Al-Furat Al-Awsat Technical University

Najaf Technical Institute

Aeronautic Technical Department

Subject

Engineering Mechanics

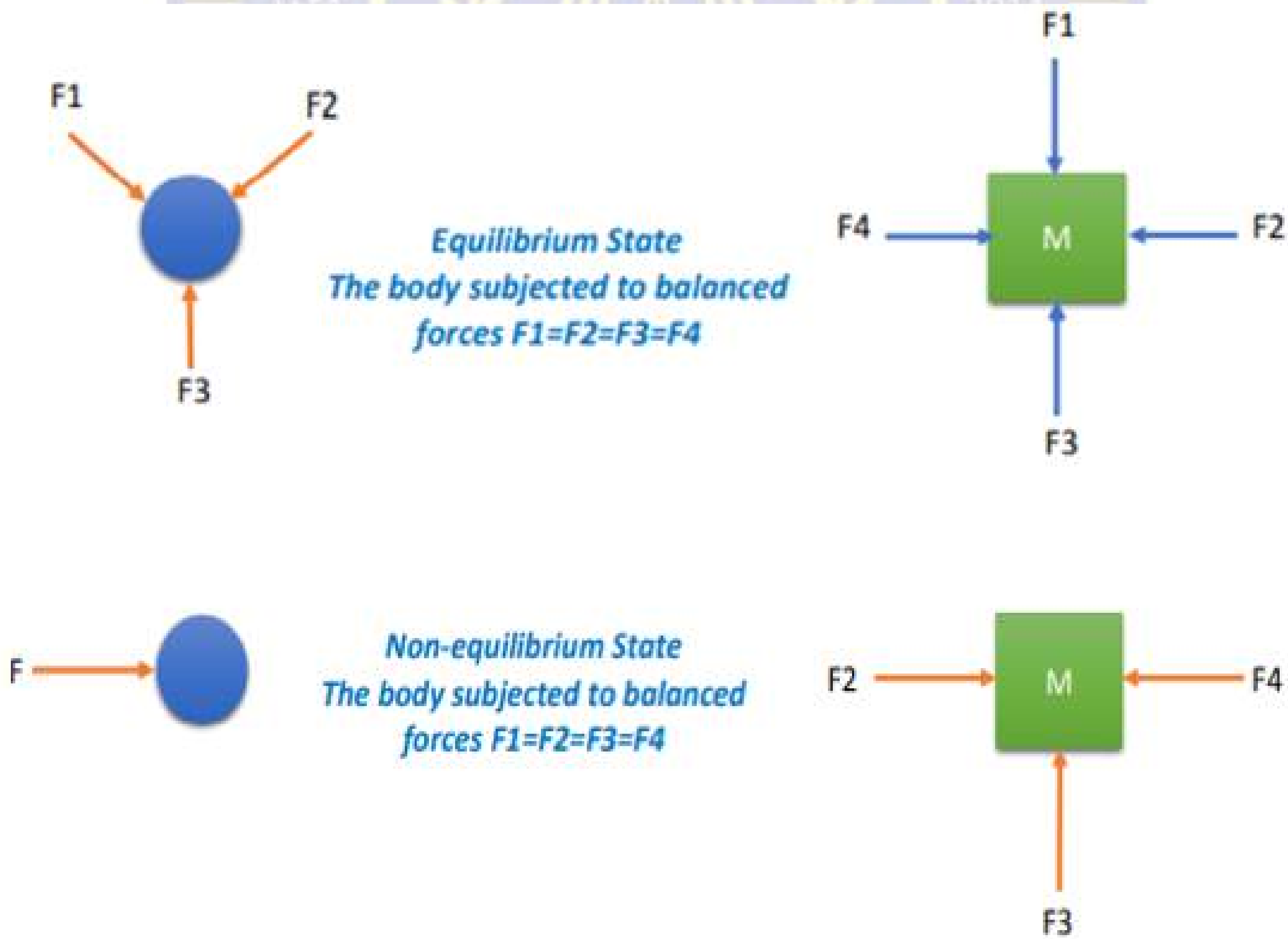
1st stage

Lecture-1-

Static, fundamental concept. Force, Scalars and Vectors, Units, Force polygon

Asst Lect. Hayder Salim

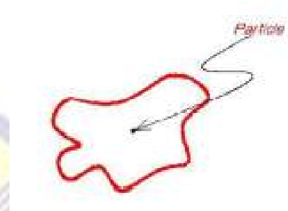
- **Engineering Mechanics:** is that branch of the physical sciences study the behavior of bodies subjected to the action of forces.
- **The aim:** study the effects of the forces on bodies statics and dynamics states, and also study the stresses and strain occurs due to the loads.
- **The subject of mechanics is divided into two parts:**
  1. **Static:** the study of objects in equilibrium (objects either at rest or moving with constant velocity.
  2. **Dynamic:** the study of objects with accelerated motion



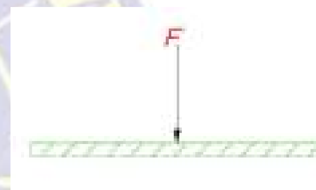
Fundamental Concept

Basic Quantities. The following four quantities are used throughout mechanics.

Particles: a particle has a mass but a size can be neglect (the geometry of body is ignored). A particle is often represented by point in space.



Rigid body: a rigid body has a mass and a size (shape) but it is assumed that any changes in shape can be neglected



Length

is defined, one can then use it to define distances and geometric

Time

Time is conceived as a succession of events. Although the principles of statics are time independent, this quantity plays an important role in the study of dynamics.

Mass.

Mass is a measure of a quantity of matter that is used to compare the action of one body with that of another. This property manifests itself as a gravitational attraction between two bodies and provides a measure of the resistance of matter to a change in velocity.

Force

force is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated.

scalar: quantity is one which has only a magnitude, such as mass, volume, etc.

Vector: quantity is one which involves both magnitude and direction so that it can be represented by a directed line segment, such as force, velocity and acceleration.

Basic units

a- International System units (SI)

1- Length (m)

2- Time (sec)

3- Mass (kg)

- **Velocity (V)** = distance change (length)/ time= m/s
- **Acceleration (a)** = change in velocity/time= (m/s)/s=m/s^2
- **Force** = mass (m)* acceleration (a) =kg*m/s^2= Newton(N)

b- US customary units

1- Length – Foot (ft.).

2- Mass- (slug = (Ib. s^2)/ft).

3- Time- Second (s).

4- Force- Pound (Ib).

Notes:

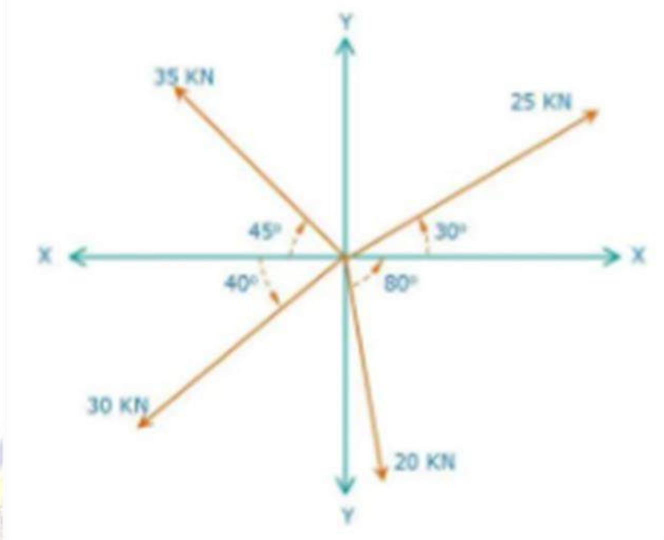
1 ft. = 0.3048m.

1 slug = 14.593 kg.

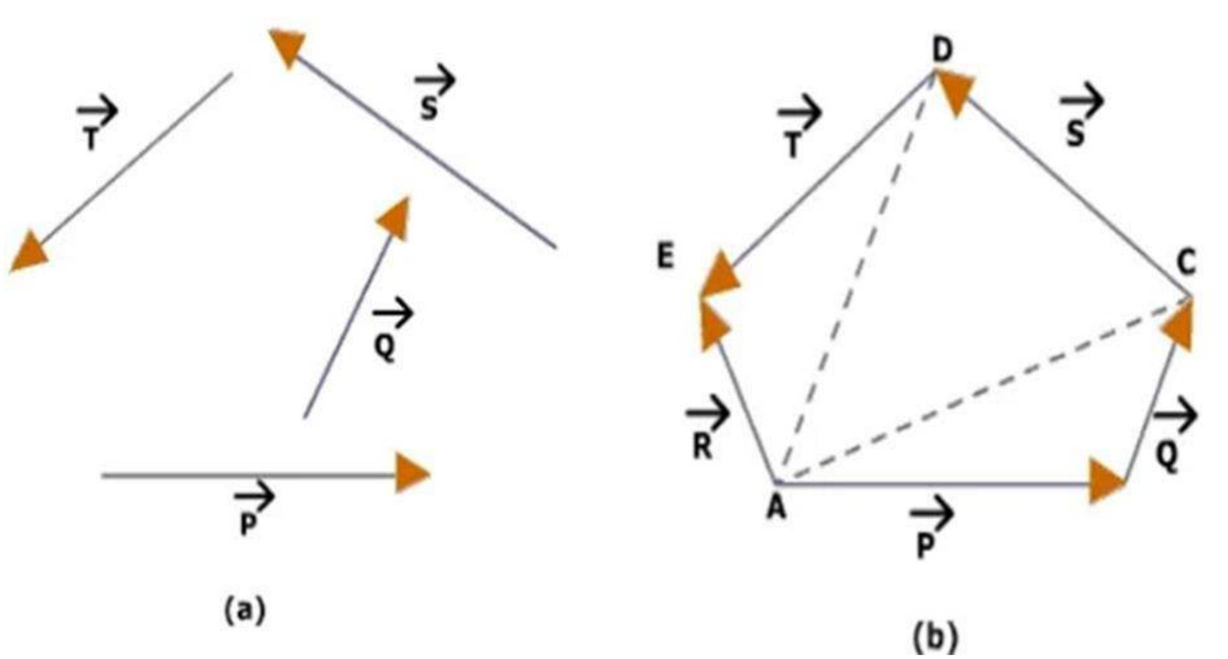
1 pound = 4.4482N.

Force polygon

Polygon Law of Forces If a number of forces acting on a point be represented by the sides of a polygon taken in order then their resultant is obtained by the closing side of the polygon taken in opposite order. The magnitude of the Resultant may be measured if drawn to scale or may also be calculated



analytical method



Graphical method

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1st stage

Lecture-2-

Analysis of Forces

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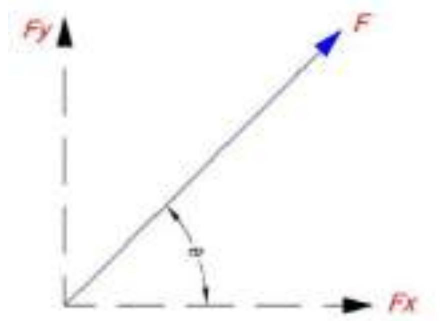
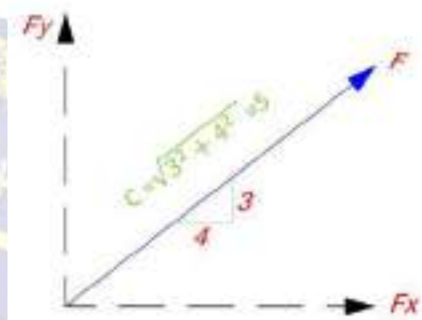
Force Analysis

For analysis inclined forces we need:

1- Slope
2- Angle of slope





$$F_x = F * cos\theta$$

$$F_y = F * sin\theta$$

$$F_x = F * \left(\frac{4}{5}\right)$$

$$F_y = F * \left(\frac{3}{5}\right)$$

Example 1 / for the figure (1) shown, find Fx and Fy?

Ans.

$F_x = F * cos\theta$

$= 130 * cos\ 30° = 112.5\ N$

$F_y = F * sin\theta$

$= 130 * \sin 30° = 65\ N$

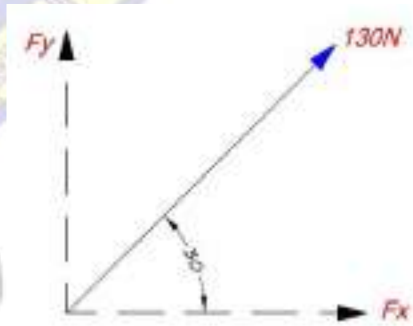


Figure (1)

Example 2 / for the figure (2) shown, find Fx and Fy?

Ans.

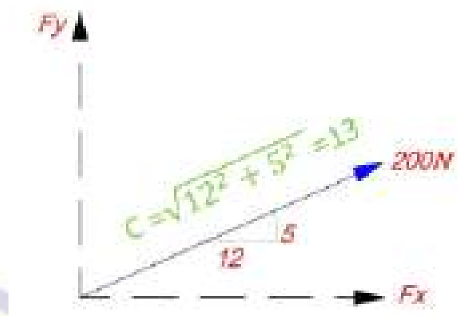


Figure (2)

$$F_x = 200 * \left(\frac{12}{13}\right) = 184 \cdot 6\ N$$

$$F_y = 200 * \left(\frac{5}{13}\right) = 76 \cdot 9\ N$$

Example 3 / for the figure (3) shown, find Fx and Fy?

Ans.

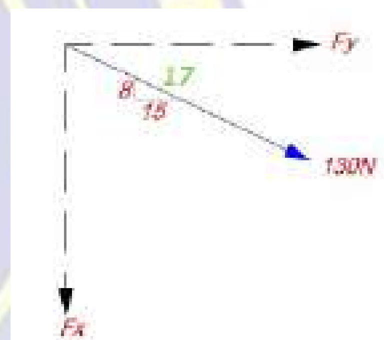


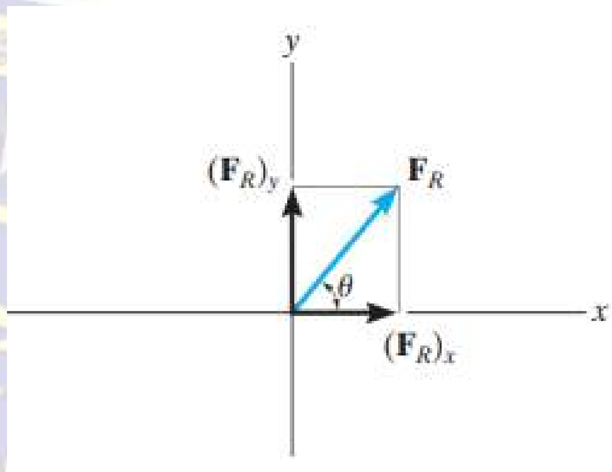
Figure 3

$$F_x = 130 * \left(\frac{15}{17}\right) = 114 \cdot 7\ N$$

$$F_y = 200 * \left(\frac{8}{17}\right) = 94 \cdot 1\ N$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition, as shown in Fig. 2–17c. From this sketch, the magnitude of FR is then found from the Pythagorean theorem; that is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

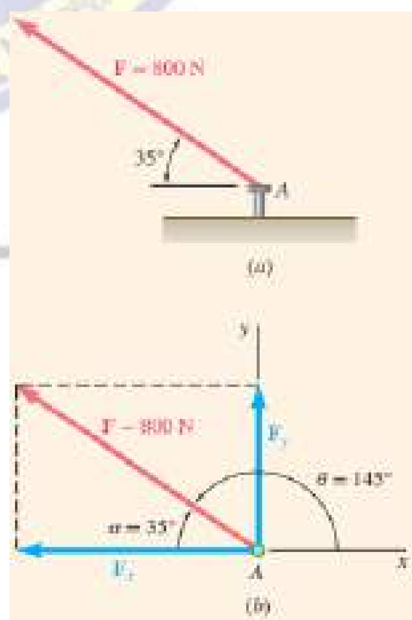


(c)

Also, the angle u, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Example 4 / for the figure (3) shown, find Fx and Fy



$F_x = -F \cos \alpha = -(800\text{ N}) \cos 35° = -655\text{ N}$

$F_y = +F \sin \alpha = +(800\text{ N}) \sin 35° = +459\text{ N}$

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Lecture-3-

RESULTANT OF CONCURRENT ,COPLANAR FORCE SYSTEM

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Resultant of concurrent

Resultant: - is a single force has the same effect of the original forces on the body. The resultant of two concurrent forces, ex. (F1&F2) can be determined by means of the parallelogram law.

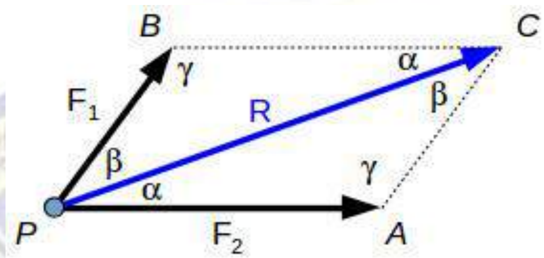


Figure 1

Tringle rule

special case of the parallelogram law, if the two vectors A and B are collinear, i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B.

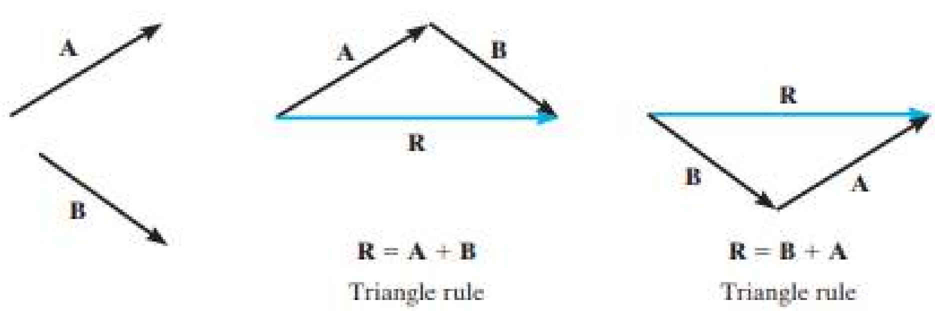


Figure 2

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

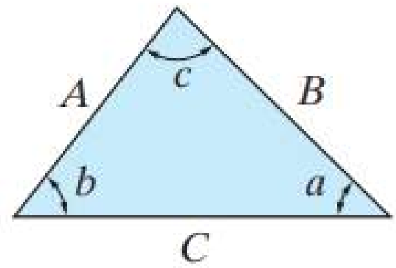


Figure 3

Example 4 / **for the figure (4) shown, use sine law to find value of θ, and use cosine law to find value of C?**

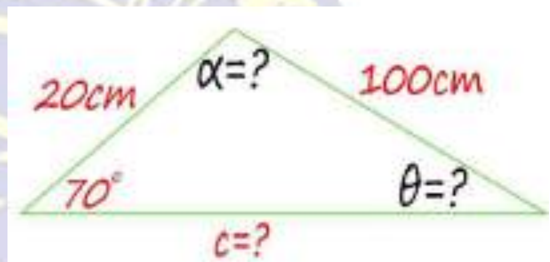


Figure 4

Ans.

$$\frac{\sin 70}{100} = \frac{\sin \theta}{20}$$

$$\sin \theta = \frac{20\, sin\, 70}{100} = 0 \cdot 1879$$

$$\theta = \; sin^{-1}\,(0 \cdot 1874) = 10 \cdot 89°$$

$$\alpha = 180 - (70 + 10 \cdot 89) = 99 \cdot 11°$$

According to cosine law:

$$C = \sqrt{100^2 \; + 20^2 \; - 2 * 100 * 20 * COS\, 99 \cdot 11}$$

$$c = 105 \cdot 03\; cm$$

Example 5 / **for the figure (5) shown, use sine law to find value of θ?**

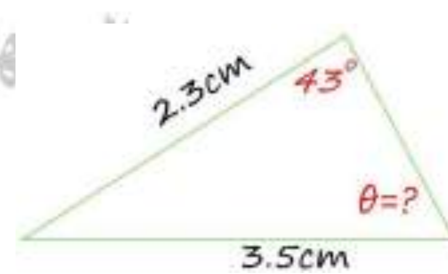
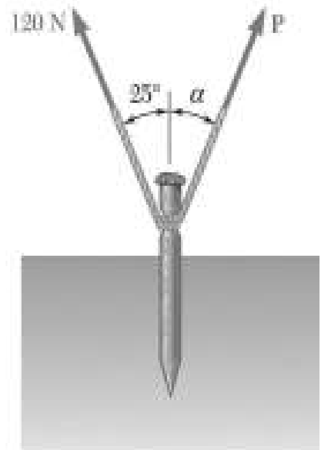


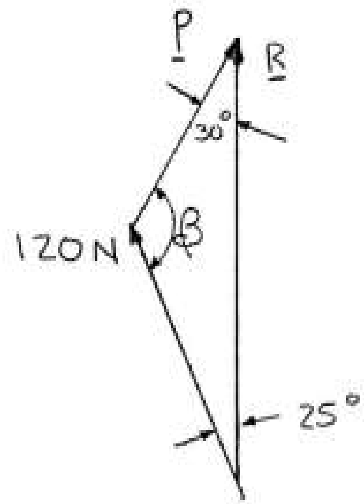
Figure 5



PROBLEM 2.5

A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$\frac{120\text{ N}}{\sin 30°} = \frac{P}{\sin 25°}$$

$P = 101.4\text{ N}$ ◀

(*b*)

$$30° + \beta + 25° = 180°$$

$$\beta = 180° - 25° - 30°$$

$$= 125°$$

$$\frac{120\text{ N}}{\sin 30°} = \frac{R}{\sin 125°}$$

$R = 196.6\text{ N}$ ◀

EXAMPLE 2.3

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

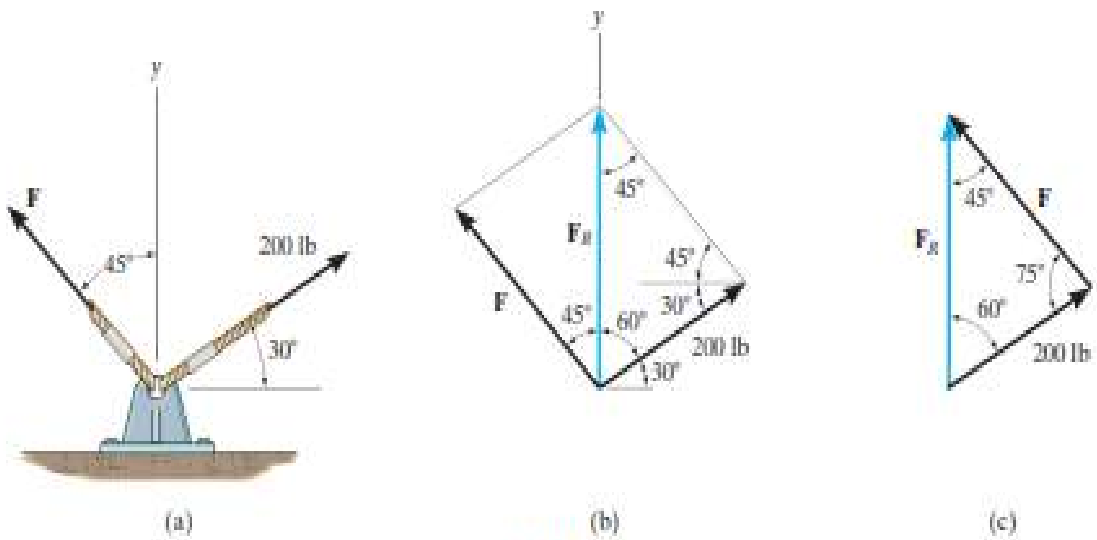


Fig. 2–13

SOLUTION

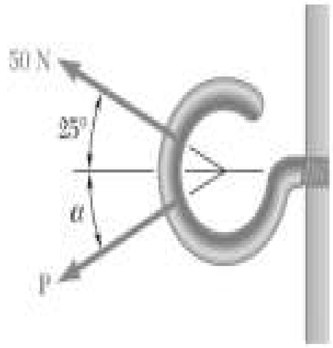
The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of $\mathbf{F}_R$ and **F** are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb} \qquad \textit{Ans.}$$

