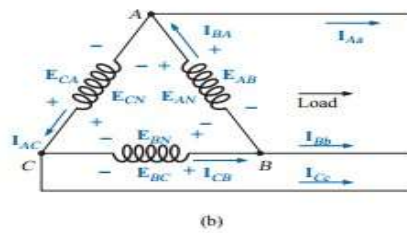
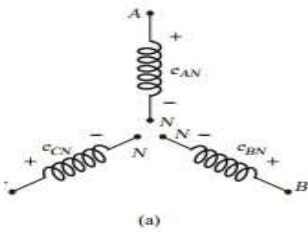
$$I_{bc} = \frac{V_{bc}}{Z_{bc}} = \frac{150 \text{ V} \angle -120^\circ}{10 \Omega \angle 53.13^\circ} = 15 \text{ A} \angle -173.13^\circ$$

$$I_{ca} = \frac{V_{ca}}{Z_{ca}} = \frac{150 \text{ V} \angle +120^\circ}{10 \Omega \angle 53.13^\circ} = 15 \text{ A} \angle 66.87^\circ$$

c. $I_L = \sqrt{3}I_\phi = (1.73)(15 \text{ A}) = 25.95 \text{ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = 25.95 \text{ A}$$

THE Δ -CONNECTED GENERATOR

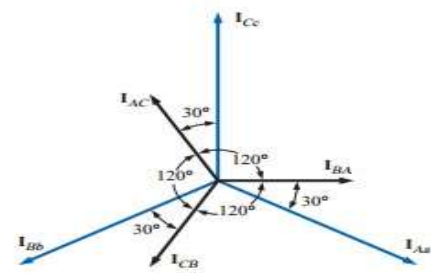


Δ -connected generator.

$$\left. \begin{array}{l} E_{AB} = E_{AN} \text{ and } e_{AN} = \sqrt{2}E_{AN}\sin \omega t \\ E_{BC} = E_{BN} \text{ and } e_{BN} = \sqrt{2}E_{BN}\sin(\omega t - 120^\circ) \\ E_{CA} = E_{CN} \text{ and } e_{CN} = \sqrt{2}E_{CN}\sin(\omega t + 120^\circ) \end{array} \right\} \begin{array}{l} \text{Phase} \\ \text{sequence} \\ \text{ABC} \end{array}$$

$$E_L = E_{\phi g}$$

$$I_L = \sqrt{3}I_{\phi g}$$

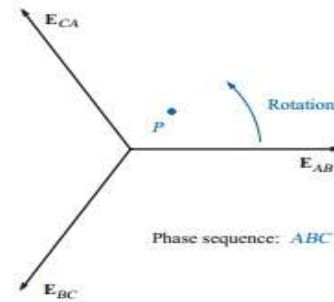


The phasor diagram of the currents of a three-phase, Δ -connected generator.

PHASE SEQUENCE (Δ -CONNECTED GENERATOR)

$$\begin{aligned} \mathbf{E}_{AB} &= E_{AB} \angle 0^\circ \\ \mathbf{E}_{BC} &= E_{BC} \angle -120^\circ \\ \mathbf{E}_{CA} &= E_{CA} \angle 120^\circ \end{aligned}$$

Determining the phase sequence for a Δ -connected, three-phase generator.