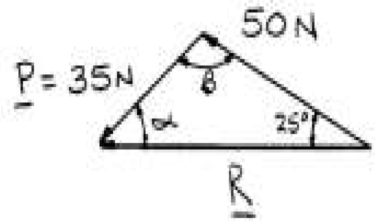


PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and law of sines:



(*a*)

$$\frac{\sin\alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$$\sin\alpha = 0.60374$$

$$\alpha = 37.138^\circ$$

$\alpha = 37.1^\circ$ ◀

(*b*)

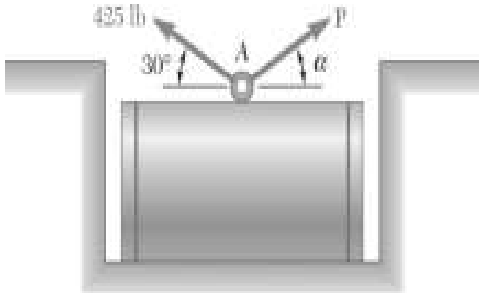
$$\alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.862^\circ$$

$$\frac{R}{\sin 117.862^\circ} = \frac{35\text{ N}}{\sin 25^\circ}$$

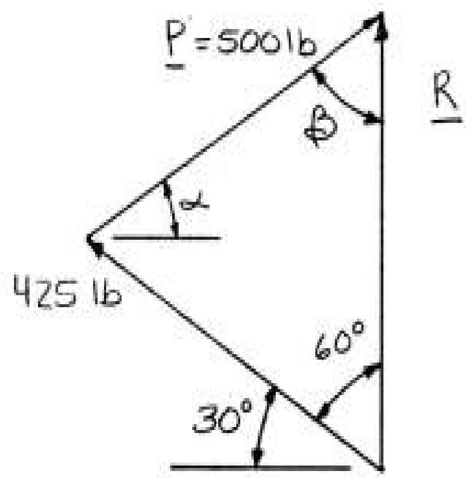
$R = 73.2\text{ N}$ ◀



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 500 lb, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$(\alpha + 30°) + 60° + \beta = 180°$$

$$\beta = 180° - (\alpha + 30°) - 60°$$

$$\beta = 90° - \alpha$$

$$\frac{\sin(90° - \alpha)}{425 \text{ lb}} = \frac{\sin 60°}{500 \text{ lb}}$$

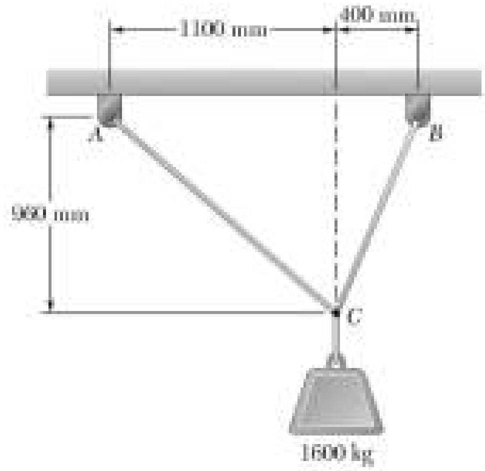
$$90° - \alpha = 47.402°$$

$\alpha = 42.6°$ ◀

(*b*)

$$\frac{R}{\sin(42.598° + 30°)} = \frac{500 \text{ lb}}{\sin 60°}$$

$R = 551$ lb ◀

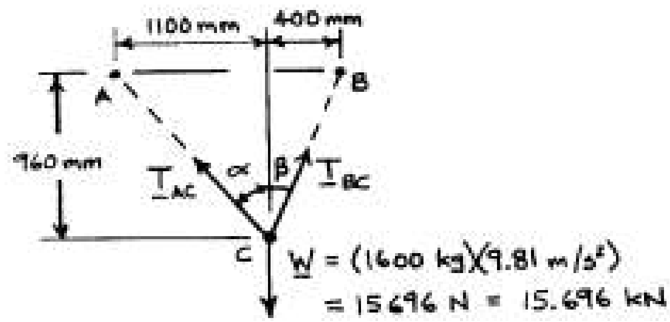


PROBLEM 2.43

Two cables are tied together at C and are loaded as shown. Determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram



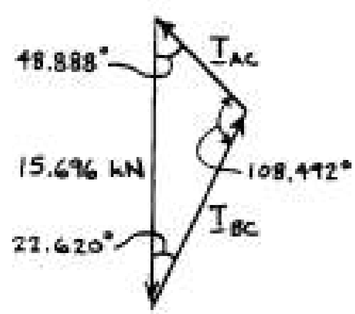
$$\tan\alpha = \frac{1100}{960}$$

$$\alpha = 48.888^\circ$$

$$\tan\beta = \frac{400}{960}$$

$$\beta = 22.620^\circ$$

Force Triangle



Law of sines:

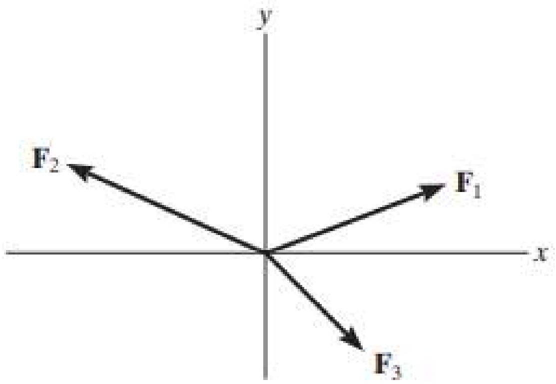
$$\frac{T_{AC}}{\sin 22.620^\circ} = \frac{T_{BC}}{\sin 48.888^\circ} = \frac{15.696\text{ kN}}{\sin 108.492^\circ}$$

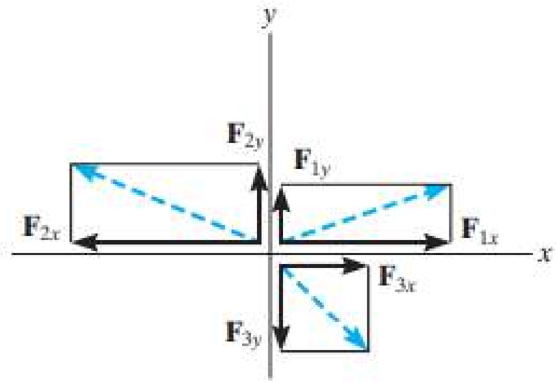
(*a*) $$T_{AC} = \frac{15.696\text{ kN}}{\sin 108.492^\circ}(\sin 22.620^\circ)$$ $T_{AC} = 6.37\text{ kN}$ ◀

(*b*) $$T_{BC} = \frac{15.696\text{ kN}}{\sin 108.492^\circ}(\sin 48.888^\circ)$$ $T_{BC} = 12.47\text{ kN}$ ◀

Coplanar force system

forces that all lie in the same plane. to determine the resultant of several coplanar forces, each force is first resolved into its x and y components, and then using algebraic sum of the x and y components of all the forces.





the positive directions of components along the x and y axes indicating with symbolic arrows, we have:

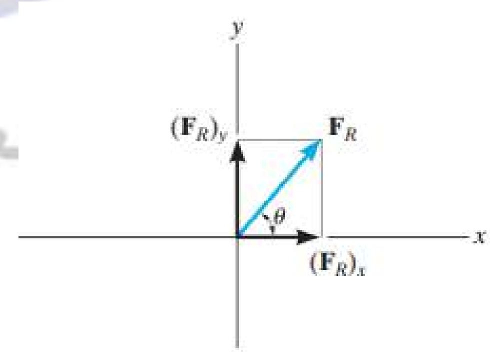
$$\xrightarrow{+} \quad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$+\uparrow \quad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

So that:

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$

the resultant force can be determined from the Pythagorean theorem:

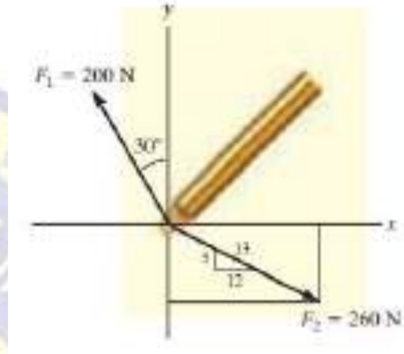
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

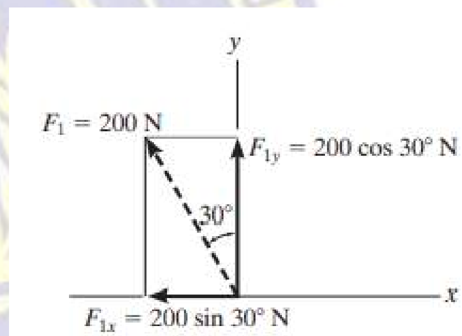


the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

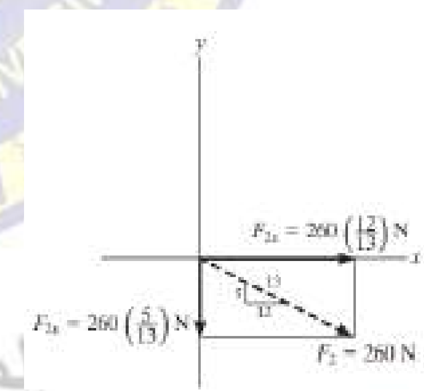
Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig.





$$F_{1x} = -200 \sin 30° \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$

$$F_{1y} = 200 \cos 30° \text{ N} = 173 \text{ N} = 173 \text{ N}\uparrow$$



$$F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$

$$F_{2x} = 240 \text{ N} = 240 \text{ N} \rightarrow$$

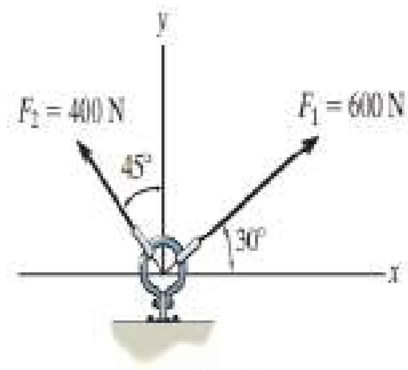
$$F_{2y} = -100 \text{ N} = 100 \text{ N}\downarrow$$

$$\sum Fx = F_{1X} + F_{2X} = -100 + 240 = 140\ N$$

$$\sum Fy = F_{1y} + F_{2y} = 173 - 100 = 73\ N$$

EXAMPLE 2.6

The link in Fig. 2–19*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



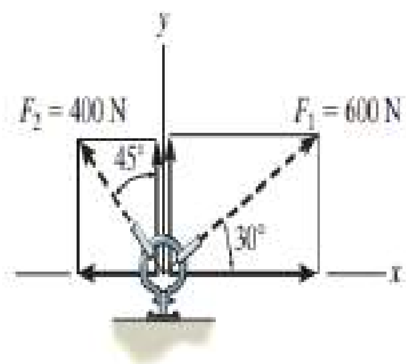
(a)

SOLUTION I

Scalar Notation. First we resolve each force into its x and y components, Fig. 2–19*b*, then we sum these components algebraically.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N}$$
$$= 236.8 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N}$$
$$= 582.8 \text{ N}\uparrow$$



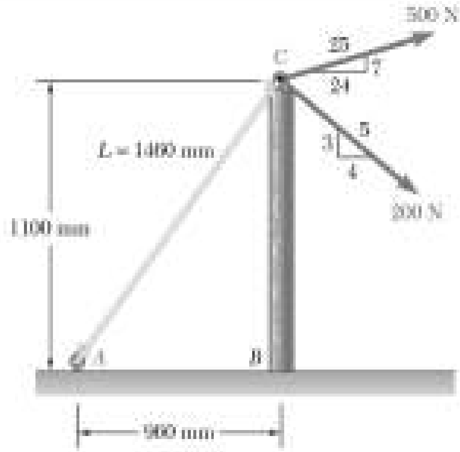
(b)

The resultant force, shown in Fig. 2–19*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2}$$
$$= 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition,

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9^\circ \qquad \textit{Ans.}$$

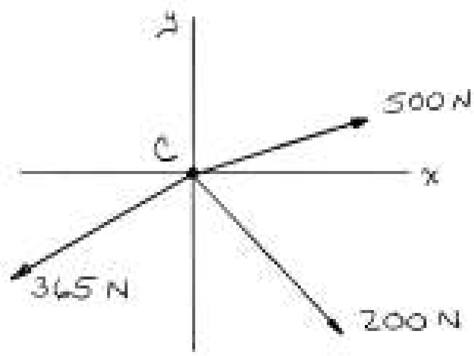


PROBLEM 2.36

Knowing that the tension in rope AC is 365 N, determine the resultant of the three forces exerted at point C of post BC.

SOLUTION

Determine force components:



Cable force AC:

$$F_x = -(365\text{ N})\frac{960}{1460} = -240\text{ N}$$

$$F_y = -(365\text{ N})\frac{1100}{1460} = -275\text{ N}$$

500-N Force:

$$F_x = (500\text{ N})\frac{24}{25} = 480\text{ N}$$

$$F_y = (500\text{ N})\frac{7}{25} = 140\text{ N}$$

200-N Force:

$$F_x = (200\text{ N})\frac{4}{5} = 160\text{ N}$$

$$F_y = -(200\text{ N})\frac{3}{5} = -120\text{ N}$$

and

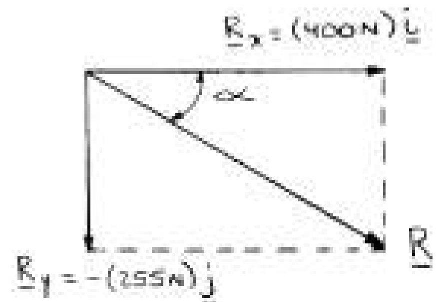
$$R_x = \Sigma F_x = -240\text{ N} + 480\text{ N} + 160\text{ N} = 400\text{ N}$$

$$R_y = \Sigma F_y = -275\text{ N} + 140\text{ N} - 120\text{ N} = -255\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(400\text{ N})^2 + (-255\text{ N})^2}$$

$$= 474.37\text{ N}$$



Further:

$$\tan\alpha = \frac{255}{400}$$

$$\alpha = 32.5^\circ$$

$\mathbf{R} = 474\text{ N}$ ⦪ 32.5° ◀

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Subject

Engineering Mechanics

1st stage

Lecture-4-

Moments

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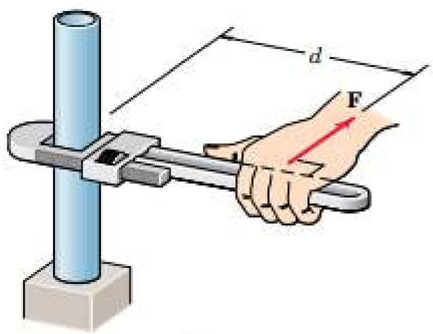
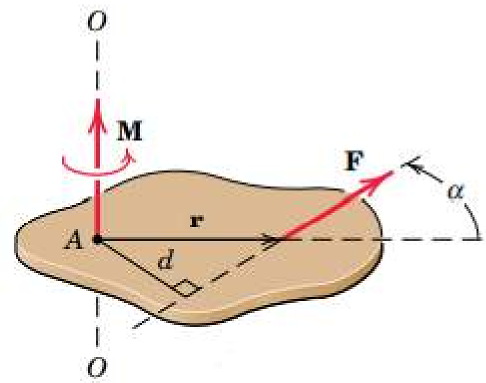
moment

force can tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the moment M of the force.





The figure shows a two-dimensional body acted on by a force F in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, which is the perpendicular distance from the axis to the line of action of the force, the magnitude of the moment is defined as:

$$M = F \times d \ (N \cdot m)$$

M= moment

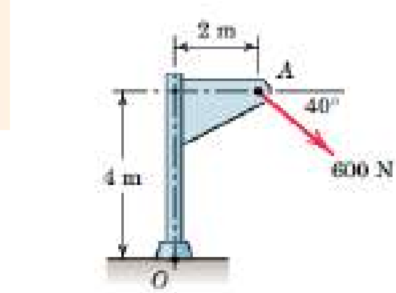
F= force that cause body to rotate (N).

d= the arm of the moment which represent the perpendicular distance between the line of action of the force and the point of rotating (m).

direction: Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (-) for clockwise moments, or vice versa.

Sample Problem 2/5

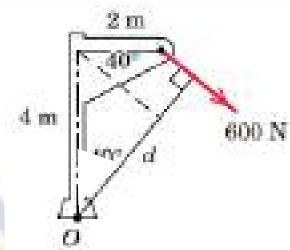
Calculate the magnitude of the moment about the base point O of the 600-N



sol.

Case 1/

The moment arm to the 600-N force is:

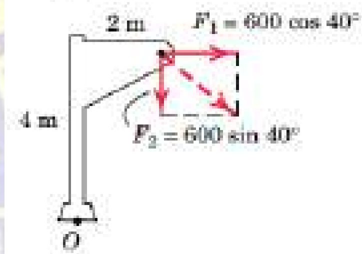


$$d = 4\cos 40 + 2\cos 50 = 4.349\ m$$

$$M = F \times d$$

$$= 600 * 4 \cdot 349 = 2609.851\ N \cdot m$$

Case 2/



analyze the force to its rectangular components at A:

$$F_X = 600\ COS\ 40° = 459.626\ N$$

$$F_Y = 600\ Sin\ 40° = 385.672\ N$$

The total moment becomes:

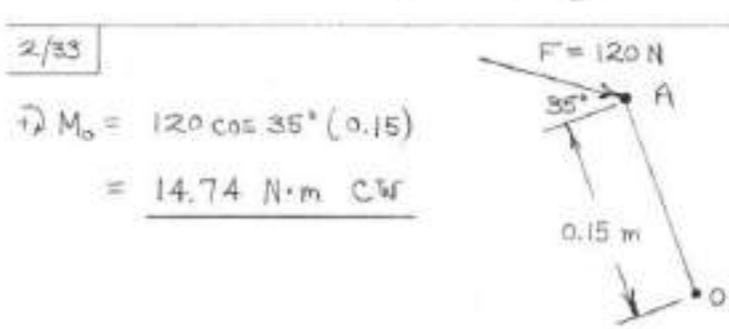
$$M_T = M_1 + M_2$$

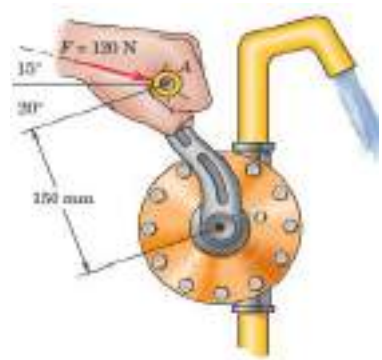
$$M_1 = F_x * d = 459.626 * 4 = 1838 \cdot 5$$

$$M_2 = F_y * d = 385.672 * 2 = 771 \cdot 34$$

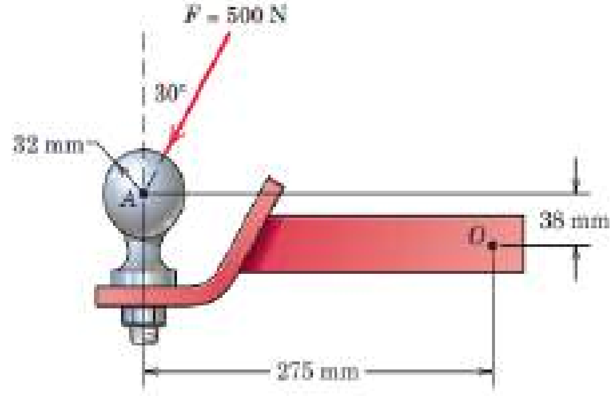
$$M_T = 1838 \cdot 5 + 771 \cdot 34 = 2609.844 N \cdot m$$

2/33 In steadily turning the water pump, a person exerts the 120-N force on the handle as shown. Determine the moment of this force about point O.





2/38 As a trailer is towed in the forward direction, the force $F = 500$ N is applied as shown to the ball of the trailer hitch. Determine the moment of this force about point O.



sol.

$F_X = 500 \sin 30° = 250\ N$

$F_Y = 500 \cos 30° = 433 \cdot 2\ N$

$$M_1 = F_x * d = 250 * \frac{38}{1000} = 9 \cdot 5\ N \cdot m$$

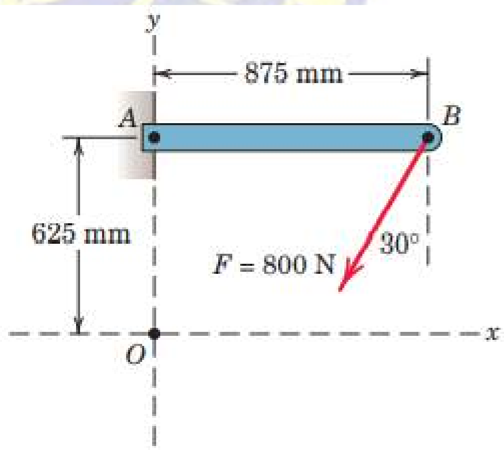
$$M_2 = F_y * d = 433 \cdot 2 * \frac{275}{1000} = 119 \cdot 13\ N \cdot m$$

$M_T = M_1 + M_2$

$M_T = 9 \cdot 5 + 119 \cdot 13 = 128 \cdot 63\ N \cdot m$

H.W

2/30 Determine the moment of the 800-N force ab point A and about point O.



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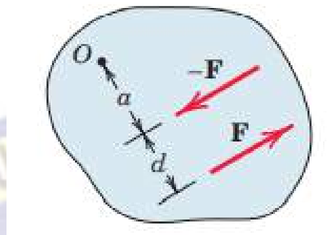
Lecture-5-

Couples

Asst Lect. Hayder Salim

couple

The moment produced by two equal, opposite, and noncollinear forces is called a couple.