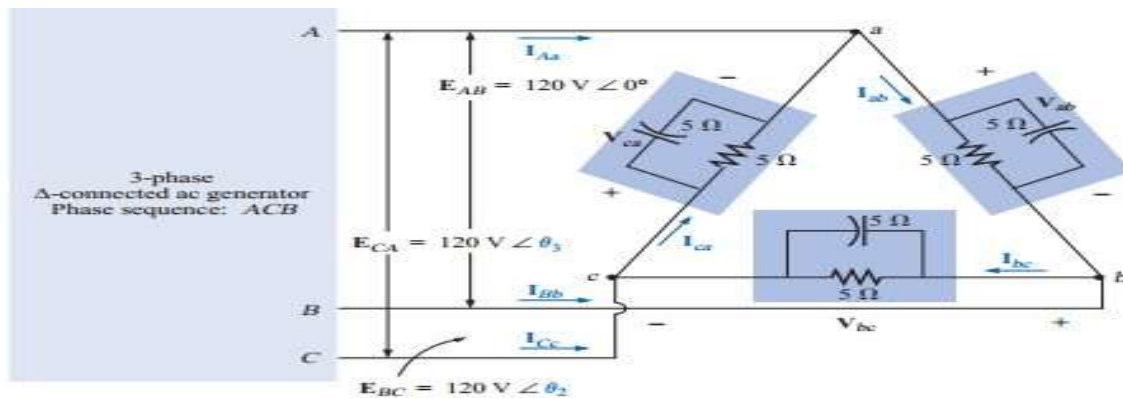


THE Δ - Δ , Δ -Y THREE-PHASE SYSTEMS

The basic equations necessary to analyze either of the two systems (Δ - Δ , Δ -Y) have been presented at least once in this chapter. We will therefore proceed directly to two descriptive examples, one with a Δ -connected load and one with a Y-connected load.

EXAMPLE For the Δ - Δ system shown in Fig.

- Find the phase angles θ_2 and θ_3 for the specified phase sequence.
- Find the current in each phase of the load.
- Find the magnitude of the line currents.



Solutions:

- For an ACB phase sequence,

$$\theta_2 = 120^\circ \quad \text{and} \quad \theta_3 = -120^\circ$$

- $\mathbf{V}_\phi = \mathbf{E}_L$. Therefore,

$$\mathbf{V}_{ab} = \mathbf{E}_{AB} \quad \mathbf{V}_{ca} = \mathbf{E}_{CA} \quad \mathbf{V}_{bc} = \mathbf{E}_{BC}$$

The phase currents are

$$\begin{aligned} \mathbf{I}_{ab} &= \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{120 \text{ V} \angle 0^\circ}{(5 \Omega \angle 0^\circ)(5 \Omega \angle -90^\circ)} = \frac{120 \text{ V} \angle 0^\circ}{25 \Omega \angle -90^\circ} \\ &= \frac{120 \text{ V} \angle 0^\circ}{3.54 \Omega \angle -45^\circ} = 33.9 \text{ A} \angle 45^\circ \\ \mathbf{I}_{bc} &= \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{bc}} = \frac{120 \text{ V} \angle 120^\circ}{3.54 \Omega \angle -45^\circ} = 33.9 \text{ A} \angle 165^\circ \\ \mathbf{I}_{ca} &= \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{ca}} = \frac{120 \text{ V} \angle -120^\circ}{3.54 \Omega \angle -45^\circ} = 33.9 \text{ A} \angle -75^\circ \end{aligned}$$

- $I_L = \sqrt{3}I_\phi = (1.73)(34 \text{ A}) = 58.82 \text{ A}$. Therefore,

$$I_{Aa} = I_{Bb} = I_{Cc} = 58.82 \text{ A}$$

EXAMPLE

For the Δ -Y system shown in Fig

Find the voltage across each phase of the load.

Find the magnitude of the line voltages.

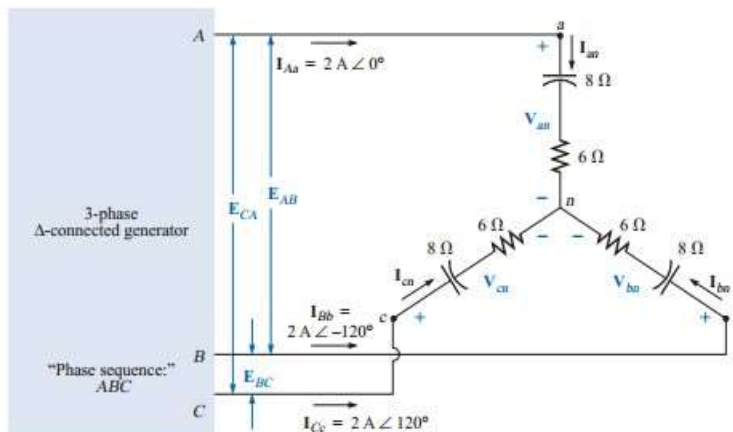
Solutions:

a. $I_{\phi L} = I_L$. Therefore,

$$I_{an} = I_{Aa} = 2 \text{ A } \angle 0^\circ$$

$$I_{bn} = I_{Bb} = 2 \text{ A } \angle -120^\circ$$

$$I_{cn} = I_{Cc} = 2 \text{ A } \angle 120^\circ$$



The phase voltages are

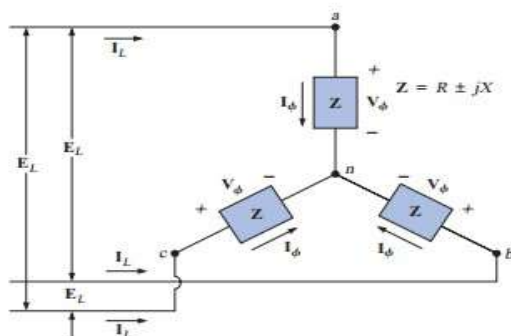
$$V_{an} = I_{an} Z_{an} = (2 \text{ A } \angle 0^\circ)(10 \Omega \angle -53.13^\circ) = 20 \text{ V } \angle -53.13^\circ$$

$$V_{bn} = I_{bn} Z_{bn} = (2 \text{ A } \angle -120^\circ)(10 \Omega \angle -53.13^\circ) = 20 \text{ V } \angle -173.13^\circ$$

$$V_{cn} = I_{cn} Z_{cn} = (2 \text{ A } \angle 120^\circ)(10 \Omega \angle -53.13^\circ) = 20 \text{ V } \angle 66.87^\circ$$

b. $E_L = \sqrt{3} V_\phi = (1.73)(20 \text{ V}) = 34.6 \text{ V}$. Therefore,

$$E_{BA} = E_{CB} = E_{AC} = 34.6 \text{ V}$$

POWER**Y-Connected Balanced Load**

Y-connected balanced load.

Average Power The average power delivered to each phase can be determined

$$P_\phi = V_\phi I_\phi \cos \theta_{V_\phi I_\phi} = I_\phi^2 R_\phi = \frac{V_R^2}{R_\phi} \quad (\text{watts, W})$$

where $\theta_{V_\phi I_\phi}$ indicates that θ is the phase angle between V_ϕ and I_ϕ . The total power to the balanced load is

$$P_T = 3P_\phi \quad (\text{W})$$

$$\text{or, since } V_\phi = \frac{E_L}{\sqrt{3}} \text{ and } I_\phi = I_L$$

$$\text{then } P_T = 3 \frac{E_L}{\sqrt{3}} I_L \cos \theta_{V_\phi I_\phi}$$

$$\text{Therefore, } P_T = \sqrt{3} E_L I_L \cos \theta_{V_\phi I_\phi} = 3 I_L^2 R_\phi \quad (\text{W})$$

Reactive Power The reactive power of each phase (in volt-amperes reactive) is

$$Q_\phi = V_\phi I_\phi \sin \theta_{V_\phi I_\phi} = I_\phi^2 X_\phi = \frac{V_X^2}{X_\phi} \quad (\text{VAR})$$

$$\text{The total reactive power of the load is } Q_T = 3Q_\phi \quad (\text{VAR})$$

$$Q_T = \sqrt{3} E_L I_L \sin \theta_{V_\phi I_\phi} = 3 I_L^2 X_\phi \quad (\text{VAR})$$

$$\text{Apparent Power} \quad \text{The apparent power of each phase is } S_\phi = V_\phi I_\phi \quad (\text{VA})$$

$$\text{The total apparent power of the load is } S_T = 3S_\phi \quad (\text{VA})$$