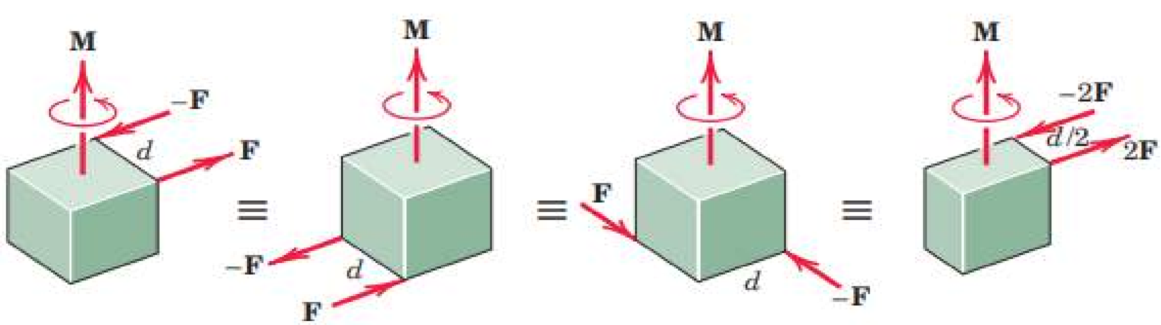


Consider the action of two equal and opposite forces F and -F a distance d apart, as shown in Fig. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple M. This couple has a magnitude:

$$M = F(a + d) - Fa$$

Equivalent couple

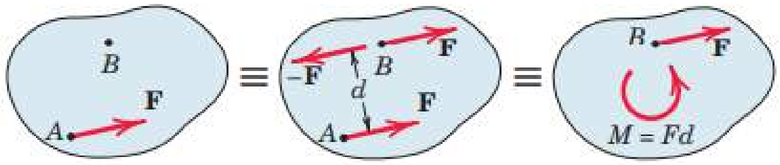
the moment of the couple is not affected if the forces are act with different values and parallel lines action. The Figure shows four different configurations of the same couple M. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.



$M = F \cdot d$ $\qquad\qquad$ $M = 2F \cdot \frac{d}{2}$

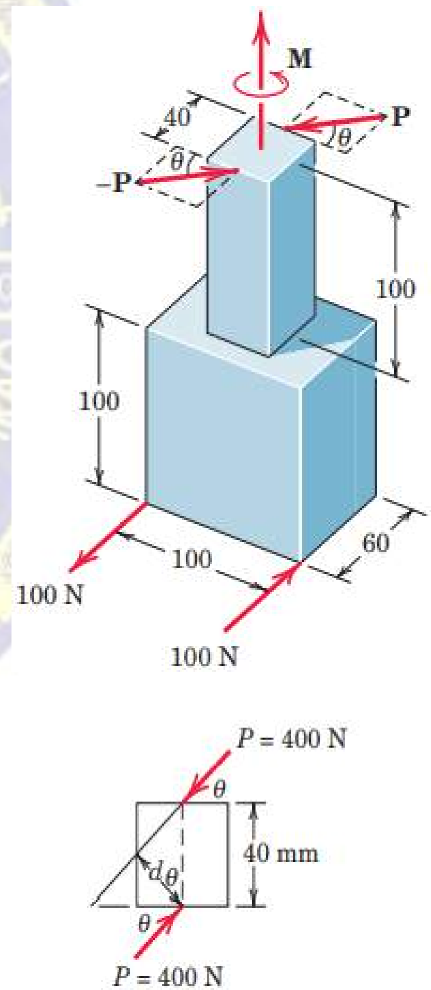
Force-couple system

Any force F acting on a rigid body can be moved to a point as a force with same magnitude and moment of couple about that point.



Example

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and −**P**, each of which has a magnitude of 400 N. Determine the proper angle θ.



Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad M = 100(0.1) = 10 \text{ N}\cdot\text{m}$

The forces **P** and −**P** produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

Equating the two expressions gives

$$10 = (400)(0.040)\cos\theta$$

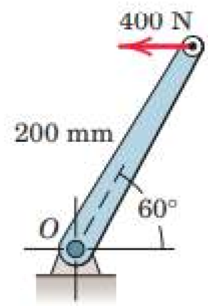
$$\theta = \cos^{-1}\frac{10}{16} = 51.3^\circ \qquad \textit{Ans.}$$

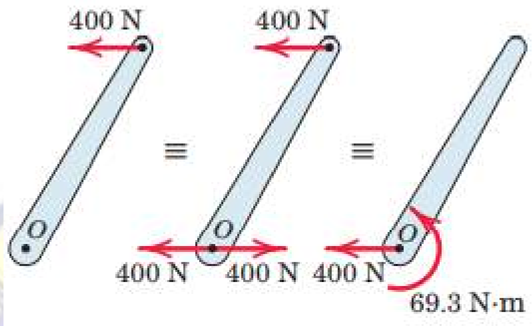
Helpful Hint

① Since the two equal couples are parallel free vectors, the only dimensions which are relevant are those which give the perpendicular distances between the forces of the couples.

Sample Problem 2/8

Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at O and a couple.





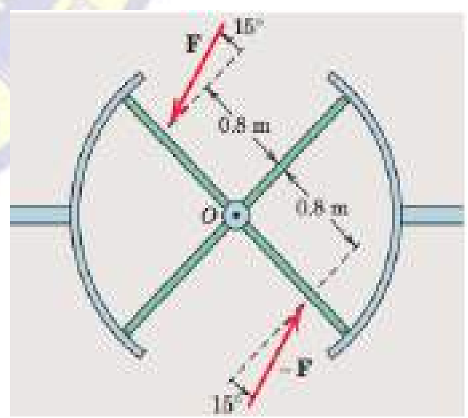
Solution. We apply two equal and opposite 400-N forces at O and identify the counterclockwise couple

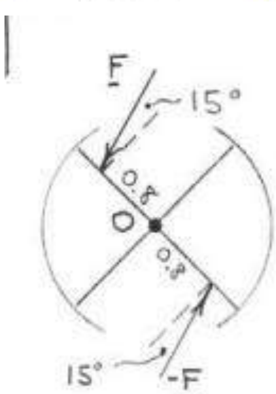
$[M = Fd]$ $\qquad M = 400(0.200 \sin 60°) = 69.3 \text{ N}\cdot\text{m}$ $\qquad$ *Ans.*

Thus, the original force is equivalent to the 400-N force at O and the 69.3-N·m couple as shown in the third of the three equivalent figures.

example/

The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert force of equal magnitudes as shown. If the resulting moment about the door pivot axis at O is 25 N·m, determine the force magnitude F.



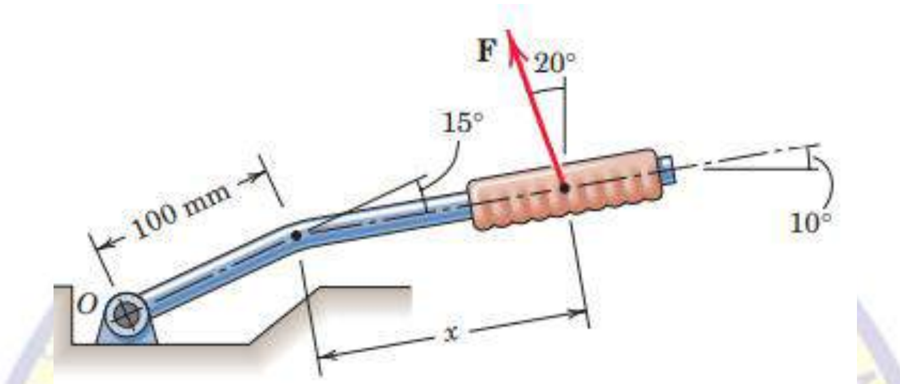


$$\curvearrowleft + \; M_o = \Sigma F d$$

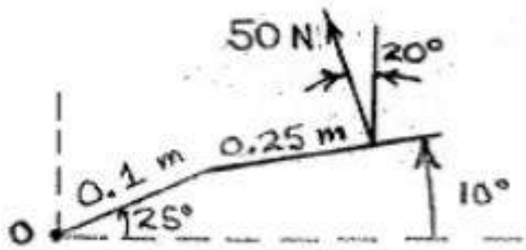
$$25 = 2F(\cos 15°)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$

A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point O.



Solution



$$\circlearrowleft \Sigma M_o = 50 \cos 20° [0.1 \cos 25° + 0.25 \cos 10°]$$

$$+ 50 \sin 20° [0.1 \sin 25° + 0.25 \sin 10°]$$

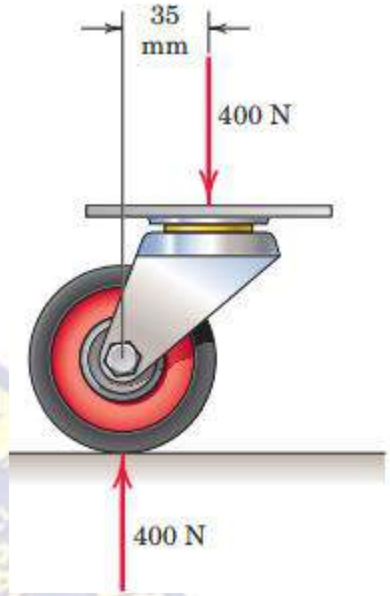
$$= 17.29 \text{ N·m}$$

Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \ \angle 110° \\ M_o = 17.29 \text{ N·m} \ \circlearrowleft \end{cases}$$

H.W

The caster unit is subjected to the pair of 400-N forces shown. Determine the moment associated with these forces.



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Lecture-6-7-

Resultant of Non-Coplanar Force System,

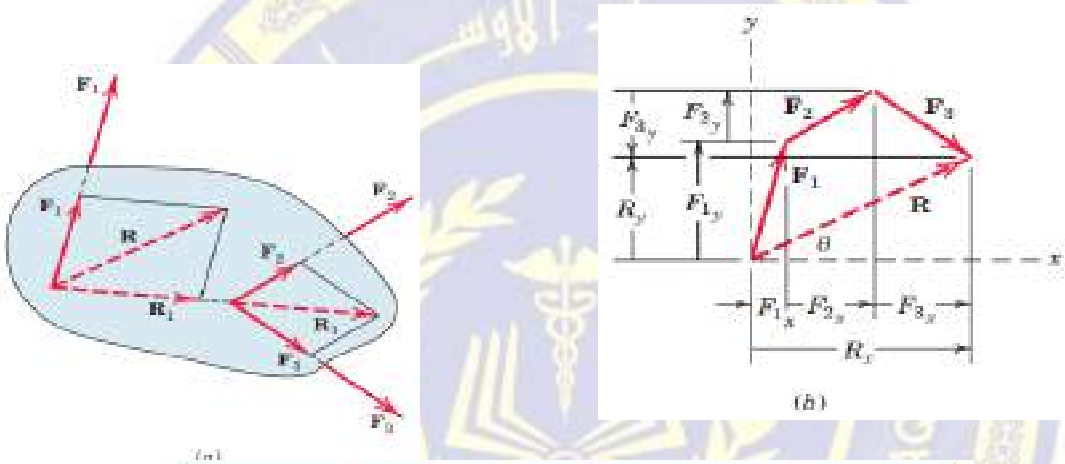
Free Body Diagram

Asst Lect. Hayder Salim

The resultant of a system of forces

The resultant of a system of forces are the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

The most common type of force system occurs when the forces all act in a single plane, say, the x-y plane, as illustrated by the system of three forces F1, F2, and F3 in Fig. a. We obtain the magnitude and direction of the resultant force R by forming the force polygon shown in part b of the fig. b.



$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

$$\mathbf{R} = \Sigma\mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

Free Body Diagram

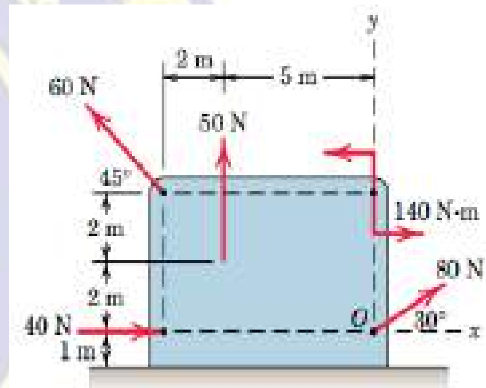
To apply the equation of equilibrium, we must account for all the known and unknown forces (F) which act on the particle. The best way to do this is to think of the particle as isolated and “free” from its surroundings. A drawing that shows the particle with all the forces that act on it is called a free-body diagram (FBD).

Procedure

- analysis all inclined forces in x and y directions.
- if the forces parallel in X-axis find Rx=ΣFx.
- if the forces parallel in y-axis find Ry=ΣFy.
- Find R= $Rx2 + Ry2$.
- Find θ=tan⁻¹ Ry/Rx.
- Find location of resultant according to specific point R.d= ΣF*d.

Problem 1

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution 1

Point O is selected as a convenient reference point for the force–couple system which is to represent the given system

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$

$\left[\theta = \tan^{-1} \frac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1} \frac{132.4}{66.9} = 63.2°$

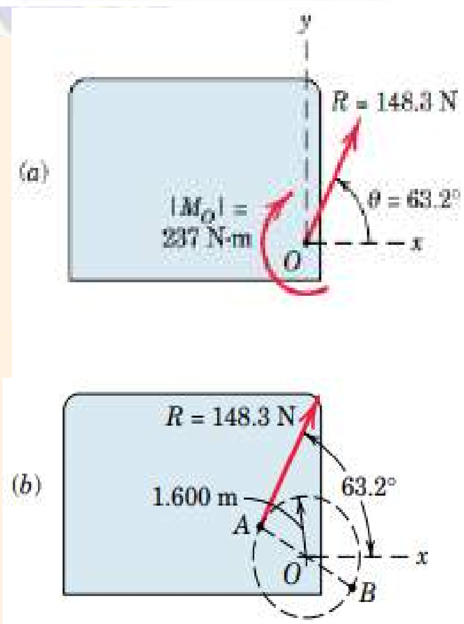
$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$\quad = -237 \text{ N·m}$

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

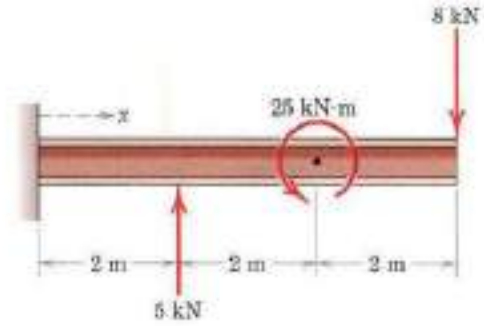
We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|]$ $\quad 148.3d = 237 \quad d = 1.600 \text{ m}$



Problem 2

Determine and locate the resultant R of the two forces and one couple acting on the I-beam

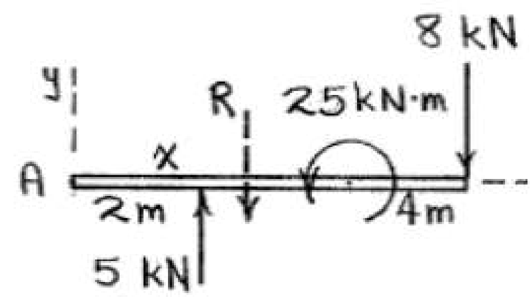


Solution

$$R = \Sigma F_y = 5 - 8 = \underline{-3\ \text{kN}}$$

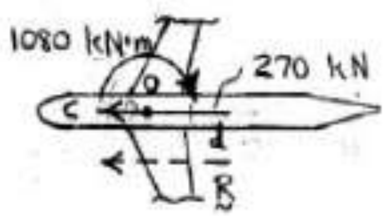
$$\circlearrowleft\ |R|x = M_A :\quad 3x = -5(2) - 25 + 8(6)$$

$$\underline{x = 4.33\ \text{m}}$$



Problem 3

A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two dimensional problem.

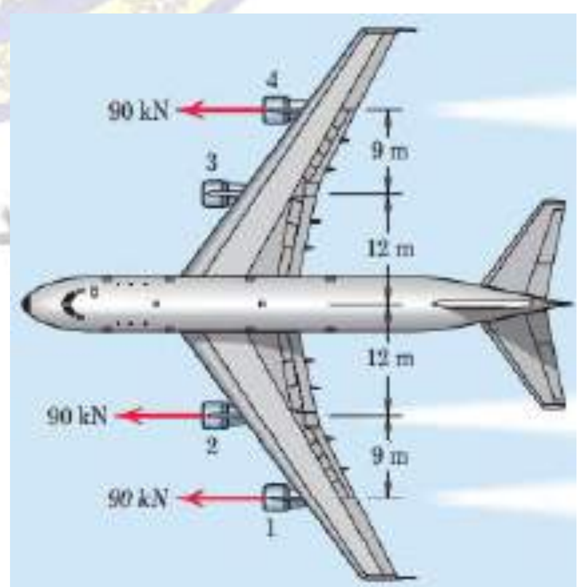


$$R = 3(90) = 270\ \text{kN}\ (\leftarrow)$$

$$\circlearrowleft M_O = 12(90) = 1080\ \text{kN}\cdot\text{m}$$

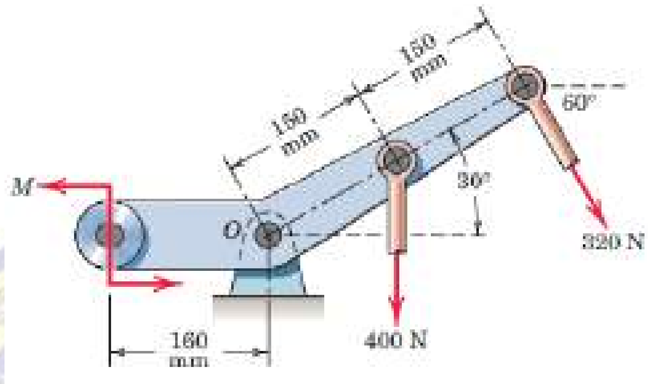
$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= \underline{4\ \text{m}}$$



Problem 4

If the resultant of the two forces and couple M passes through point O, determine M.



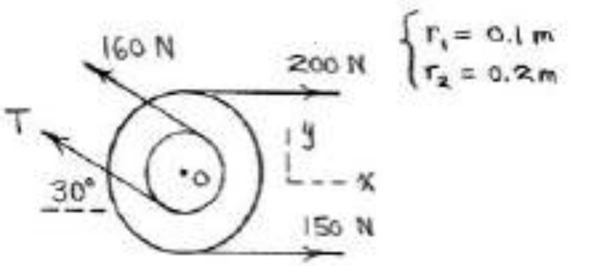
$M_o = 0$, so

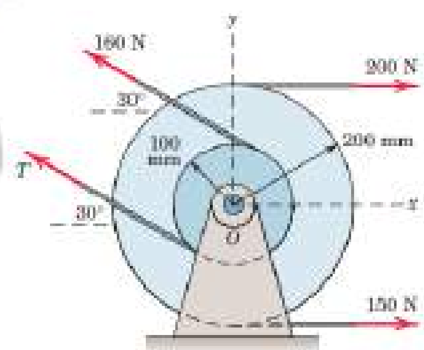
$$\circlearrowleft^{+} M - 400(0.150 \cos 30^\circ) - 320(0.300) = 0$$

$$\underline{M = 148.0 \ N \cdot m}$$

Problem 5

Two integral pulleys are subjected to the belt tensions shown. If the resultant R of these forces passes through the center O, determine T and the magnitude of R and the counterclockwise angle θ it makes with the x-axis.





$$\circlearrowleft^{+} M_o = 0: \ 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$$

$$\underline{T = 60 \ N}$$

$$R_x = \Sigma F_x = 200 + 150 - (160 + 60)\cos 30^\circ = 159.5 \ N$$

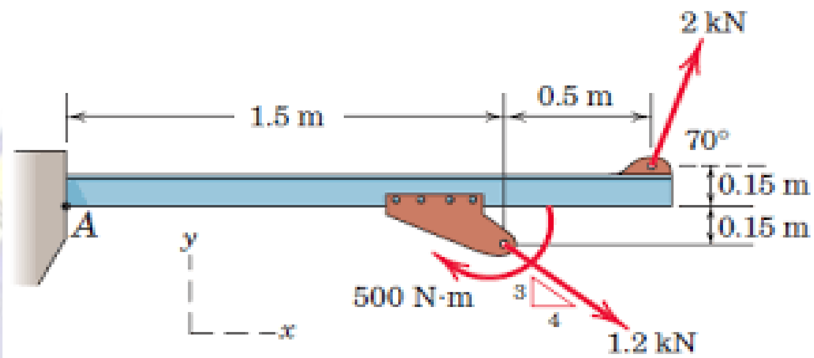
$$R_y = \Sigma F_y = (160 + 30)\sin 30^\circ = 110 \ N$$

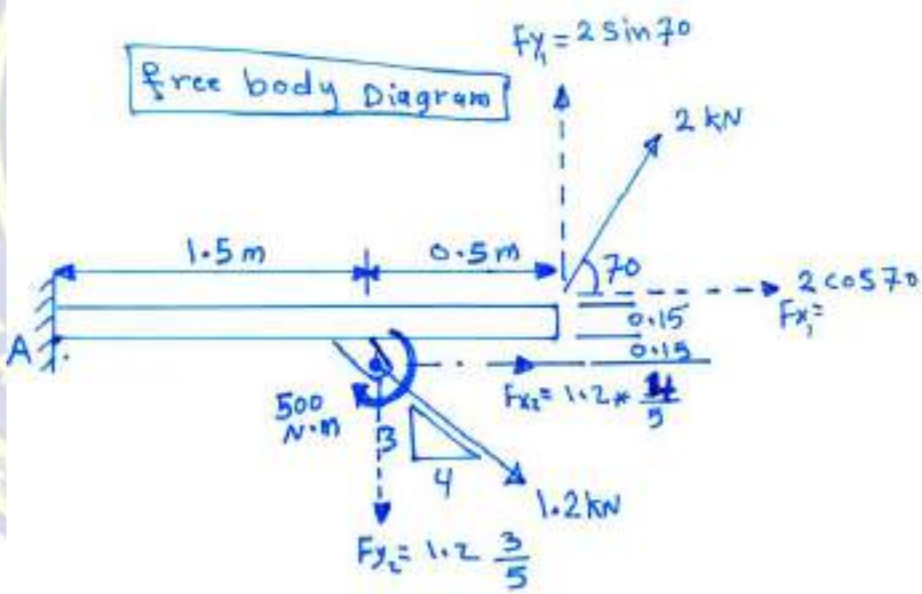
$$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \ N}$$

$$\theta = \tan^{-1}(R_y / R_x) = \underline{34.6^\circ}$$

Problem 6

The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown, and their effect on the design of the attachment at A must be determined. Replace the two forces and couple by an equivalent couple .M and resultant force R at A.





$$R_x = \Sigma F_x = 2\cos 70^\circ + 1.2\left(\frac{4}{5}\right) = 1.644 \text{ kN}$$

$$R_y = \Sigma F_y = 2\sin 70^\circ - 1.2\left(\frac{3}{5}\right) = 1.159 \text{ kN}$$

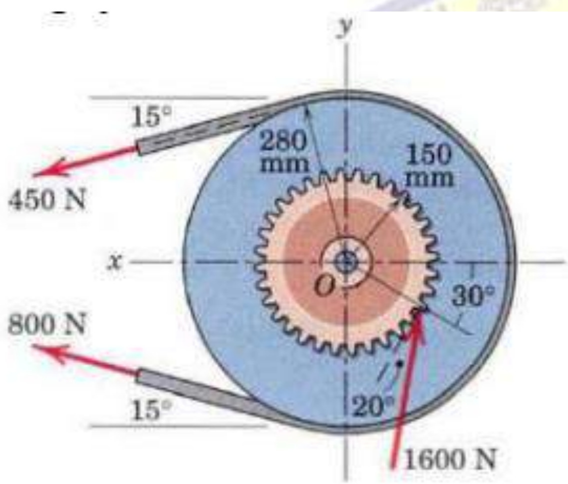
$$\curvearrowleft + M_A = -2\cos 70^\circ (0.15) + 2\sin 70^\circ (1.5 + 0.5)$$

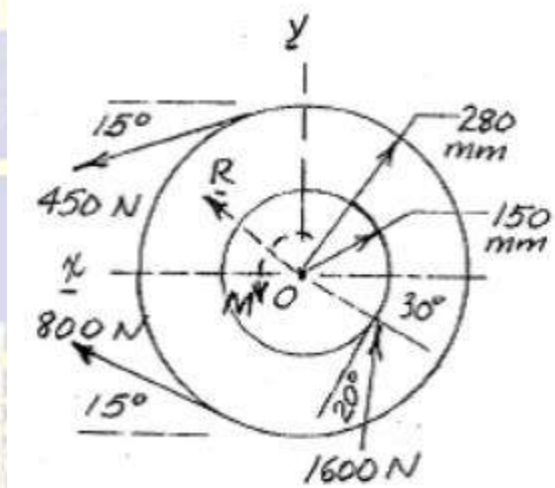
$$+1.2\left(\frac{4}{5}\right)(0.15) - 1.2\left(\frac{3}{5}\right)(1.5) - 0.5$$

$$= 2.22 \text{ kN}\cdot\text{m} \quad \text{CCW}$$

Problem 7

The gear and attached V-belt pulley are turning counterclockwise and are subjected to the tooth load of 1600 N and the 800-N and 450-N tens ions in the V-belt. Represent the action of these three forces by a resultant force R at O and a couple of magnitude M. Is the unit slowing down or speeding up?





$$R_x = \Sigma F_x = (800 + 450)\cos 15° - 1600 \sin(30° - 20°)$$

$$= 1207 - 278 = 929.6 \text{ N}$$

$$R_y = \Sigma F_y = 1600 \cos 10° + (800 - 450)\sin 15°$$

$$= 1576 + 90.6 = 1666 \text{ N}$$

$$M = \Sigma M_O \circlearrowleft; \quad M = 1600 \cos 20° (0.150) + (450 - 800) 0.280$$

$$= 225.5 - 98.0 = \underline{127.5 \text{ N·m CCW}}$$

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Subject

Engineering Mechanics

1st stage

Lecture-8-

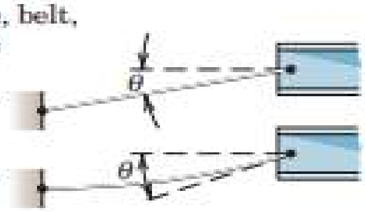
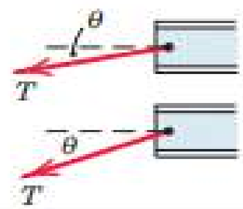
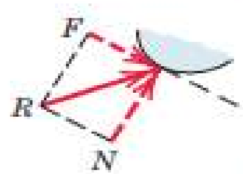
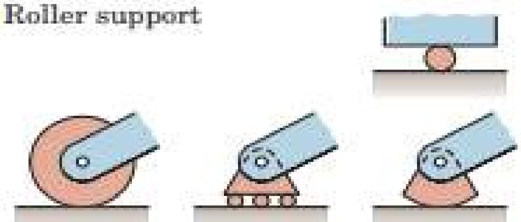
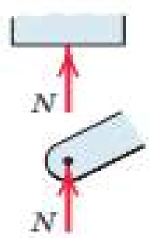
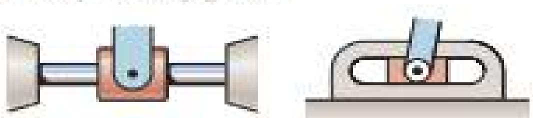
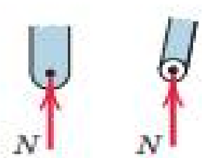
Equilibrium

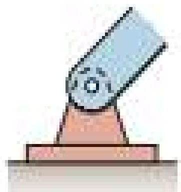
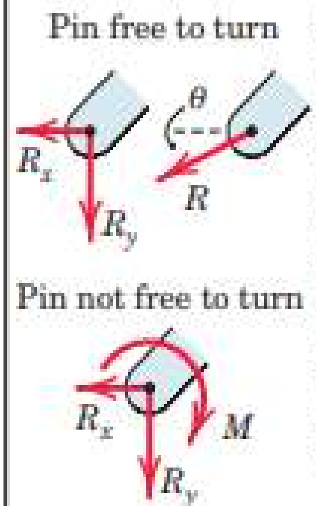
Asst Lect. Hayder Salim

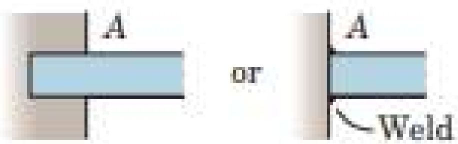
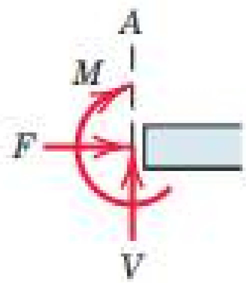
Equilibrium: - When the force and the couple are both equal to zero, the external forces form a system equivalent to zero, and the rigid body is said to be in equilibrium. we defined equilibrium as the condition in which the result ant of all forces and moments acting on a body is zero.

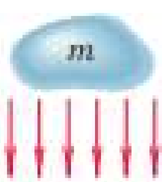
Equations of equilibrium in two dimensions

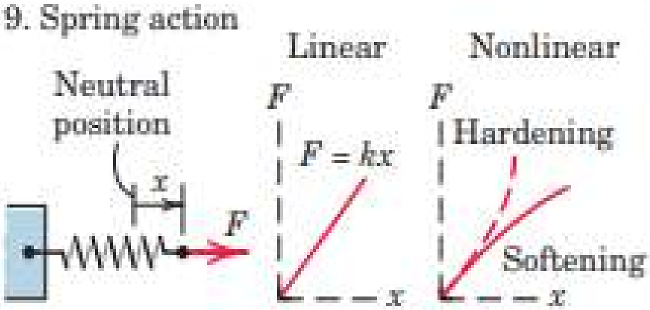
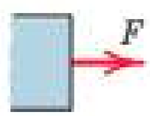
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0$$

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

<table>
<tr><th colspan="2">MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)</th></tr>
<tr><th>Type of Contact and Force Origin</th><th>Action on Body to Be Isolated</th></tr>
<tr><td>6. Pin connection
</td><td>

A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.</td></tr>
<tr><td>7. Built-in or fixed support

</td><td>

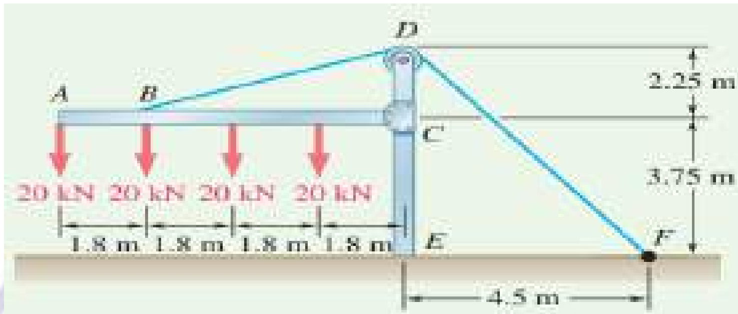
A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.</td></tr>
<tr><td>8. Gravitational attraction

</td><td>

The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.</td></tr>
<tr><td>9. Spring action

</td><td>

Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.</td></tr>
</table>

Example 1/

The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 kN, determine the reaction at the fixed end E.



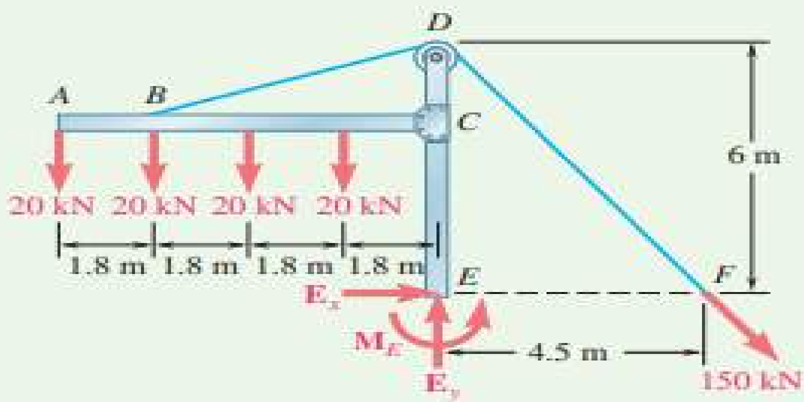


Fig. 1 Free-body diagram of frame.

ANALYSIS:

Equilibrium Equations. First note that

$$DF = \sqrt{(4.5\text{ m})^2 + (6\text{ m})^2} = 7.5\text{ m}$$

Then, you can write the three equilibrium equations and solve for the reactions at E.

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $\quad E_x + \frac{4.5}{7.5}(150\text{ kN}) = 0$

$E_x = -90.0\text{ kN} \qquad \mathbf{E}_x = 90.0\text{ kN} \leftarrow$ ◄

$+\uparrow\Sigma F_y = 0$: $\quad E_y - 4(20\text{ kN}) - \frac{6}{7.5}(150\text{ kN}) = 0$

$E_y = +200\text{ kN} \qquad \mathbf{E}_y = 200\text{ kN} \uparrow$ ◄

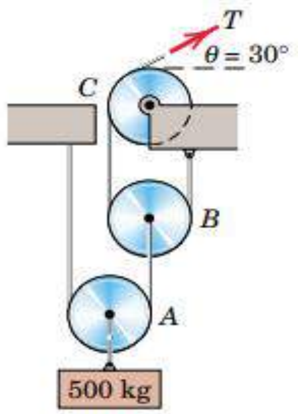
$+\circlearrowleft\Sigma M_E = 0$: $\quad (20\text{ kN})(7.2\text{ m}) + (20\text{ kN})(5.4\text{ m}) + (20\text{ kN})(3.6\text{ m})$

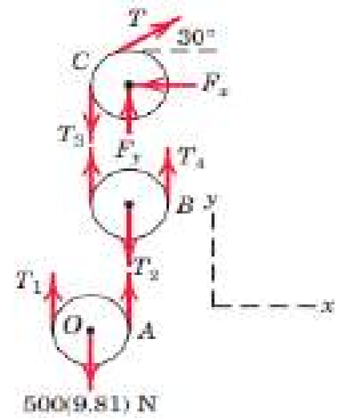
$+(20\text{ kN})(1.8\text{ m}) - \frac{6}{7.5}(150\text{ kN})(4.5\text{ m}) + M_E = 0$

$M_E = +180.0\text{ kN}\cdot\text{m} \qquad \mathbf{M}_E = 180.0\text{ kN}\cdot\text{m} \circlearrowleft$ ◄

example 2/

Calculate the tension T in the cable which supports the 1000 lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.





$[\Sigma M_O = 0]$ $\quad T_1 r - T_2 r = 0 \quad T_1 = T_2$

$[\Sigma F_y = 0]$ $\quad T_1 + T_2 - 500(9.81) = 0 \quad 2T_1 = 500(9.81) \quad T_1 = T_2 = 2450 \text{ N}$

From the example of pulley *A* we may write the equilibrium of forces on pulley *B* by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$

For pulley *C* the angle $\theta = 30°$ in no way affects the moment of *T* about the center of the pulley, so that moment equilibrium requires

$$T = T_3 \quad \text{or} \quad T = 1226 \text{ N} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the *x*- and *y*-directions requires

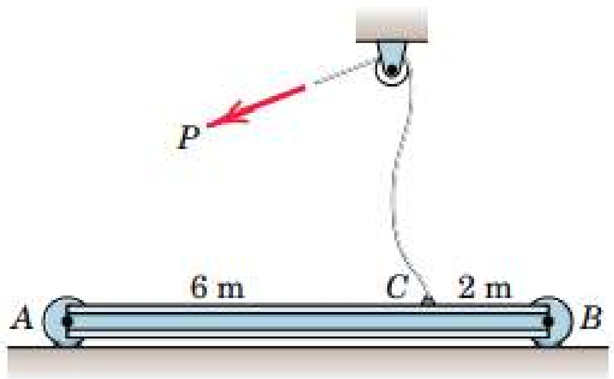
$[\Sigma F_x = 0]$ $\quad 1226 \cos 30° - F_x = 0 \quad F_x = 1062 \text{ N}$

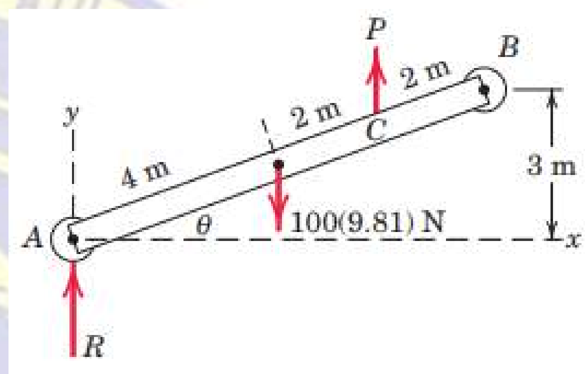
$[\Sigma F_y = 0]$ $\quad F_y + 1226 \sin 30° - 1226 = 0 \quad F_y = 613 \text{ N}$

$[F = \sqrt{F_x^2 + F_y^2}]$ $\quad F = \sqrt{(1062)^2 + (613)^2} = 1226 \text{ N} \qquad \textit{Ans.}$

example 3/

Problem 3 The uniform 100-kg I-beam is supported initially by its end roller s on the horizontal surface at A and B. By mean s of the cable at C it is desired to elevate end B to a position 3 m above end A. Determine the required tension P, the reaction at A, and the angle 8 made by the beam with the horizontal in the elevated position





Moment equilibrium about A eliminates force R and gives

$[\Sigma M_A = 0]$ $\quad P(6 \cos \theta) - 981(4 \cos \theta) = 0 \quad P = 654$ N

Equilibrium of vertical forces requires

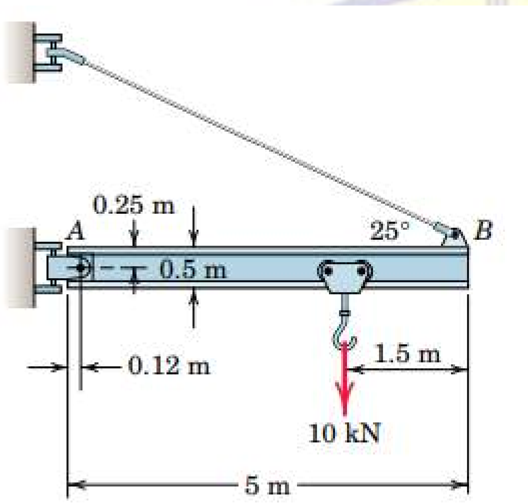
$[\Sigma F_y = 0]$ $\quad 654 + R - 981 = 0 \quad R = 327$ N

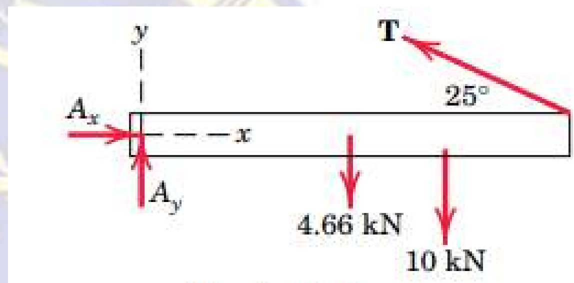
The angle θ depends only on the specified geometry and is

$$\sin \theta = 3/8 \qquad \theta = 22.0°$$

example 4/

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.





Free-body diagram

$[\Sigma M_A = 0]$ $\quad (T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$

from which $\quad T = 19.61$ kN $\quad$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

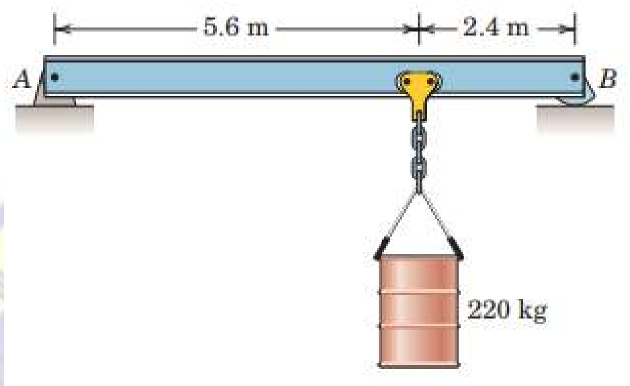
$[\Sigma F_x = 0]$ $\quad A_x - 19.61 \cos 25° = 0 \quad A_x = 17.77$ kN

$[\Sigma F_y = 0]$ $\quad A_y + 19.61 \sin 25° - 4.66 - 10 = 0 \quad A_y = 6.37$ kN

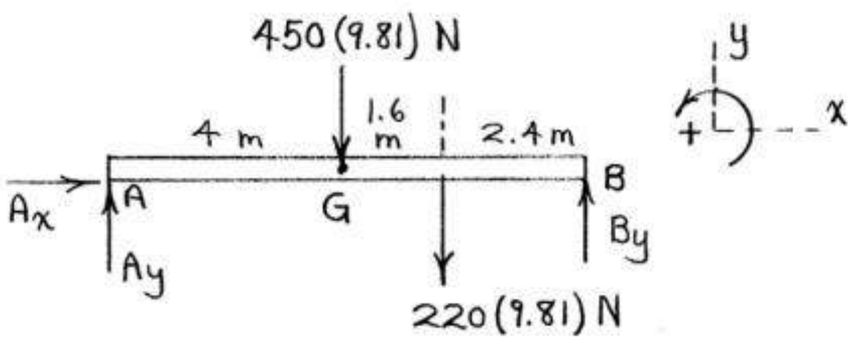
$[A = \sqrt{A_x{}^2 + A_y{}^2}]$ $\quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88$ kN $\quad$ *Ans.*

example 5/

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



Solution



From $\Sigma F_x = 0, \quad A_x = 0$

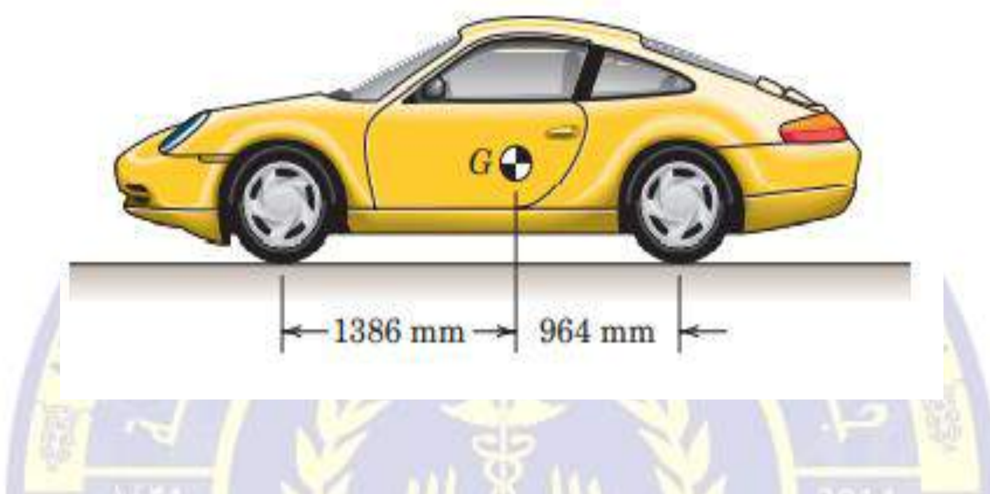
$$\Sigma M_A = 0: \; -450(9.81)4 - 220(9.81)(5.6) + B_y(8) = 0, \quad \underline{B_y = 3720 \text{ N}}$$

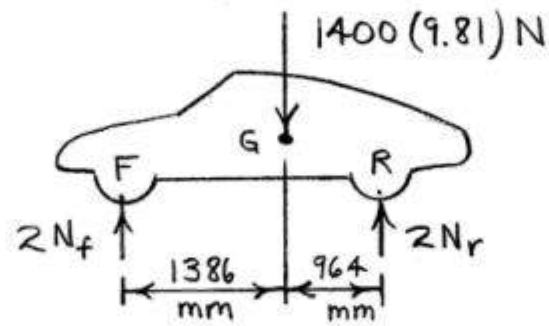
$$\Sigma F_y = 0: \; A_y - 450(9.81) - 220(9.81) + 3720 = 0$$

$$\underline{A_y = 2850 \text{ N}}$$

example 6/

The mass center G of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.





$$+\uparrow \Sigma F = 0 : \quad 2N_f + 2N_r - 1400(9.81) = 0$$

$$\circlearrowleft + \Sigma M_F = 0 : \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

Al-Furat Al-Awsat Technical University

Najaf Technical Institute

Aeronautic Technical Department

Subject

Engineering Mechanics

1st stage

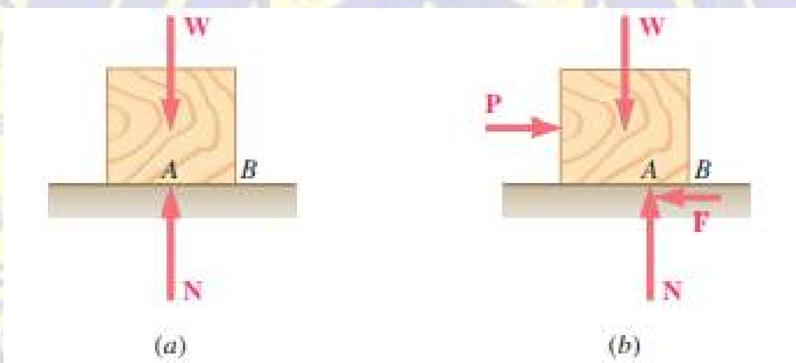
Lecture-9-10

Friction

المعهد التقني / النجف

Asst Lect. Hayder Salim

Friction: - represent as a tangential force between the contacting surface which always effect in opposite to the direction of sliding (motion) or sliding tendency. Place a block of weight W on a horizontal plane surface (Fig. a). The forces acting on the block are its weight W and the reaction of the surface. Because the weight has no horizontal component, the reaction of the surface also has no horizontal component; the reaction is therefore normal to the surface and is represented by N in Fig. a. Now suppose that you apply a horizontal force P to the block (Fig. b). If P is small, the block does not move; some other horizontal force must therefore exist, which balances P. This other force is the static-friction force F, which is actually the resultant of a great number of forces acting over the entire surface of contact between the block and the plane.



Assume that the contacting surfaces have some roughness.

Static friction $$F_m = \mu_s N$$

where μ_s is a constant called the coefficient of static friction

Kinetic friction $$F_k = \mu_k N$$

where μ_k is a constant called the coefficient of kinetic friction.

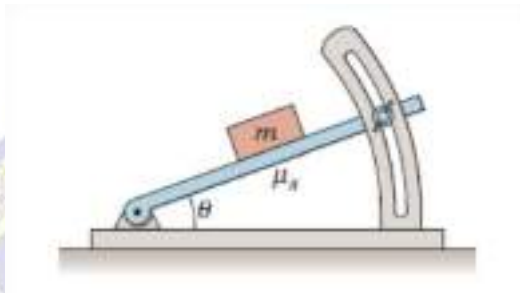
Angle of static friction

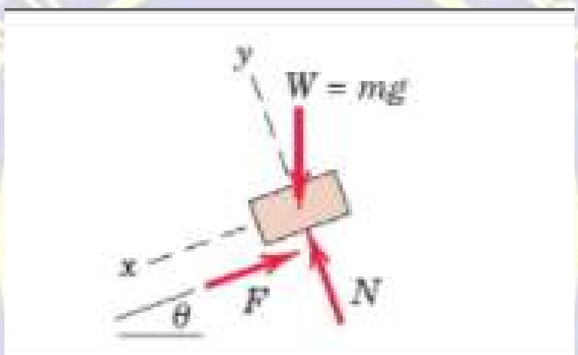
$$\tan \phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan \phi_s = \mu_s$$

Example1/

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s





Sol.