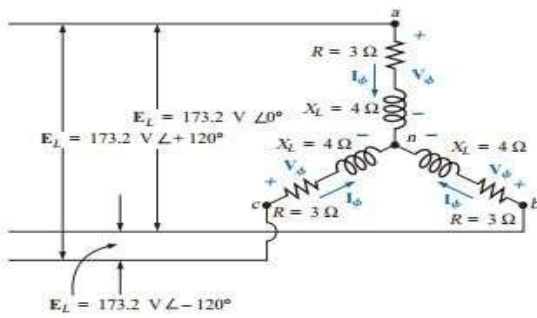
$$\text{or } S_T = \sqrt{3} E_L I_L \quad (\text{VA})$$

Power Factor The power factor of the system is given by

$$F_p = \frac{P_T}{S_T} = \cos \theta_{V_\phi I_\phi} \quad (\text{leading or lagging})$$

EXAMPLE

For the Y-connected load of Fig.



- Find the average power to each phase and the total load.
- Determine the reactive power to each phase and the total reactive power.
- Find the apparent power to each phase and the total apparent power.
- Find the power factor of the load.

Solutions:

- a. The average power is

$$P_{\phi} = V_{\phi} I_{\phi} \cos \theta_{V_{\phi}} = (100 \text{ V})(20 \text{ A}) \cos 53.13^{\circ} = (2000)(0.6) = \mathbf{1200 \text{ W}}$$

$$P_{\phi} = I_{\phi}^2 R_{\phi} = (20 \text{ A})^2 (3 \Omega) = (400)(3) = \mathbf{1200 \text{ W}}$$

$$P_{\phi} = \frac{V_R^2}{R_{\phi}} = \frac{(60 \text{ V})^2}{3 \Omega} = \frac{3600}{3} = \mathbf{1200 \text{ W}}$$

$$P_T = 3P_{\phi} = (3)(1200 \text{ W}) = \mathbf{3600 \text{ W}}$$

or

$$P_T = \sqrt{3} E_L I_L \cos \theta_{V_{\phi}} = (1.732)(173.2 \text{ V})(20 \text{ A})(0.6) = \mathbf{3600 \text{ W}}$$

- b. The reactive power is

$$Q_{\phi} = V_{\phi} I_{\phi} \sin \theta_{V_{\phi}} = (100 \text{ V})(20 \text{ A}) \sin 53.13^{\circ} = (2000)(0.8) = \mathbf{1600 \text{ VAR}}$$

$$\text{or } Q_{\phi} = I_{\phi}^2 X_{\phi} = (20 \text{ A})^2 (4 \Omega) = (400)(4) = \mathbf{1600 \text{ VAR}}$$

$$Q_T = 3Q_{\phi} = (3)(1600 \text{ VAR}) = \mathbf{4800 \text{ VAR}}$$

$$\text{or } Q_T = \sqrt{3} E_L I_L \sin \theta_{V_{\phi}} = (1.732)(173.2 \text{ V})(20 \text{ A})(0.8) = \mathbf{4800 \text{ VAR}}$$

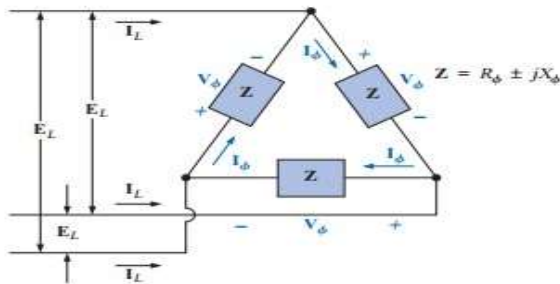
- c. The apparent power is $S_{\phi} = V_{\phi} I_{\phi} = (100 \text{ V})(20 \text{ A}) = \mathbf{2000 \text{ VA}}$
 $S_T = 3S_{\phi} = (3)(2000 \text{ VA}) = \mathbf{6000 \text{ VA}}$

$$\text{or } S_T = \sqrt{3} E_L I_L = (1.732)(173.2 \text{ V})(20 \text{ A}) = \mathbf{6000 \text{ VA}}$$

- d. The power factor is

$$F_p = \frac{P_T}{S_T} = \frac{3600 \text{ W}}{6000 \text{ VA}} = \mathbf{0.6 \text{ lagging}}$$

Δ-Connected Balanced Load



Δ-connected balanced load.