Equilibrium in the x- and y-directions requires

$\Sigma F_x = 0 \qquad mg \sin\theta - F = 0 \qquad F = mg \sin\theta$

$\Sigma F_y = 0 \qquad -mg \cos\theta + N = 0 \qquad N = mg \cos\theta$

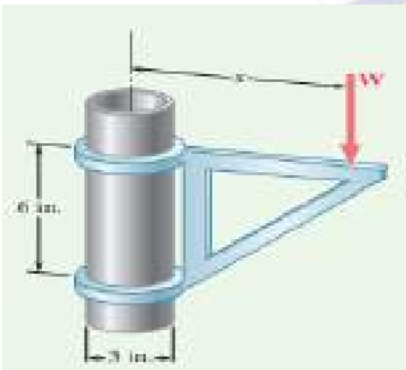
$F/N = \tan\theta$

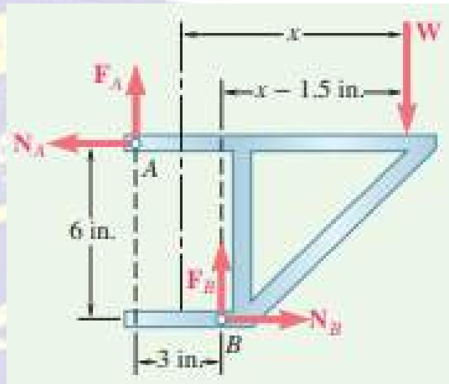
$F = F_{max} = \mu_s N$

$\mu_s = \tan\theta_{max}$ or $\theta_{max} = \tan^{-1}\mu_s$,

Example2/

The movable bracket shown may be placed at any height on the 3-in.-diameter pipe. If the coefficient of static friction between the pipe and bracket is 0.25, determine the minimum distance x at which the load W can be supported. Neglect the weight of the bracket.





$$F_A = \mu_s N_A = 0.25\ N_A$$
$$F_B = \mu_s N_B = 0.25\ N_B$$

ANALYSIS:

Equilibrium Equations.

$\xrightarrow{+} \Sigma F_x = 0$: $\quad N_B - N_A = 0$

$$N_B = N_A$$

$+\uparrow \Sigma F_y = 0$: $\quad F_A + F_B - W = 0$

$$0.25N_A + 0.25N_B = W$$

Because N_B is equal to N_A,

$$0.50N_A = W$$
$$N_A = 2W$$

$+\circlearrowleft \Sigma M_B = 0$: $\quad N_A(6\text{ in.}) - F_A(3\text{ in.}) - W(x - 1.5\text{ in.}) = 0$

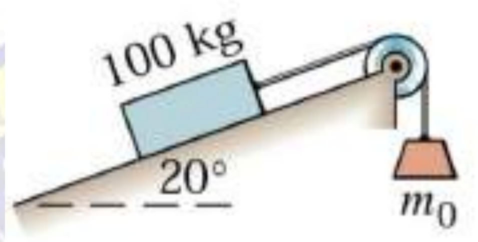
$$6N_A - 3(0.25N_A) - Wx + 1.5W = 0$$
$$6(2W) - 0.75(2W) - Wx + 1.5W = 0$$

Dividing through by W and solving for x, you have

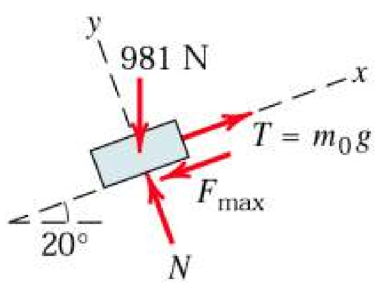
$$x = 12\text{ in.}$$

Example3/

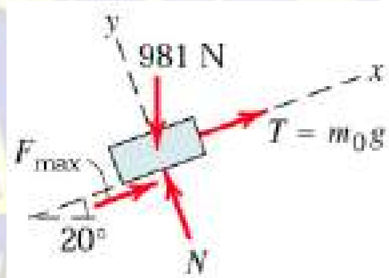
Determine the range of values which the mass m_o may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



Sol.



Case I



Case II

bounded m_o values → block start moving → $F = \mu_s N$

$$\left[\sum F_y = 0\right] \quad N - 100g\cos 20 = 0, \quad N = 922\ \text{N}$$

Case I: max m_o, start moving up, friction downward

$$\left[\sum F_x = 0\right] \quad m_o g - \mu_s N - 100g\sin 20 = 0, \quad m_o = 62.4\ \text{kg}$$

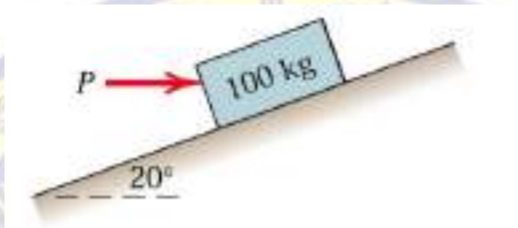
Case II: min m_o, start moving down, friction upward

$$\left[\sum F_x = 0\right] \quad m_o g + \mu_s N - 100g\sin 20 = 0, \quad m_o = 6.0\ \text{kg}$$

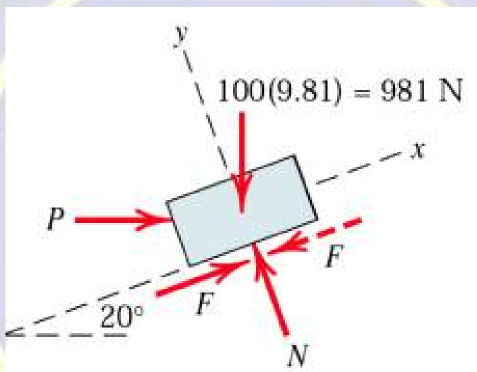
$$\therefore 6.0 \le m_o \le 62.4\ \text{kg} \quad \text{and } F \le F_{max} = 277\ \text{N up/downward}$$

Example 4/

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, P = 500 N and, second, P = 100 N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Sol.



P = 500 N: assume the block tends to move up → friction downward

$\left[\sum F_y = 0\right]$ $N - 500\sin 20 - 100g\cos 20 = 0,\ \ N = 1092.85\ \text{N}$

max supportable friction $= \mu_s N = 218.6\ \text{N}$

$\left[\sum F_x = 0\right]$ $500\cos 20 - F - 100g\sin 20 = 0,\ \ F = 134.3\ \text{N} < \mu_s N$

$\therefore$ the assumption is valid

P = 100 N: assume the block tends to slide down → friction upward

$\left[\sum F_y = 0\right]$ $N - 100\sin 20 - 100g\cos 20 = 0,\ \ N = 956.04\ \text{N}$

max supportable friction $= \mu_s N = 191.21\ \text{N}$

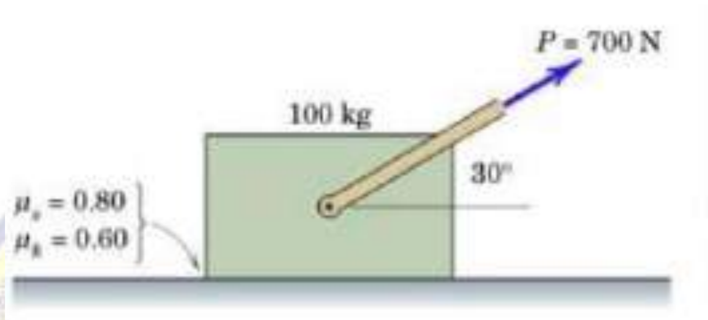
$\left[\sum F_x = 0\right]$ $F + 100\cos 20 - 100g\sin 20 = 0,\ \ F = 241.55\ \text{N} > \mu_s N$

$\therefore$ the assumption is invalid, block is moving downward

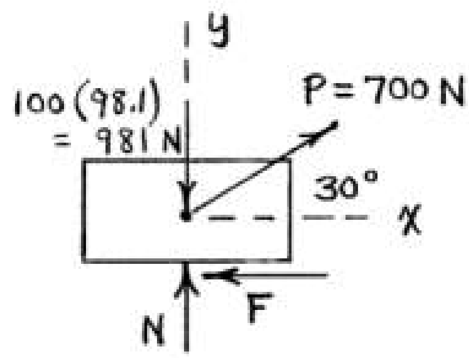
kinetic friction upward $= \mu_k N = 162.5\ \text{N}$

Example 5/

The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block.



Sol.



$$\Sigma F_x = 0: \quad 700 \cos 30^\circ - F = 0, \qquad F = 606 \text{ N}$$

$$\Sigma F_y = 0: \quad N - 981 + 700 \sin 30^\circ = 0, \qquad N = 631 \text{ N}$$

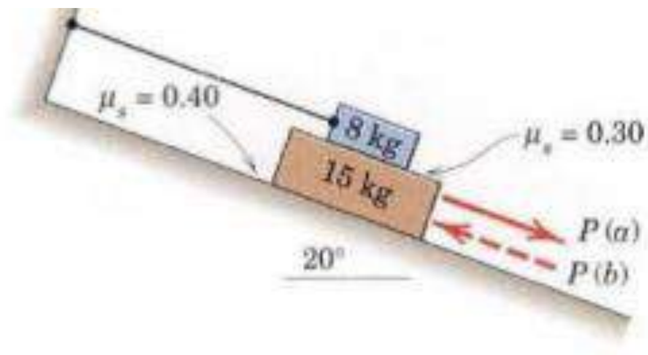
$$F_{max} = \mu_s N = 0.8(631) = 505 \text{ N} < F = 606 \text{ N}$$

Assumption invalid, motion occurs.

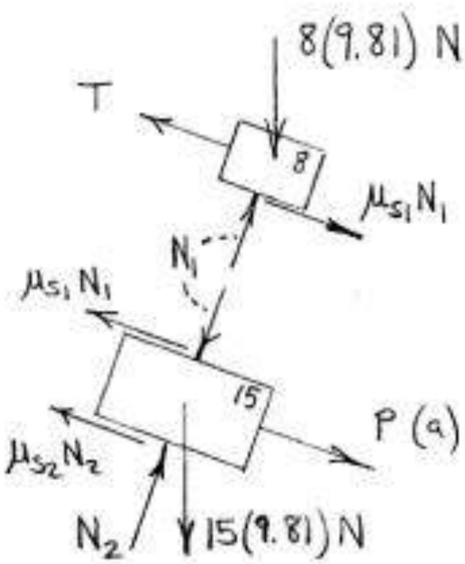
$$F = \mu_k N = 0.6(631) = \underline{379 \text{ N}}$$

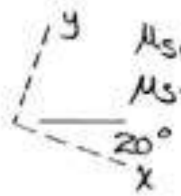
Example 6/

The two blocks are placed on the incline with the cab le taut. Determine the force P required to initiate motion of the 15-kg block if P is applied down the incline.



Solution





$\mu_{s_1} = 0.30$

$\mu_{s_2} = 0.40$

From $\Sigma F_y = 0$,

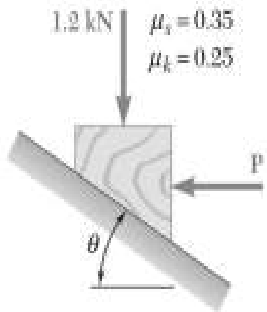
$$\begin{cases} N_1 = 73.7 \text{ N} \\ N_2 = 212 \text{ N} \end{cases}$$

(throughout)

$\Sigma F_x = 0$:

$$\left.\begin{array}{l} -T + 8(9.81)\sin 20^\circ + \mu_{s_1} N_1 = 0 \\ -\mu_{s_1} N_1 - \mu_{s_2} N_2 + 15(9.81)\sin 20^\circ + P = 0 \end{array}\right\}$$

Solution : $\underline{P = 56.6 \text{ N},}$ $T = 49.0$ N



PROBLEM 8.1

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 750$ N.

SOLUTION

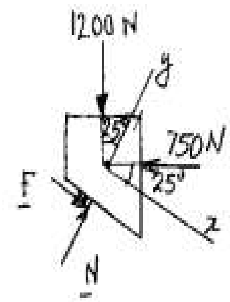
Assume equilibrium:

$$\searrow^{+} \Sigma F_x = 0: \quad F + (1200 \text{ N})\sin 25° - (750 \text{ N})\cos 25° = 0$$

$$F = +172.6 \text{ N} \qquad\qquad \mathbf{F} = 172.6 \text{ N} \searrow$$

$$+\nearrow \Sigma F_y = 0: \quad N - (1200 \text{ N})\cos 25° - (750 \text{ N})\sin 25° = 0$$

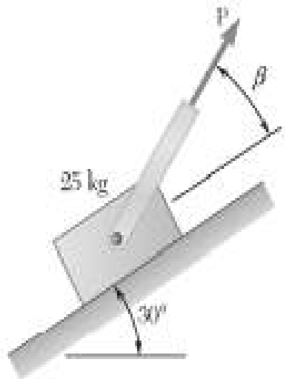
$$N = 1404.5 \text{ N}$$



Maximum friction force: $F_m = \mu_s N = 0.35(1404.5 \text{ N}) = 491.6 \text{ N}$

Since $F < F_m$, <u>block is in equilibrium</u> ◀

Friction force: $\mathbf{F} = 172.6 \text{ N} \nwarrow 25.0°$ ◀

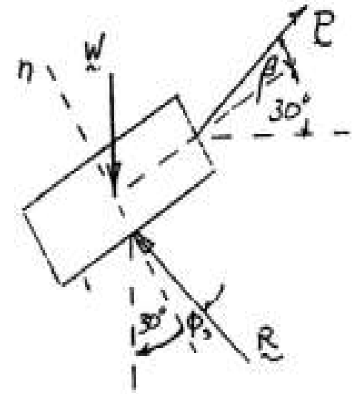


PROBLEM 8.6

Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P required to start the block moving up the incline, (*b*) the corresponding value of β.

SOLUTION

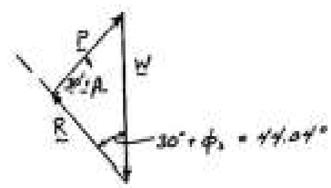
FBD block (Impending motion up)



$$W = mg$$
$$= (25\text{ kg})(9.81\text{ m/s}^2)$$
$$= 245.25\text{ N}$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.04°$$

(*a*) (*Note:* For minimum P, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$.)

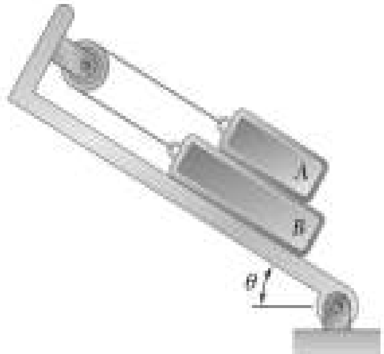


Then

$$P = W\sin(30° + \phi_s)$$
$$= (245.25\text{ N})\sin 44.04°$$

$P_{min} = 170.5\text{ N}$ ◀

(*b*) We have $\beta = \phi_s$ $\beta = 14.04°$ ◀



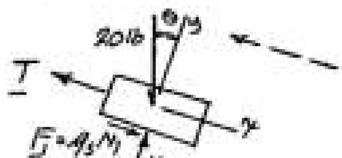
PROBLEM 8.12

The 20-lb block *A* and the 30-lb block *B* are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

Free body: Block *A*



Impending motion:

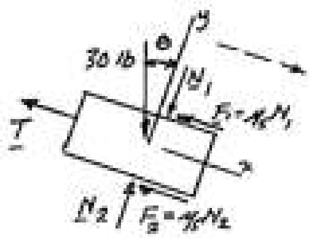
$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$

$+\nwarrow \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$

$T = 20\sin\theta + 0.15(20\cos\theta)$

$T = 20\sin\theta + 3\cos\theta \qquad (1)$

Free body: Block *B*



Impending motion:

$+\nearrow \Sigma F_y = 0: \quad N_2 - 30\cos\theta - N_1 = 0$

$N_2 = 30\cos\theta + 20\cos\theta = 50\cos\theta$

$F_2 = \mu_s N_2 = 0.15(50\cos\theta) = 6\cos\theta$

$+\nwarrow \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 + \mu_s N_2 = 0$

$T = 30\sin\theta - 0.15(20\cos\theta) - 0.15(50\cos\theta)$

$T = 30\sin\theta - 3\cos\theta - 7.5\cos\theta \qquad (2)$

Eq. (1) subtracted by Eq. (2): $\quad 20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta + 7.5\cos\theta = 0$

$13.5\cos\theta = 10\sin\theta, \quad \tan\theta = \dfrac{13.5^\circ}{10}$ $\qquad \theta = 53.5^\circ$ ◀

Al-Furat Al-Awsat Technical University

Najaf Technical Institute

Aeronautic Technical Department

Subject

Engineering Mechanics

1st stage

Lecture-11-12

Center of Gravity (simple area)

& (composite area)

Asst Lect. Hayder Salim

Center of Gravity (simple area)

The Centre of gravity of a body is defined as the point where the whole weight of the body is supposed to act. A body supported at its center of gravity remains balanced and it is in mechanical equilibrium.

Triangle

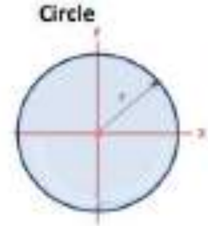
$A = b * d$

$\bar{x} = \frac{1}{2}b$

$\bar{y} = \frac{1}{2}d$

Centroid=(b/2,d/2)

Circle



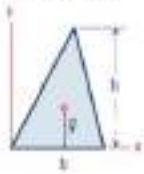
$A = \pi r^2$

$\bar{x} = 0$

$\bar{y} = 0$

Centroid =(0, 0)

Rectangular

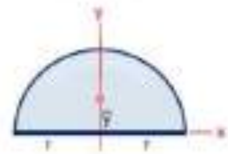


$A = \frac{1}{2}b * h$

$\bar{y} = \frac{1}{3}h$

Centroid =(b/3,h/3) for right angle triangle

Semicircle



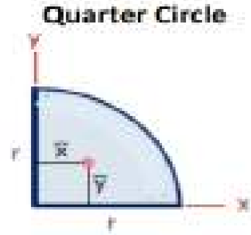
$A = \frac{\pi r^2}{2}$

$\bar{x} = 0$

$\bar{y} = \frac{4r}{3\pi}$

Centroid $= (0, \frac{4r}{3\pi})$

Quarter Circle

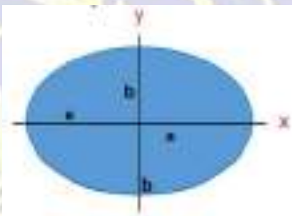


$A = \frac{\pi r^2}{4}$

$\bar{x} = \frac{4r}{3\pi}$

$\bar{y} = \frac{4r}{3\pi}$

Centroid $= (\frac{4r}{3\pi}, \frac{4r}{3\pi})$



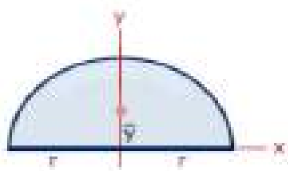
$A = \pi * a * b$

$\bar{x} = 0$

$\bar{y} = 0$

Centroid=(0,0)

Semicircle



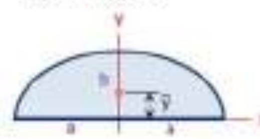
$A = \frac{\pi r^2}{2}$

$\bar{x} = 0$

$\bar{y} = \frac{4r}{3\pi}$

Centroid $= (0, \frac{4r}{3\pi})$

Half Ellipse



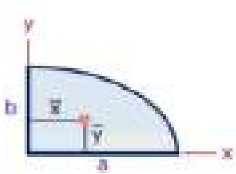
$A = \frac{\pi ab}{2}$

$\bar{x} = 0$

$\bar{y} = \frac{4b}{3\pi}$

Centroid $= (0, \frac{4b}{3\pi})$

Quarter Ellipse



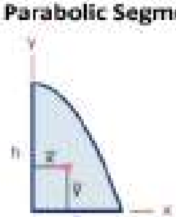
$A = \frac{\pi ab}{4}$

$\bar{x} = \frac{4a}{3\pi}$

$\bar{y} = \frac{4b}{3\pi}$

Centroid $= (\frac{4a}{3\pi}, \frac{4b}{3\pi})$

Parabolic Segment



$A = \frac{2}{3}bh$

$\bar{x} = \frac{3}{8}b$

$\bar{y} = \frac{2}{5}h$

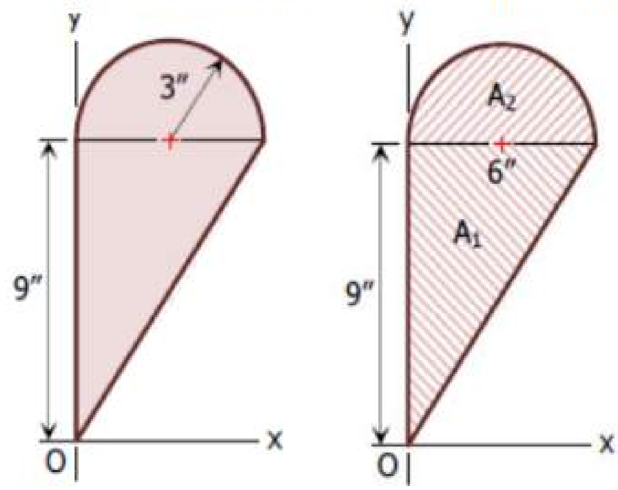
Centroid $= (\frac{3}{8}b, \frac{2}{5}h)$

Centroids of Composite Figures

Procedure

- **Divided shape into shapes with known dimensions and centers.**
- **Find the individual area for all shapes (a).**
- **Find summation of area =Σa=A.**
- **Find Σax, and Find Σay.**
- $\bar{x}$**= (Σax/A).**
- $\bar{y}$**= (Σay/A).**
- **Centroid = (**$\bar{x}$**,** $\bar{y}$**).**

Example 1 **Determine the coordinates of the centroid of the area shown in Figu below with respect to the given axes.?**



$a1=(1/2)*6*9=27 in^2$

$x1=(1/3)*6=2$ in

$y1=(2/3)*9=6$ in

$a2=\frac{\pi r^2}{2}=\frac{\pi 3^2}{2}= 14.14 in^2$

$X2=3$ in

$y2= 9+\frac{4r}{3\pi}$ ⟹ $y2= 9+\frac{4*3}{3\pi}=10.27$ in

$A=a1+a2=27+14.14= 41.14 in^2$

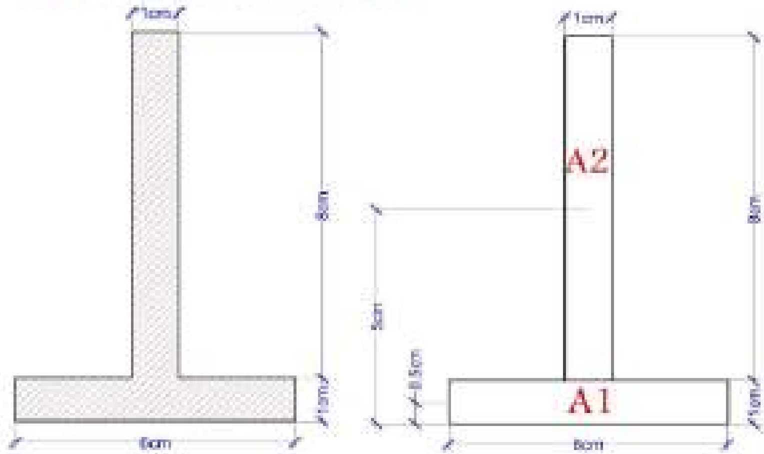
- $\bar{x}$= (Σax/A) ⟹ $\bar{x}$= (27*2+14.14*3)/41.14=2.34 in

- $\bar{y}$= (Σay/A) ⟹ $\bar{y}$= (27*6+14.41*10.27)/41.14=7.47 in

Coordinates of the centroid is at $(2.34, 7.47)$.