Average Power

$$P_{\phi} = V_{\phi} I_{\phi} \cos \theta_{V_{\phi}} = I_{\phi}^2 R_{\phi} = \frac{V_R^2}{R_{\phi}} \quad (\text{W})$$

$$P_T = 3P_{\phi}$$

Reactive Power

$$Q_{\phi} = V_{\phi} I_{\phi} \sin \theta_{V_{\phi}} = I_{\phi}^2 X_{\phi} = \frac{V_X^2}{X_{\phi}} \quad (\text{VAR})$$

$$Q_T = 3Q_{\phi} \quad (\text{VAR})$$

Apparent Power

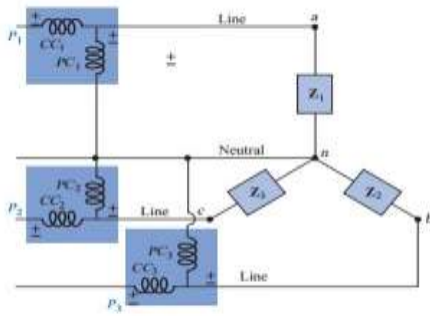
$$S_{\phi} = V_{\phi} I_{\phi} \quad (\text{VA})$$

$$S_T = 3S_{\phi} = \sqrt{3} E_L I_L \quad (\text{VA})$$

Power Factor

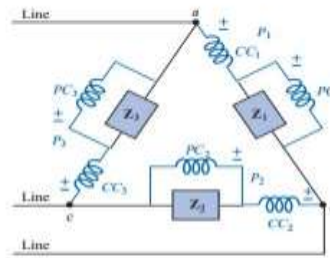
$$F_p = \frac{P_T}{S_T}$$

THE THREE-WATTMETER METHOD



Three-wattmeter method for a Y-connected load.

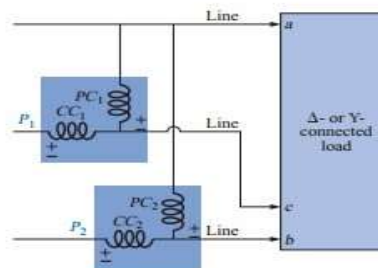
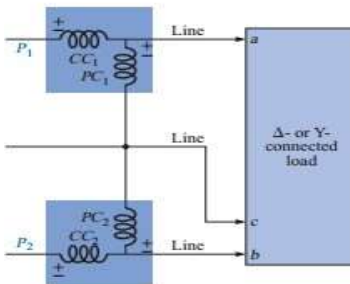
$$P_{TY} = P_1 + P_2 + P_3$$



Three-wattmeter method for a Δ-connected load.

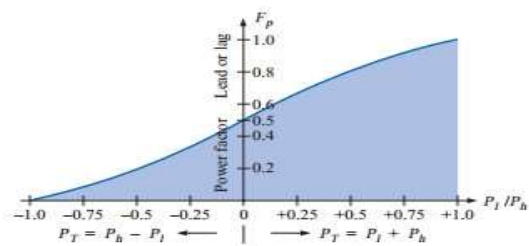
$$P_{T\Delta} = P_1 + P_2 + P_3$$

THE TWO-WATTMETER METHOD



$$P_T = P_h \pm P_l = \sqrt{3} E_L I_L \cos \theta_{\phi}^V$$

$$F_p = \cos \theta_{\phi}^V = \frac{P_h \pm P_l}{\sqrt{3} E_L I_L}$$



Determining whether the readings obtained using the two-wattmeter method should be added or subtracted.