Example 2 **The dimensions of the T-section of a cast-iron beam are shown in Figure-below. How far is the centroid of the area? above the base?**

Ans.



$a1=6*1=6cm^2$

$y1=0.5cm$

$a2=8*1=8cm^2$

$y2=4+1=5cm$

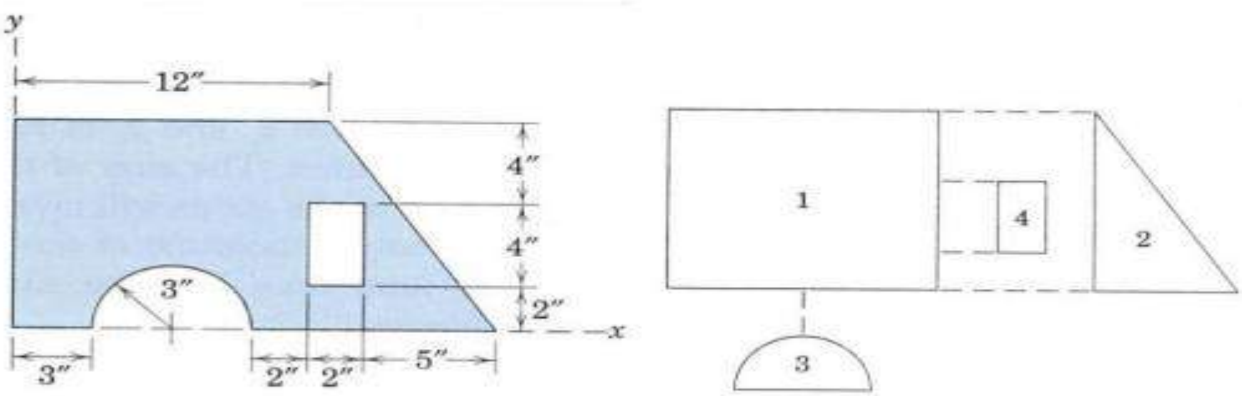
$A=a1+a2=8+6=14cm^2$

$\bar{y}=(\Sigma ay/A)$

$\bar{y}=(6*0.5+8*5)/14$

$\bar{y}$= 3.07 cm above the base

Example 3/ Locate the centroid the shaded area

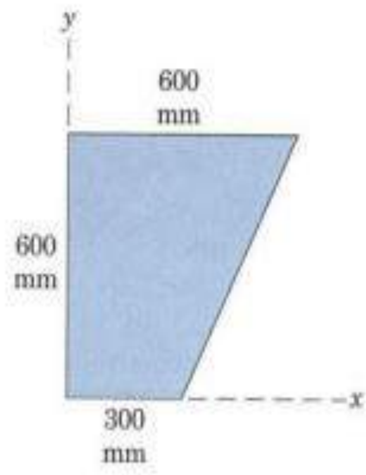


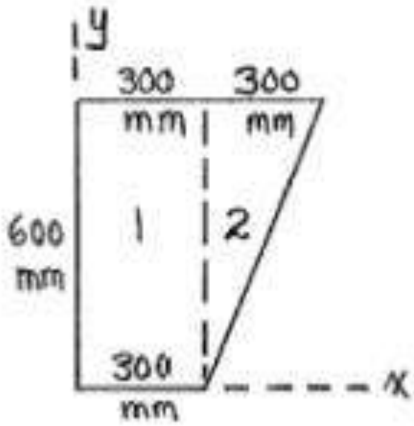
PART	A in.^{2}	$\bar{x}$ in.	$\bar{y}$ in.	$\bar{x}A$ in.^{3}	$\bar{y}A$ in.^{3}
1	120	6	5	720	600
2	30	14	10/3	420	100
3	−14.14	6	1.273	−84.8	−18
4	−8	12	4	−96	−32
TOTALS	127.9			959	650

$$\left[\bar{X}=\frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X}=\frac{959}{127.9}=7.50 \text{ in.}$$

$$\left[\bar{Y}=\frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y}=\frac{650}{127.9}=5.08 \text{ in.}$$

Example 4/ Determine the coordinates of the trapezoidal area shown.





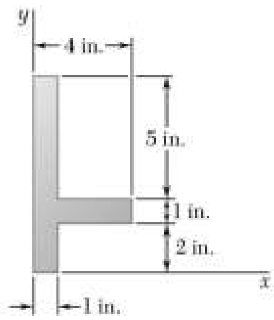
$$A_1 = 18(10^4)\ \text{mm}^2, \quad \bar{x}_1 = 150\ \text{mm}, \quad \bar{y}_1 = 300\ \text{mm}$$

$$A_2 = 9(10^4)\ \text{mm}^2, \quad \bar{x}_2 = 300 + \frac{1}{3}(300) = 400\ \text{mm}$$

$$\bar{y}_2 = 400\ \text{mm}$$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{18(10^4)(150) + 9(10^4)(400)}{18(10^4) + 9(10^4)} = \underline{233\ \text{mm}}$$

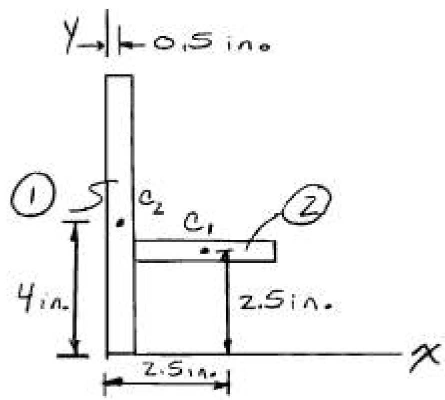
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{18(10^4)(300) + 9(10^4)(400)}{18(10^4) + 9(10^4)} = \underline{333\ \text{mm}}$$



PROBLEM 5.1

Locate the centroid of the plane area shown.

SOLUTION



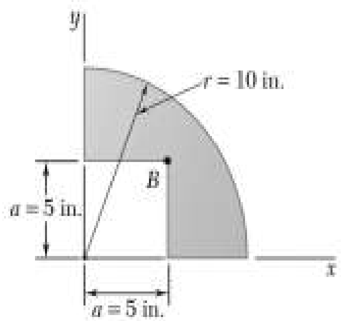
	A, in^2	$\bar{x}$, in	$\bar{y}$, in	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	8	0.5	4	4	32
2	3	2.5	2.5	7.5	7.5
Σ	11			11.5	39.5

$$\bar{X}\Sigma A = \bar{x}A$$

$$\bar{X}(11\ \text{in}^2) = 11.5\ \text{in}^3 \qquad \bar{X} = 1.045\ \text{in.} \blacktriangleleft$$

$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

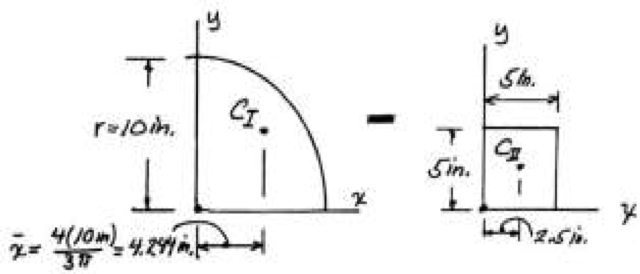
$$\bar{Y}(11) = 39.5 \qquad \bar{Y} = 3.59\ \text{in.} \blacktriangleleft$$



PROBLEM 5.5

Locate the centroid of the plane area shown.

SOLUTION

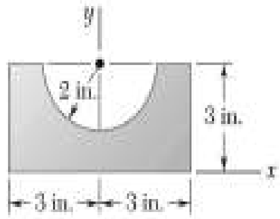


By symmetry, $\bar{X} = \bar{Y}$

	Component	A, in^2	$\bar{x}$, in.	$\bar{x}A$, in^3
I	Quarter circle	$\frac{\pi}{4}(10)^2 = 78.54$	4.2441	333.33
II	Square	$-(5)^2 = -25$	2.5	−62.5
Σ		53.54		270.83

$\bar{X}\Sigma A = \Sigma \bar{x} A$: $\quad \bar{X}(53.54\text{ in}^2) = 270.83\text{ in}^3$

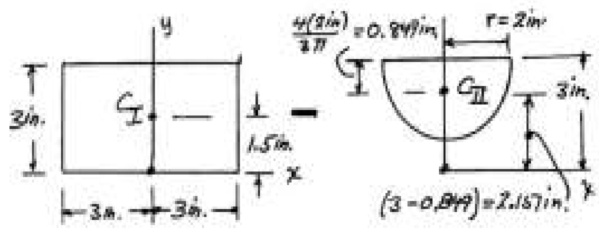
$\bar{X} = 5.0585$ in. $\qquad \bar{X} = \bar{Y} = 5.06$ in. ◀



PROBLEM 5.7

Locate the centroid of the plane area shown.

SOLUTION



By symmetry, $\bar{X} = 0$

	Component	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
I	Rectangle	$(3)(6) = 18$	1.5	27.0
II	Semicircle	$-\frac{\pi}{2}(2)^2 = -6.28$	2.151	−13.51
Σ		11.72		13.49

$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}(11.72 \text{ in.}^2) = 13.49 \text{ in}^3$$

$$\bar{Y} = 1.151 \text{ in.}$$

$\bar{X} = 0$

$\bar{Y} = 1.151$ in. ◀

Al-Furat Al-Awsat Technical University

Najaf Technical Institute

Aeronautic Technical Department

Subject

Engineering Mechanics

1st stage

Lecture-13-

moment of Inertia (simple &Composite area)

Asst Lect. Hayder Salim

moment of Inertia

consider the plate in Fig. 10-1, which is submerged in a fluid and subjected to the pressure p. this pressure varies linearly with depth, such that p = $\gamma\, y$, where γ is the specific weight of the fluid. Thus, the force acting on the differential area dA of the plate is dF = p dA = (γ y)dA. The moment of this force about the x axis is therefore dM = y dF = $\gamma\, y^2$dA,and so integrating dM over the entire area of the plate yields M = $\gamma \int y^2$ dA. The integral $\int y^2$ dA is sometimes referred to as the “second moment” of the area about an axis (the x axis), but more often it is called the moment of inertia of the area. The word “inertia” is used here since the formulation is similar to the mass moment of inertia, $\int y^2$ dm,

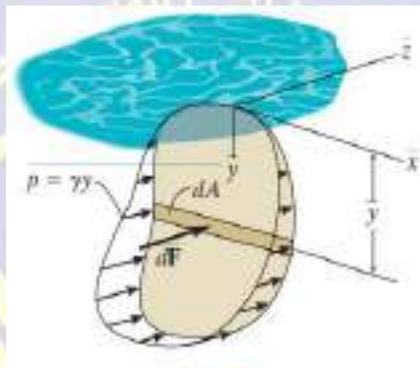


Fig. 10–1

Moment of Inertia. By definition, the moments of inertia of a differential area dA about the x and y axes are dIx = y2 dA and dIy = x2 dA, respectively, Fig. 10–2. For the entire area The moments of inertia are determined by integration

$$I_x = \int_A y^2\, dA$$
$$I_y = \int_A x^2\, dA$$

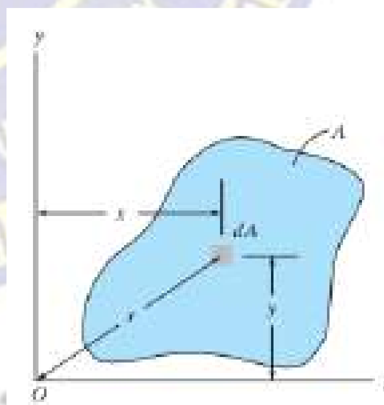


Fig. 10–2

we can also formulate this quantity for *dA* about the “pole” O or z axis, Fig. 10–2. This is referred to as **the polar moment of inertia**. It is defined as $dJ_O = r^2\, dA$, where r is the perpendicular distance from the pole (z axis) to the element *dA*. For the entire area the polar moment of inertia is

$$J_O = \int_A r^2\, dA = I_x + I_y$$

This relation between J_O and Ix, Iy is possible since $r^2 = x^2 + y^2$, Fig. 10–2

From the above formulations it is seen that Ix, Iy, and J_O will always be positive since they involve the product of distance squared and area. Furthermore, the units for moment of inertia involve length raised to the fourth power, e.g., m^4, mm^4, or ft^4, in^4

parallel-axis theorem for an Area

The parallel-axis theorem can be used to find the moment of inertia of an area about any axis that is parallel to an axis passing through the centroid and about which the moment of inertia is known. we will consider finding the moment of inertia of the shaded area shown in Fig.10–3 about the x axis. To start, we choose a differential element dA located at an arbitrary distance y' from the centroid x' axis. If the distance between the parallel x and x' axis is dy, then the moment of inertia of dA about the x axis is $dIx = (y' + dy)^2\, dA$ for the entire area

$$I_x = \int_A (y' + d_y)^2\, dA$$
$$= \int_A y'^2\, dA + 2d_y \int_A y'\, dA + d_y^2 \int_A dA$$

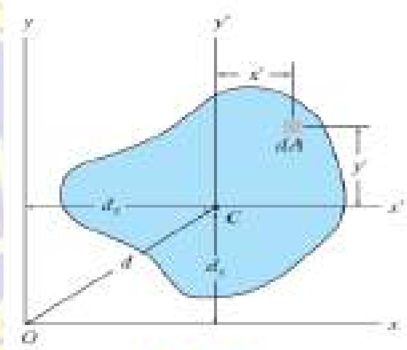


Fig. 10–3

The first integral represents the moment of inertia of the area about the centroidal axis, Ix'. The second integral is zero since the x' axis passes through the area's centroid C; i.e., $\int y'\, dA = y \int dA = 0$ since $y' = 0$. Since the third integral represents the total area A, the final result is therefore

$$I_x = \bar{I}_{x'} + Ad_y^2$$

A similar expression can be written for I_y; i.e.,

$$I_y = \bar{I}_{y'} + Ad_x^2$$

And finally, for the polar moment of inertia, since $\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'}$ and $d^2 = d_x^2 + d_y^2$, we have

$$J_O = \bar{J}_C + Ad^2$$

Example/1

Determine the moment of inertia for the rectangular area shown in Fig. 10–5 with respect to (a) the centroidal x' axis, (b) the axis x_b passing through the base of the rectangle, and (c) the pole or z' axis perpendicular to the x' - y' plane and passing through the centroid C.

SOLUTION

(a)

At the centroidal x' axis

integrate from $y' = -h/2$ to $y = h/2$. Since $dA = b\,dy'$ then

$$\bar{I}_{x'} = \int_A y'^2\,dA = \int_{-h/2}^{h/2} y'^2(b\,dy') = b\int_{-h/2}^{h/2} y'^2\,dy'$$

$$\bar{I}_{x'} = \frac{1}{12}bh^3$$

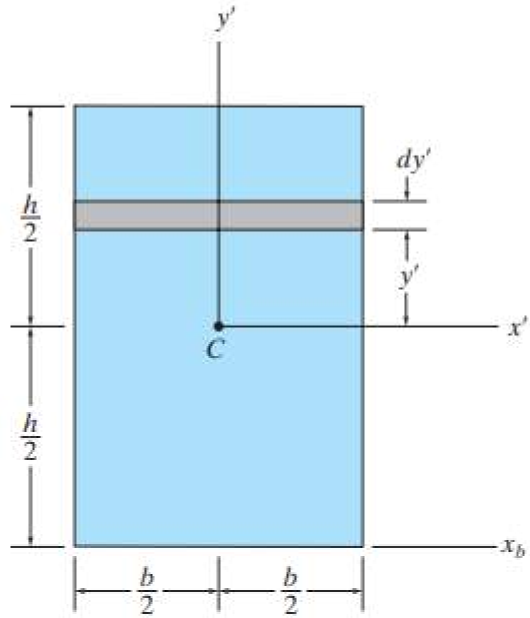


Fig. 10–5

(b)

At the centroidal x_b axis

$$I_{x_b} = \bar{I}_{x'} + A d_y^2$$

$$= \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3$$

(c)

To obtain the polar moment of inertia about point C, we must first obtain Iy', which may be found by interchanging the dimension's b and h in the result of part (a)

$$\bar{I}_{y'} = \frac{1}{12}hb^3$$

the polar moment of inertia about C is

$$\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'} = \frac{1}{12}bh(h^2 + b^2)$$

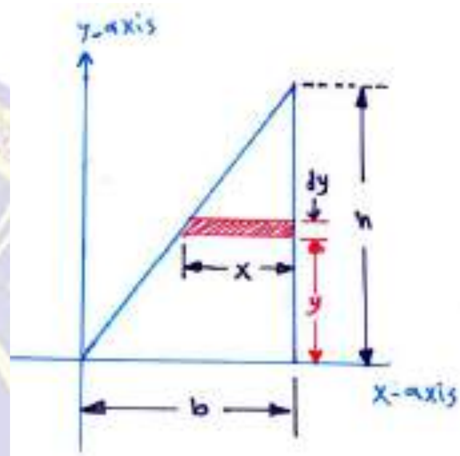
Example/2

Determine the moment of inertia for the triangle of base (b) and altitude (h) with respect to (a) an axis coinciding with its base and (b) a centroidal axis parallel to its base.

$$Ix = \int y^2 \, dA$$

$$dA = Xdy$$

$$Ix = \int y^2 \, (Xdy) = \int Xy^2 \, dy$$



From the triangle

$$\frac{h-y}{x} = \frac{h}{b} \Rightarrow x = \frac{b}{h}(h-y)$$

$$Ix = \frac{b}{h}\int (h-y)y^2 * dy = \frac{b}{h}\int (hy^2 - y^3)\, dy$$

$$Ix = \frac{b}{h}\left\{\int_0^h hy^2\, dy - \int_0^h y^3 dy\right\} = \frac{b}{h}\left\{h\int_0^h y^2\, dy - \int_0^h y^3 dy\right\}$$

$$Ix = \frac{b}{h}\left\{h\left[\frac{y^3}{3}\right]_0^h - \left[\frac{y^4}{4}\right]_0^h\right\} = \frac{b}{h}\left\{\left(h\frac{h^3}{3} - 0\right) - \left(\frac{h^4}{4} - 0\right)\right\}$$

$$Ix = \frac{b}{h}\left\{\frac{h^4}{3} - \frac{h^4}{4}\right\} = \frac{b}{h}\left\{\frac{4h^4 - 3h^4}{12}\right\} = \frac{b}{h}\left\{\frac{h^4}{12}\right\}$$

$$Ix = \frac{bh^3}{12}$$

$$I = I_C + Ad^2$$

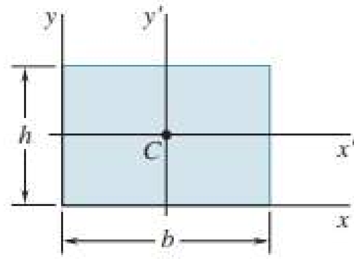
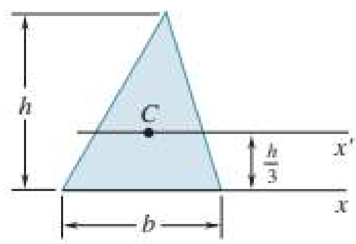
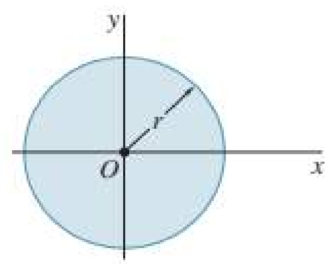
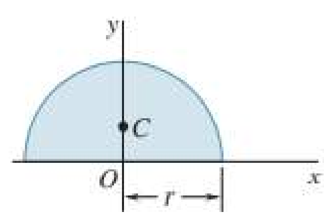
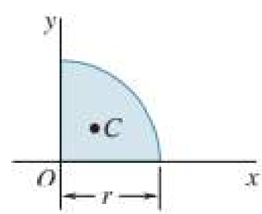
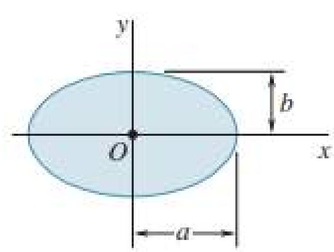
$$Ix = Ix_c + Ad^2$$

$$\frac{bh^3}{12} = Ix_c + \frac{1}{2}bh(h/3)^2$$

$$\frac{bh^3}{12} = Ix_c + \frac{bh^3}{18}$$

$$Ix_c = \frac{bh^3}{12} - \frac{bh^3}{18}$$

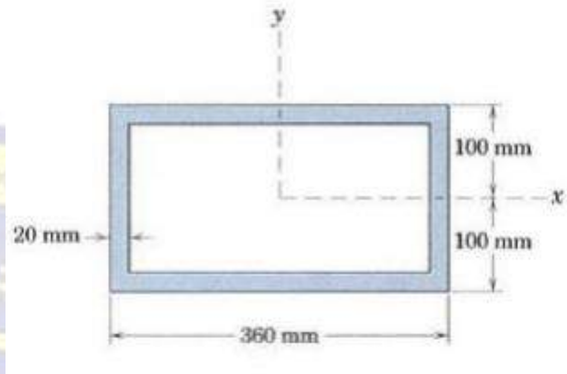
$$Ix_c = \frac{3bh^3 - 2bh^3}{36} \Rightarrow Ix_c = \frac{bh^3}{36}$$

Rectangle		$\overline{I}_{x'} = \frac{1}{12} bh^3$ $\overline{I}_{y'} = \frac{1}{12} b^3h$ $I_x = \frac{1}{3} bh^3$ $I_y = \frac{1}{3} b^3h$ $J_C = \frac{1}{12} bh(b^2 + h^2)$
Triangle		$\overline{I}_{x'} = \frac{1}{36} bh^3$ $I_x = \frac{1}{12} bh^3$
Circle		$\overline{I}_x = \overline{I}_y = \frac{1}{4} \pi r^4$ $J_O = \frac{1}{2} \pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8} \pi r^4$ $J_O = \frac{1}{4} \pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16} \pi r^4$ $J_O = \frac{1}{8} \pi r^4$
Ellipse		$\overline{I}_x = \frac{1}{4} \pi ab^3$ $\overline{I}_y = \frac{1}{4} \pi a^3b$ $J_O = \frac{1}{4} \pi ab(a^2 + b^2)$

Example/3

Determine the moment of inertia of the shaded area about the X -axis in two ways. The wall thickness is 20 mm on all four sides of the rectangle

$Ix_{T}=Ix_1-Ix_2$



$$Ix = \left(\frac{bh^3}{12}\right)_1 - \left(\frac{bh^3}{12}\right)_2$$

$$Ix = \left(\frac{360 * 200^3}{12}\right)_1 - \left(\frac{320 * 160^3}{12}\right)_2$$

$Ix = 130.8 * 10^6\ mm^4$

Example/4

Determine the moment of inertia of the area shown in Fig. 10–8a about the x axis.

properties formulae for circular and rectangular.

Circular= $I_x = \frac{1}{4}\pi r^4$

Rectangular= $I_x = \frac{1}{12}bh^3$

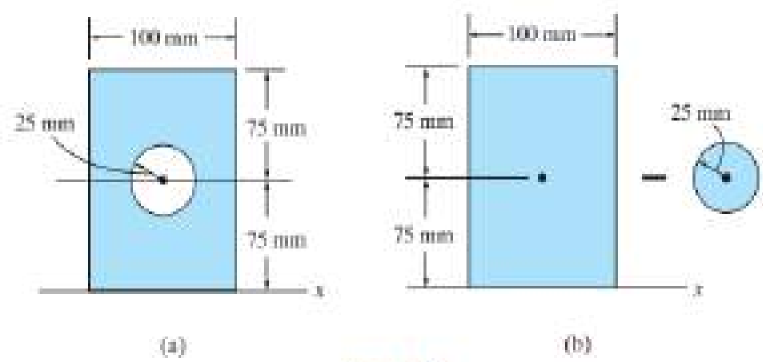


Fig. 10–8

Circle

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$= \frac{1}{4}\pi(25)^4 + \pi(25)^2(75)^2 = 11.4(10^6)\ mm^4$$

Rectangle

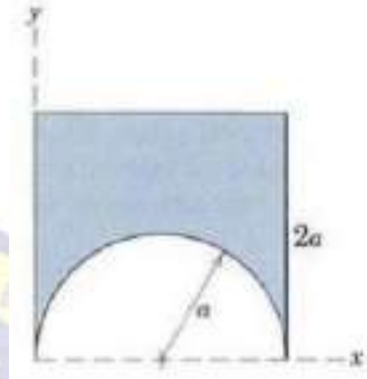
$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$= \frac{1}{12}(100)(150)^3 + (100)(150)(75)^2 = 112.5(10^6)\ mm^4$$

Summation. The moment of inertia for the area is therefore

$$I_x = -11.4(10^6) + 112.5(10^6)$$
$$= 101(10^6)\ mm^4$$

Example/5

Determine the moments of inertia of the shaded area about the x - and y -axes



$$Ix_{combine} = Ix_{squar} - Ix_{semicircle}$$

$$Ix_{squar} = Iy_{squar} = \frac{bh^3}{3} = \frac{2a(2a)^3}{3} = \frac{16a^4}{3}$$

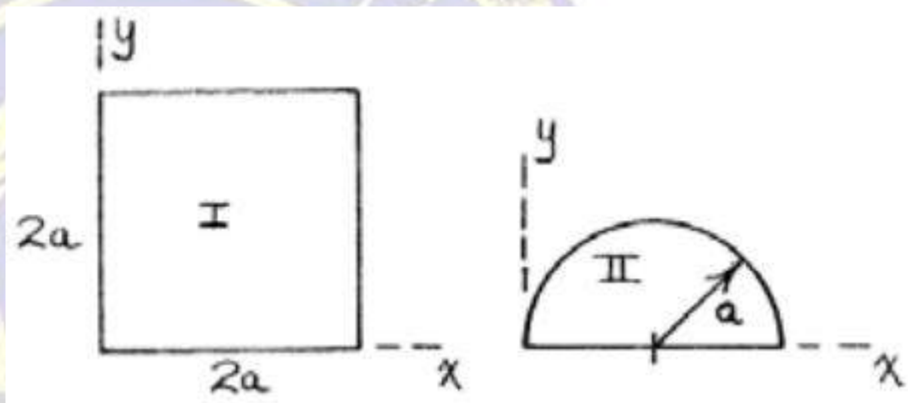
$$Ix_{semicircle} = \frac{\pi r^4}{8} = \frac{\pi a^4}{8}$$

$$Iy_{semicircle} = \frac{\pi r^4}{8} + Ad^2$$

$$= \frac{\pi a^4}{8} + \frac{\pi a^2}{2}(a^2)$$

$$= \frac{\pi a^4 + 4\pi a^4}{8}$$

$$= \frac{5\pi a^4}{8}$$



$$Ix_{combine} = \frac{16a^4}{3} - \frac{\pi a^4}{8}$$

$$= \frac{128a^4 - 3\pi a^4}{24}$$

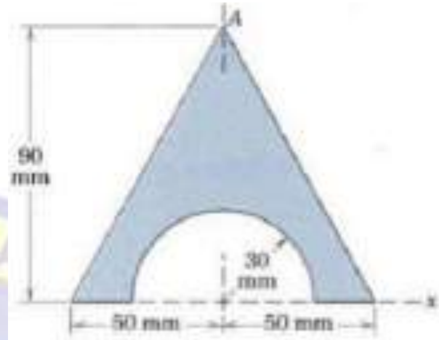
$$= \frac{128a^4 - 3\pi a^4}{24} = 4.94a^4$$

$$Iy_{combine} = \frac{16a^4}{3} - \frac{5\pi a^4}{8}$$

$$= \frac{128a^4 - 15\pi a^4}{24} = 3.37a^4$$

Example/6

Calculate the moment of inertia of the shaded area about the .x-axis



$$Ix = Ix_1 - Ix_2$$

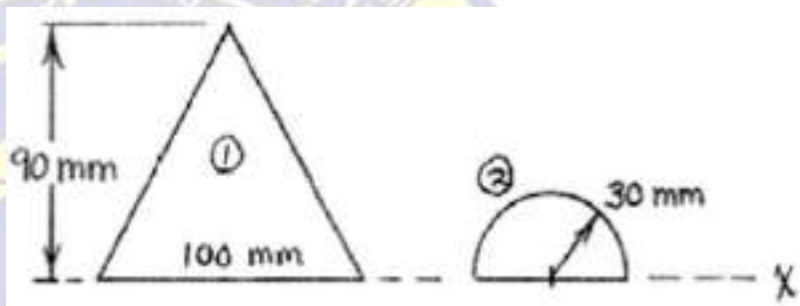
$$Ix_1 = \frac{bh^3}{12} = \frac{100 * 90^3}{12} = 6.08 * 10^6 mm^4$$

$$Ix_2 = \frac{\pi r^4}{8} = \frac{\pi 30^4}{8}$$

$$Ix_2 = 0.318\ 10^6 mm^4$$

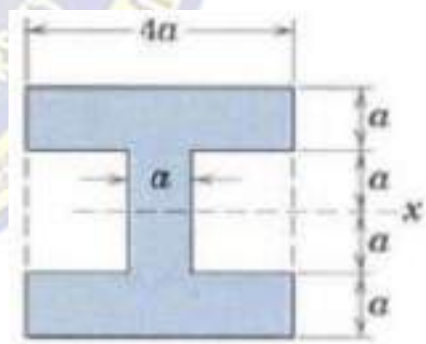
$$Ix = 6.08 * 10^6 - 0.318\ 10^6$$

$$Ix = 5.76 * 10^6 mm^4$$



Example/7

Determine the moment of inertia of the shaded area about the .x-axis

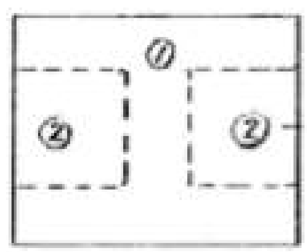


$$Ix_1 = \frac{bh^3}{12}$$

$$= \frac{4a(4a)^3}{12} = \frac{64a^4}{3}$$

$$Ix_2 = \frac{3a * (2a)^3}{12}$$

$$= 2a^4$$



$$Ix_{total=} Ix_1 - Ix_2$$

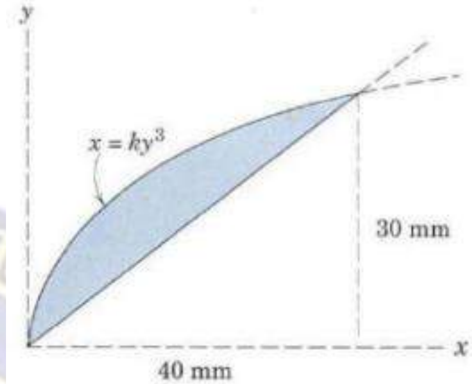
$$= \frac{64a^4}{12} - 2a^4$$

$$= \frac{58a^4}{3}$$

Example/8

Calculate the moment of inertia of the shaded area about the x-axis

for x=40 mm & y=30 mmm

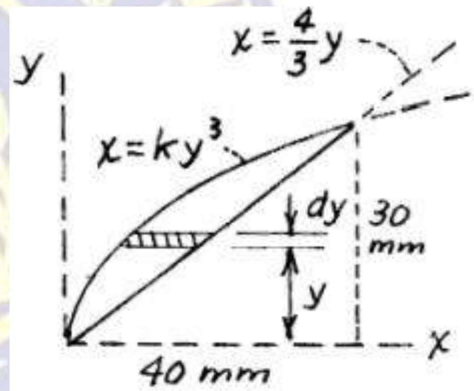


x=ky^3

40=k30^3

K=$\frac{40}{27*10^3}$

Ix =$y^2 dA = y^2(X_2 - X1)dy$



$$\text{Ix} = y^2(\frac{4}{3}y - \frac{40}{27000}y^3)dy$$

$$\text{Ix} = \int_0^{30}(\frac{4}{3}y^3 - \frac{4}{2700}y^5)dy$$

$$\text{Ix} = \frac{4}{3}\int_0^{30} y^3 dy - \frac{4}{2700}\int_0^{30} y^5\, dy$$

$$\text{Ix} = \frac{4}{3}\left[\frac{y^4}{4}\right]_0^{30} - \frac{4}{2700}\left[\frac{y^6}{6}\right]_0^{30}$$

$$\text{Ix} = \frac{1}{3} * 30^4 - \frac{4}{2700} * \frac{30^6}{6}$$

=9*10^4 mm^4

Engineering Mechanics

First Stage

Lecture-14-15

Dynamics type of Motion. Linear Motion with constant(Speed & acceleration)

Asst Lect. Hayder Salim

Dynamics

Dynamics is that branch of mechanics which deals with the motion of particles, lines and bodies. So, the dynamics is divided into two parts:

1 – **Kinematics**: which deals with the motion of the bodies without consideration of the forces required to produce the motion including the position, speed, velocity, acceleration … etc.

2 –**Kinetics:** which deals with the motion of the bodies with consideration of the forces required to produce the motion.

Definition

Motion: the change of position of one body with respect to another. The rate of change is the speed

Speed: change in distance with respect to time. Speed is a scalar rather than a vector quantity; i.e., the speed of a body tells one how fast the body is moving but not the direction of the motion.

Velocity: change in displacement with respect to time. Displacement is the vector counterpart of distance, having both magnitude and direction. Velocity is therefore also a vector quantity. The magnitude of velocity is known as the speed of a body.

linear motion with constant speed

Does linear motion have constant speed?

In uniform linear motion the distance and displacement are equal at every particular time interval. Therefore, the speed and velocity at every particular time interval are equal. So we can say that in uniform linear motion the speed and velocity are constant

Uniform motion

is motion at a constant speed in a straight line. can be described by a few simple equations. The distance (S) covered by a body moving with velocity (V) during a time (t) is given by $S = V.t$, If the velocity is changing, either in direction or magnitude, it is called Accelerated motion. If (a) is the acceleration, (V_o) the original velocity, and (V_f) the final velocity, then the final velocity is given by

distance:

total length of actual path covered in travelling from intial to final positions

Displacement:

short distance between initial and final position

Speed

Speed is the **rate of change** in **distance**.

Formula:

$$v = \frac{d}{t}$$

v = speed
d = distance travelled
t = time taken

Unit: **ms^{-1}**

Type of quantity: **Scalar quantity**

Velocity

Velocity is the rate of change in displacement.

Formula:

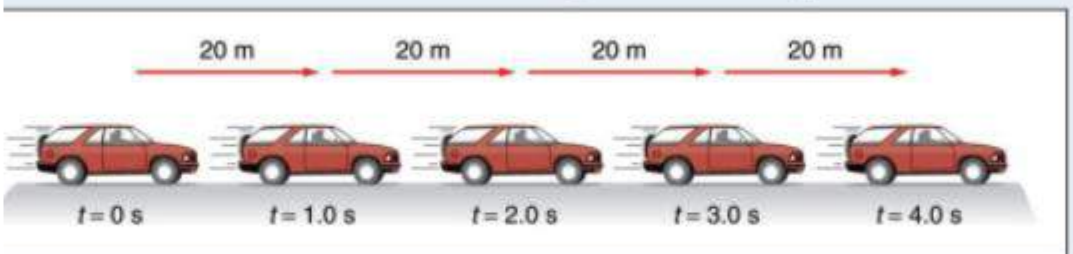
$$v = \frac{s}{t}$$

v = velocity
s = displacement
t = time taken

Unit: **ms^{-1}**

Type of quantity: **Vector quantity**

Constant Velocity - Example



- *Describe the speed of the car above.*
- GIVEN: d = 80 m & t = 4.0 s
- EQUATION: $\overline{v} = d/t$
- SOLVE: d/t = 80 m/4.0 s = 20 m/s = average speed
- Velocity would be described as 20 m/s in the direction of motion (east?)

Example /1

A car travels at a constant 10 m/s. Determine distance after 10 seconds and 60 seconds.

Solution

Constant speed 10 meters/second means car travels 10 meters every 1 second.

$$v = \frac{d}{t}$$

For t=10sec

$d = v * t$=10*10=100 m

For t= 60

$d = v * t$=10*60=600 m

Example /2

A car travels along a straight road at constant 72 km/h. Determine the car’s distance after 2 minutes and 5 minutes.

Solution

72 km/h = (72) (1000 meters) / 3600 seconds = 72,000 / 3600 seconds = 20 meters/second.

The constant speed at 20 meters/second means car travels 20 meters every 1 second.

After 120 seconds = 2 minutes, car travels

$d = v * t$

20 x 120 = 2400 meters= 2.4 km

After 300 seconds or 5 minutes, car travels 20 x 300 = 6000 meters=6 km.

Example /3

A body travels along a straight road for 100 meters in 50 seconds. Determine the speed of the body.

Solution

$$v = \frac{d}{t}$$

$$v = \frac{100}{50} = 2\ m/sec$$

Example /4

Cars A and B approach each other on parallel tracks. When the distance between the two cars is 100 meters, car A moves at a constant speed of 10 m/s, car B moves at a constant speed of 40 m/s. Determine (a) time interval before car B passing car A (b). the distance of car A before passing car B.

Solution

$d_A + d_B = d_T$

$v_A * t + v_B * t = 100$

t (10+40) =100

$$t = \frac{100}{50} = 2\ sec$$

$$d_A = v_a * t$$

$$d_A = 10 * 2 = 20\ m/sec$$

$$d_b = v_b * t$$

$$d_b = 40 * 2 = 80\ m/sec$$

Kinematics of a Particle

* لدينا ثلاث معادلات يمكن استخدامها لدراسة حركة جسيم يتحرك بعجلة متغيرة:

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

ومن هاتين المعادلتين يمكن استنتاج المعادلة الثالثة كما يلي:

$$v = \frac{ds}{(dv/a)} \quad \Rightarrow \quad v\,dv = a\,ds$$

Constant Acceleration

إذا كانت العجلة ثابتة ($a = a_c$) فبتكامل المعادلات الثلاثة السابقة نحصل على:

$$\int_{v_0}^{v} dv = \int_0^t a_c \; dt$$

$$\boxed{v = v_0 + a_c\, t}$$

$$\int_{s_0}^{s} ds = \int_0^t v \; dt \; = \int_0^t (v_0 + a_c\, t) \; dt$$

$$\boxed{s = s_0 + v_0\, t + \frac{1}{2}\, a_c\, t^2}$$

$$\int_{v_0}^{v} v\, dv = \int_{s_0}^{s} a_c\, ds$$

$$\boxed{v^2 = {v_0}^2 + 2\, a_c\, (s - s_0)}$$

Example /5

A bicyclist starts from rest and after traveling along a straight path a distance of 20 m reaches a speed of 30 km/h. Determine his acceleration if it is *constant*. Also, how long does it take to reach the speed of 30 km/h?

Solution:

$$v^2 = v_o^{\,2} + 2\,a\,(s - s_o)$$

$$\left(\frac{(30 * 1000)}{60 * 60}\right)^2 = 0 + 2\,a\,(20 - 0)$$

$$a = 1.736\ m/s^2$$

$$v = v_o + a\,t$$

$$\frac{(30 * 1000)}{(60 * 60)} = 0 + 1.736\,t$$

$$t = 4.80\ s$$

The following table explain the three equations of motion with constant acceleration according to the type of a motion :

Rectilinear motion (horizontal motion)	*Vertical motion*	
	Upward motion	*Downward motion*
$V_f = V_o + at$	$V_f = V_o - gt$	$V_f = V_o + gt$
$S = V_o.t + \frac{1}{2}a\ t^2$	$h = V_o.t - \frac{1}{2}g\ t^2$	$h = V_o.t + \frac{1}{2}g\ t^2$
$V_f^{\,2} = V_o^{\,2} + 2aS$	$V_f^{\,2} = V_o^{\,2} - 2gh$	$V_f^{\,2} = V_o^{\,2} + 2gh$

V_f = final velocity (m / sec) , Vo = initial velocity (m / sec) , a = linear acceleration (m / sec^2)
S = distance (displacement) (m) , g = gravitational acceleration (m / sec^2) ,

Example /6

A body is fall down from (5 m) high , In what time does reach the earth ?

Solution :

$$V_f^{\,2} = V_o^{\,2} + 2\,g\,h = 0 - 2 * 10 * 5 = \sqrt{100} = 10 \quad m/sec$$

$$V_f = V_o + gt$$

$10 = 0 + 10 * t$ .... $t = 1$ sec

Example /7

A stone is thrown vertically upward returns to the earth during (5 sec) , How high does it go?

Solution :

$$h = \frac{1}{2}\,.g\,.t^2$$

$$h_1 = 0.5 * 10 * t_1^2 = 5\,t_1^2$$

$$h_2 = 0.5 * 10 * (5 - t_1)^2 = 5\,(25 - 10\,t_1 + t_1^2) = 125 - 50t_1 + 5t_1^2$$

$$h_1 = h_2$$

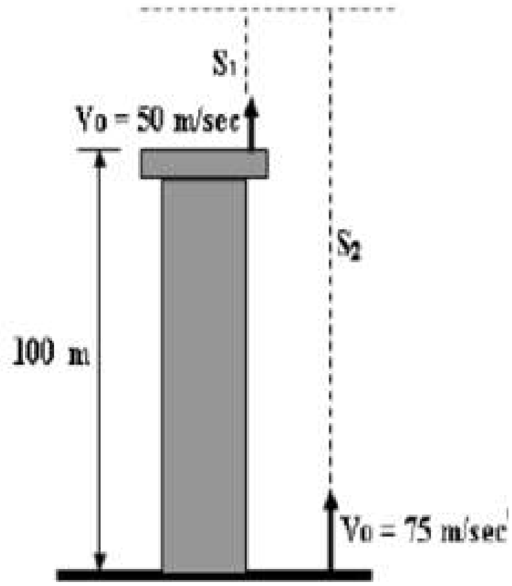
$$5t_1^2 = 125 - 50\,t_1 + 5\,t_1^2$$

$$5t_1^2 - 125 + 50\,t_1 - 5t_1^2 = 0 \longrightarrow 50\,t_1 = 125 \longrightarrow t_1 = 2.5\ sec$$

$$h = \tfrac{1}{2} * 10 * (2.5)^2 = 31.25\ m$$

Example /8

A stone is thrown vertically into the air from a tower (100 m) high , at the same instant that a second stone is thrown upward from the ground . The initial velocity of the first stone is (50 m / sec) and that of the second stone is (75 m / sec) , When and Where will the two stones be at the same height from the ground ?



Solution :

$$h = V_o \cdot t - \frac{1}{2} g \; t^2$$

$h_1 = 50 * t - ½ * 10 * t^2 = 50t - 5t^2$ .... (1)

$h_2 = 75 * t - ½ * 10 * t^2 = 75t - 5t^2$ .... (2)

$\pm h_1 = - 50t \pm 5t^2$

$h_2 = + 75t - 5t^2$

-------------------------------- adding

$h_2 - h_1 = 25t$

Then , $h_2 - h_1 = 100\ m$

$\therefore 100 = 25t \longrightarrow t = 4\ sec$

$h_1 = 50 * 4 - 5(4)^2 = 200 - 80 = - 120\ m$

$h_2 = 75 * 4 - 5(4)^2 = 300 - 80 = 220\ m$

Example /9

A body moving with constant acceleration 4 m/sec², the position of the body is x = 5 m at t = 0 with initial velocity 3 m/sec.

a- Determine the position and the velocity after 2 sec

b- Where the body maybe in position when its velocity 5 m/sec

Solution :

a-

$$x = x_0 + Vo.t + \frac{1}{2}a.t^2$$

$$= 5 + 3*2 + \frac{1}{2}*4*(2)^2 = 19\ m$$

$$Vf = Vo + a.t$$

$$= 3 + 4*2 = 11\ m/\sec$$

b-

$$Vf^2 = Vo^2 + 2a(x - x_0)$$

$$5^2 = 3^2 + 2*4(x-5)$$

$$25 = 9 + 8(x-5)$$

$$x = \frac{56}{8} = 7\ m$$

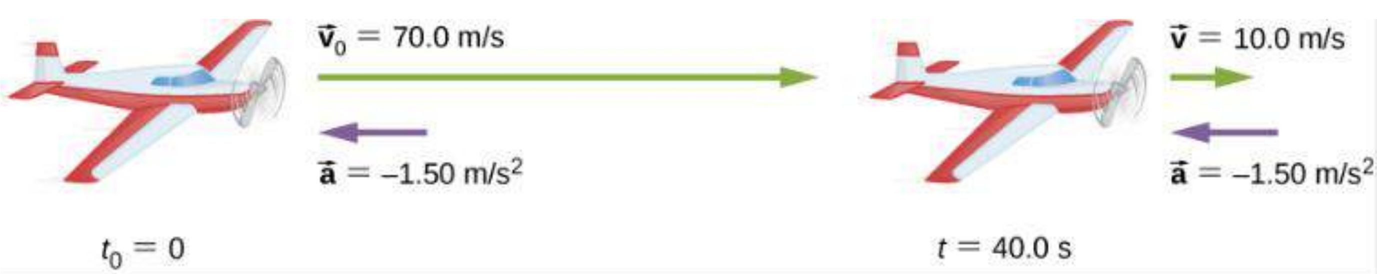
Example /10

An airplane lands with an initial velocity of 70.0 m/s and then accelerates opposite to the motion at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Solution

Substitute the known values and solve:

v=v0+at=70.0m/s+(−1.50m/s2)(40.0 s)=10.0 m/s.



Al-Furat Al-Awsat Technical University

Najaf Technical Institute

Aeronautic Technical Department

Subject

Engineering Mechanics

1st stage

Lecture-16-

newton's second law

Asst Lect. Hayder Salim

Newton's Laws (Empirical laws governing motion) :

Newton's First Law :

Every body continues in its state of rest or of motion at a constant velocity unless acted on by an unbalanced force (F).

i.e. if $F = 0$ ***the*** $a = 0$

(Force is something which changes the state of motion of a body)

The total force F can be the sum of several forces

Newton's Second Law :

If an unbalanced force (F) acts on a body it produces an acceleration (a) where :

$\boxed{F = m \cdot a}$ ***F = resultant force , m = mass of the body , a = linear acceleration***

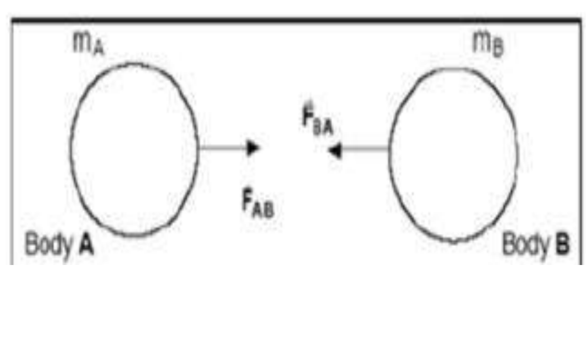
Mass ***: defined by this law is a measure of a body's resistance to motion (inertia) and is called the inertial mass. At a fixed place on the earth's surface:*** $F = W = m \cdot g$

(W) is called the weight of a body and (g) is the gravitational acceleration at the earth's surface.

Newton's Third Law :

"To every action there is an equal and opposite reaction".

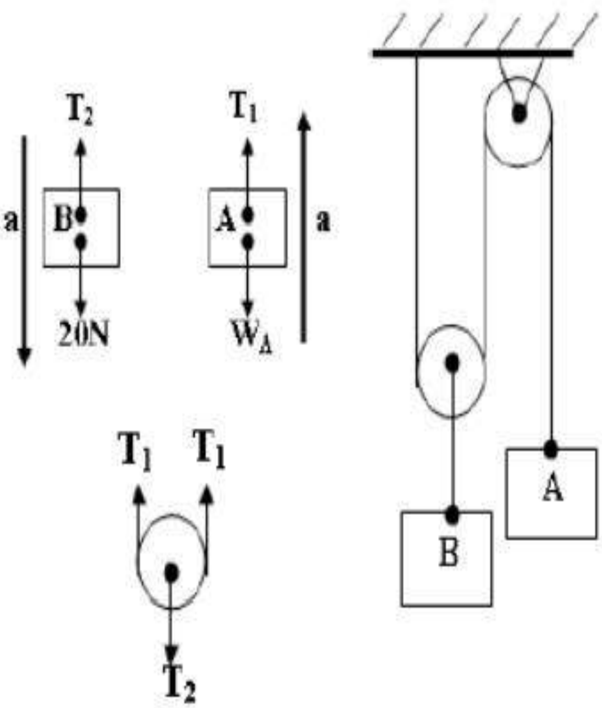
$F_{AB} = F_{BA}$



Example 1

Determine the weight of the body (A) to give the body (B) of (20 N) weight a downward acceleration of 0.5 m/sec² ?

<u>Solution :</u>



For the body (A) : $F = m \cdot a$

$$T_1 - W_A = \frac{W_A}{10} * a_1$$

$$10T_1 - 10W_A = W_A * a_1 \quad \ldots\ldots\ldots \ (1)$$

For the body (B) : $F = m \cdot a$

$$20 - T_2 = \frac{20}{10} * a_2$$

$$20 - T_2 = 2 * a_2$$

$$20 - T_2 = 2 * 0.5$$

$$20 - T_2 = 1 \Rightarrow 20 - 1 = T_2 \Rightarrow T_2 = 19 \quad N$$

For the pulley :

$T_2 = 2\ T_1 \longrightarrow 19 = 2\ T_1 \longrightarrow\ T_1 = 9.5 \quad N$

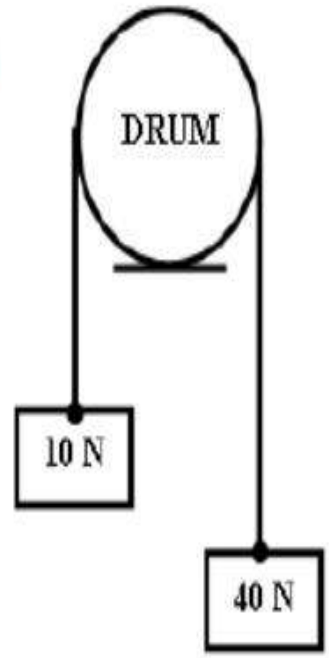
$a_2 = 0.5a_1 \quad a_1 = 1$

Subst. In (1) :

$W_A = 8.63\ N$

Example 2

Determine the acceleration of each block , and the tension in the cord , if the fixed drum is smooth ?



Solution :

For the first block (10 N) weight :

$F = m \cdot a$

$$T - 10 = \frac{10}{10} \cdot a$$

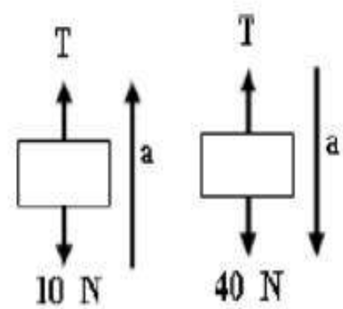
$$T - 10 = a \quad ....(1)$$

For the second block (40 N) weight :

$F = m \cdot a$

$$40 - T = \frac{40}{10} \cdot a$$

$$40 - T = 4\,a \quad .... (2)$$



Subst. (1) in (2) :

$40 - T = 4\,(T - 10) \rightarrow .... 40 - T = 4\,T - 40 \rightarrow ..... T = 16 \;\; N$

Subst. In (1) : $16 - 10 = a$ .... $a = 6 \;\; m/sec^2$

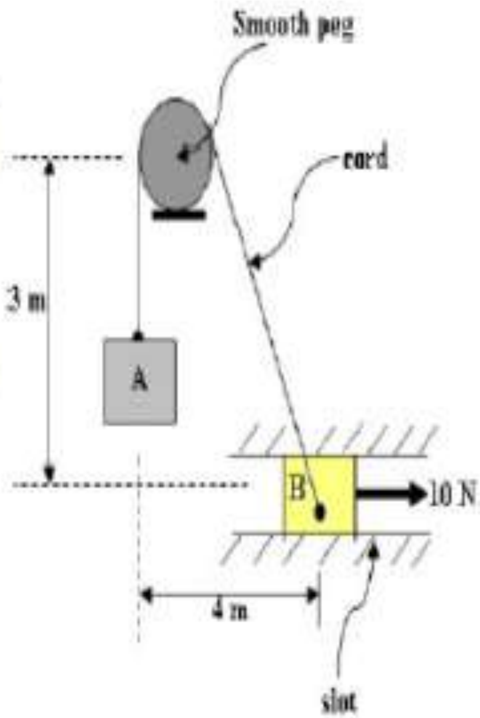
Example 3

Block (A) weighs (8 N) , Block (B) weighs (16 N) , The horizontal slot is smooth , the length of the slot is (7.2 m) , Determine :

a – the tension in the cord .

b – the acceleration of each block .

c – the required time for the block (B) to complete its motion along the slot . ($g = 10\ m/sec^2$)



Solution :

a – For the block (A) : $F = m \cdot a$

$$T - 8 = \frac{8}{10} \cdot a \quad \ldots\ldots \rightarrow \quad 10\,T - 80 = 8 \cdot a \text{ ------- } (1)$$

For the block (B) : $F = m \cdot a$

$$10 - \frac{4}{5}T = \frac{16}{10} \cdot a \quad \ldots\ldots \rightarrow \quad 500 - 40\,T = 80\,a \text{ -------- } (2)$$

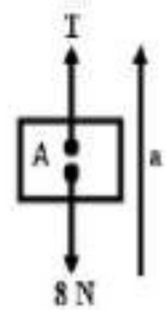
From (2) :

$$a = \frac{500 - 40\,T}{80} \text{ -------- } (3)$$

Subst. (3) in (1) :

$$10\,T - 80 = 8\left(\frac{500 - 40\,T}{80}\right)$$

$$100\,T - 800 = 500 - 40\,T \quad \ldots\ldots \rightarrow \quad T = 9.2\,N$$



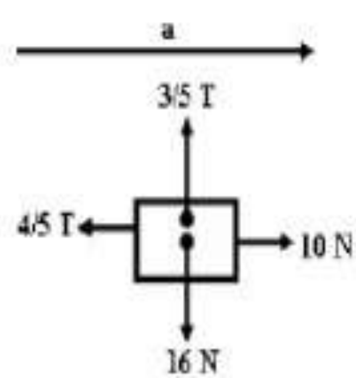
b- Subst. In (3) :

$$a = \frac{500 - 40 * 9.2}{80} = 1.6\ m/sec^2$$

c- $S = V_o \cdot t + \frac{1}{2} a\, t^2$

S= the length of slot

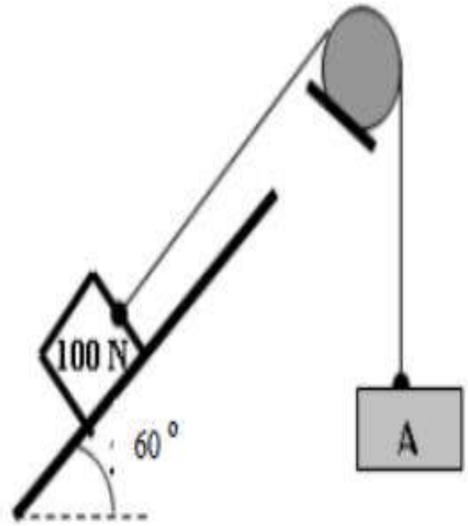
$$7.2 = 0 * t + \frac{1}{2}(1.6)\,t^2 \ \ldots\ldots \rightarrow t = 3\ sec$$



4

Example 4

Determine the weight of the body (A) to give the block (100 N) an acceleration of (0.5 m / sec^2) , $\mu = 0.1$?



<u>*Solution :*</u>

<u>*For the block (100 N) :*</u>

$F_f = \mu \cdot N = 0.1 * 100 \cos 60 = 5\ N$

$F = m \cdot a$

$T - 100 \sin 60 - F_f = (100 / 10) \cdot a$

$T - 91.6 = 10 \cdot a$

$T - 91.6 = 10 \cdot * 0.5$

$T = 96.6\ N$

<u>*For the block (A) :*</u>

$F = m \cdot a$

$W_A - T = (W_A / 10) \cdot a$

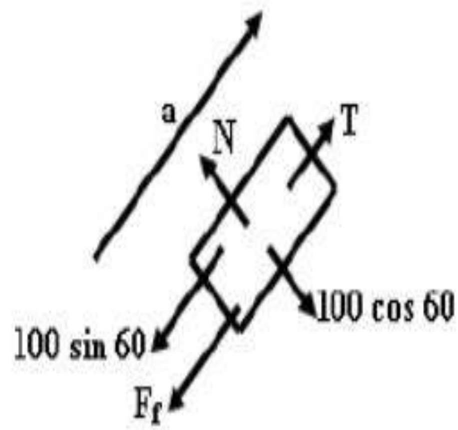
$W_A - 96.6 = (W_A / 10) * 0.5$

$10\ W_A - 960.6 = 0.5 * W_A$

$10\ W_A - 0.5\ W_A = 960.6$

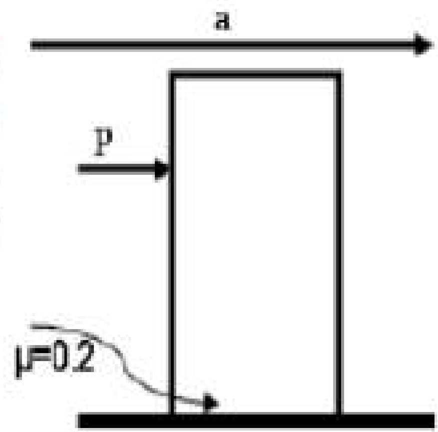
$9.5\ W_A = 960.6$

$W_A = 960.6 / 9.5 = 193.2\ N$



Example 5

A man wants to slide the homogeneous (100 N) box shown in fig . across the floor by pushing on it with the force (P) , the coefficient of friction between the box and the floor is (0.2) . Determine the force (P) to give the box an acceleration of (8 m / sec^2) ?



<u>*Solution*</u> *:*

$\sum Fy = 0 \longrightarrow N - 100 = 0 \longrightarrow N = 100$

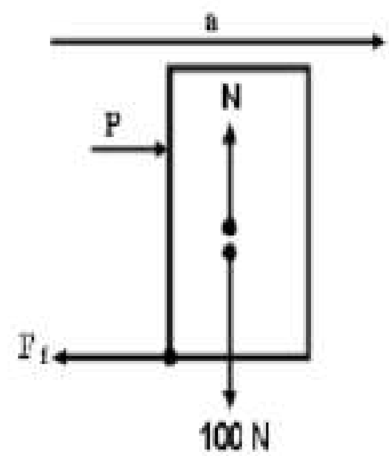
$F_f = \mu . N = 0.2 * 100 = 20\ N$

$F = m . a$

$P - F_f = (100 / 10) * 8$

$P - 20 = 10 * 8$

$P - 80 + 20 - 100\ N$



Al-Furat Al-Awsat Technical University

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Aeronautic Technical Department

Subject

Engineering Mechanics

1st stage

Lecture-17-18-19

Angular motion&
Work, energy, power

Asst Lect. Hayder Salim

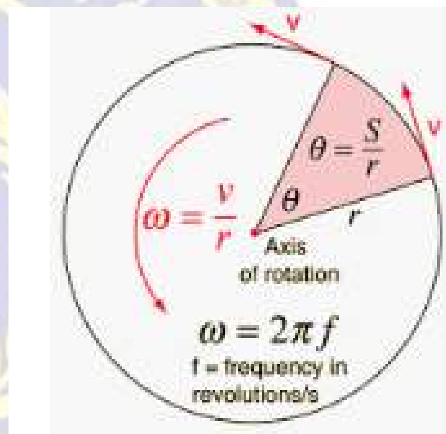
Angular motion

Angular motion is defined as, the motion of a body about a fixed point or fixed axis. It is equal to the angle passed over at the point or axis by a line drawn to the body.

The simplest angular motion is one in which the body moves along a curved path at a constant angular velocity, as when a runner travels along a circular path or an automobile rounds a curve.

An angle at any moment is determined by the angle θ made by its linear position vector r with the x-axis. If the particle is at position θ1 at moment t1, then it is at position θ2 at moment t2,

Then we find that it is an angular motion



$\Delta\theta = \theta_1 - \theta_2$

at time

$\Delta t = t_2 - t_1$

And then we know the angular velocity by the relationship

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

ω = angular velocity rad/sec

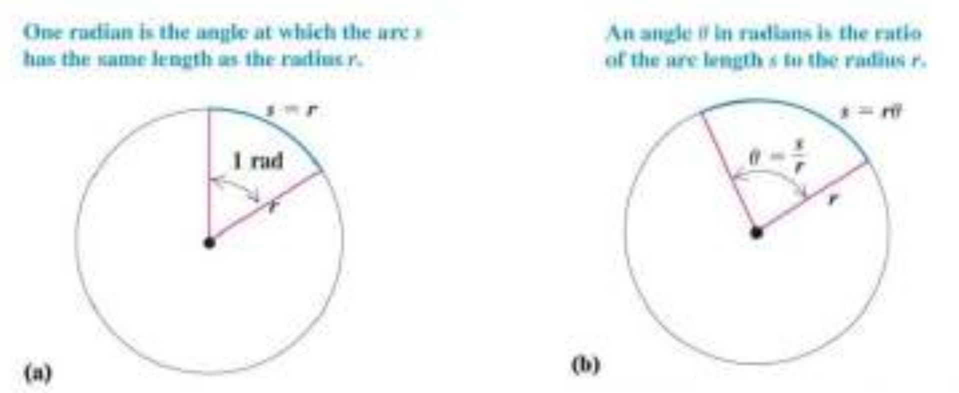
Δθ=change in angular rotation

Δt =change in time sec

Angular Velocity	Linear Velocity
$\omega = \frac{\theta}{t}$ ω in radians per unit time θ in radians	$v = \frac{s}{t}$ $v = \frac{r\theta}{t}$ $v = r\omega$

➜ measuring θ in degrees turns out to be a poor choice

➜ **radians** are a more natural choice of angular unit



$$1\,\text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

Example 8

The wheels of moving car rotate at 13.5 revolutions per second. What is the angular velocity of a point on the wheel that is not on the axis of rotation of the wheel

1 rev= 2π rad

ω =13.5 *2 π=27 π rad= 84.8 rad/sec

$$\omega \approx 85\, rad/sec$$

Example 9

The radius of a car wheel is 0.25 m/s and its linear velocity Calculate the angular velocity of the wheel

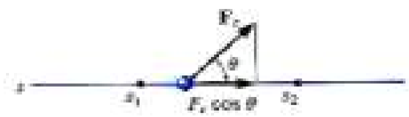
$$\omega = \frac{v}{r}$$

$$\omega = \frac{25}{0.25} = 100\, rad/sec$$

Work by Forces

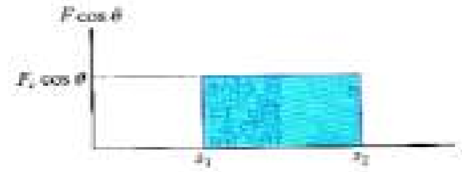
work of a constant force:

$$U_{1-2} = F_c \cos\theta \, (s_2 - s_1)$$



فإذا كانت القوة بنفس اتجاه الحركة:

$$U_{1-2} = F_c \, (s_2 - s_1)$$



work of a weight:

$$U_{1-2} = |W \Delta y|$$

وتكون موجبة إذا كانت Δy لأسفل، وسالبة إذا كانت Δy لأعلى.

POWER :

The rate of work done by the body per time .

P : Power , F : Applied force , S : displacement , V : velocity

$$P = \frac{W}{t} = \frac{F \, . \, S}{t} = F \, . \, V$$

ENERGY

The ability to do the work . Divided into two parts :

1 – Potential Energy (P.E) : The energy which stored in or loosed from the body .

$P.E = W \, . \, h = m \, . \, g \, . \, h$

W : the weight of the body , h : the high of the body

2 – Kinetic Energy (K.E) : The ability of the body to do the work due to its velocity .

$K.E = \frac{1}{2} \, . \, m \, . \, V^2$

m= mass of the body , V = velocity

Principle of Work and Energy

$$T_1 + \sum U_{1-2} = T_2$$

means that the particle's intial kinetic energy plus the work done by all the forces acting on the particle as it moves equals to the final kinetic energy

where :

$$T_1 = \frac{1}{2} m \upsilon_1^2, \qquad T_2 = \frac{1}{2} m \upsilon_2^2$$

Example 1:

Find the work done by a body of (5 N) weight which is fallen down from (10 m) height ?

Solution :

Work done = P.E = m . g . h = (5/10) X 10 X 10 = 50 J

Example 2:

Determine the total energy of (10 tones) airplane mass when it is flying with (15 m/sec) and of (2500 m) high?

Solution :

P.E = m . g . h = 10 X 1000 X 10 X 2500 = 250 000 000 J

K.E = ½ . m . V_f^2 = ½ X 10 000 X (15)^2 = 1125000 J

E_T = P.E + K.E = 250 000 000 + 1125 000 = 26125 000 J

Example 3

The diesel engine of a 400-Mg train increases the train's speed uniformly from rest to 10 m/s in 100 s along a horizontal track. Determine the average power developed.

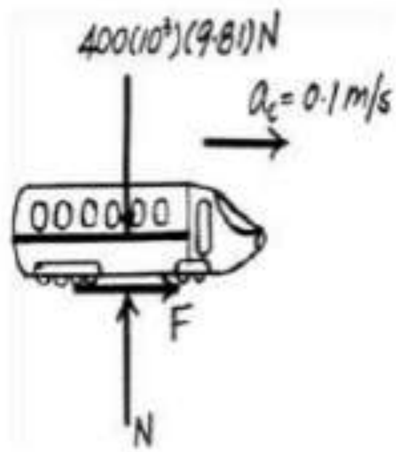
- **Solution:**

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + U_{1-2} = \frac{1}{2}(400)(10)^3(10)^2$$

$$U_{1-2} = 20(10)^6 \text{ J}$$

$$P_{avg} = \frac{U_{1-2}}{t} = \frac{20(10)^6}{100} = 200 \text{ kW}$$



Also,

$$v = v_0 + a_c t$$

$$10 = 0 + a_c(100)$$

$$a_c = 0.1 \text{ m/s}^2$$

Engineering Mechanics

First Stage

Lecture-20

Strength of material, Fundamental concepts, Hook's Law, stress-strain curve

Asst Lect. Hayder Salim

Strength of material

In mechanics of materials, the strength of a material is its ability to withstand an applied load without failure or plastic deformation. Strength of materials basically considers the relationship between the external loads applied to a material and the resulting deformation or change in material dimensions

Stress: -Every material is elastic in nature That is why whenever some external system of farce Some deformation as the body undergoes deformation its molecules set up some resistance to deformation This Resistant Per unit area to deformation stress acts body it under goes set up Some is known as

$$stress\ \sigma = \frac{p}{A}$$

Where

P: - load or force acting on the body

A: - cross-section area of the body

In S.I the unite of stress is Pascal (pa) = 1 N/m^2

Mega Pascal (Mpa) = $N/mm^2 * 10^6$

Giga Pascal (Gpa) = KN/mm2 * 10^9

Classification of Stress

Generally following engineering stresses are classified in strength of materials studies.

Type of stress

1- normal stress 2- shearing stress 3- bearing stress

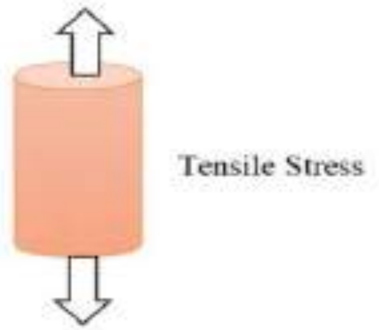
Normal Stress

- When the applied force is perpendicular to cross-section area of the specimen (axial load), then the corresponding stress produced in the material is known as normal stress.
- Many times force applied on the surface is not uniform; in that case, we take an average of the applied force.

$$\text{Normal Stress} = \frac{p}{A}$$

1- Tensile Stress

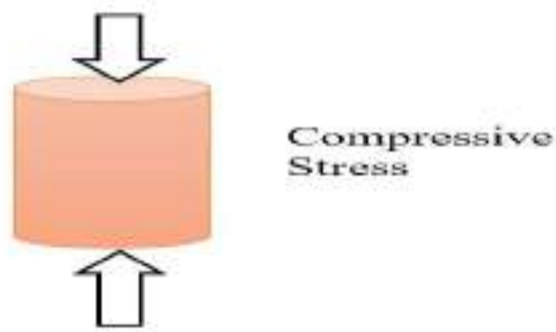
When a section is subjected to two equal and opposite pulls and the body tends to increase its length as shown in fig. The stress is induced is called tensile stress the corresponding strain is called tensile strain as a result of the tensile strass the cross-section area of the body gets reduced



Strength of Materials : Tensile Stress

2-Compressive Stress

When a- section is subjected to two equal opposite pushes and the body tends to shorten its length as in fig. the stress induce is called compressive stress. the corresponding strain is called compressive strain .as result of the compressive stress, the cross section area of body gets increased



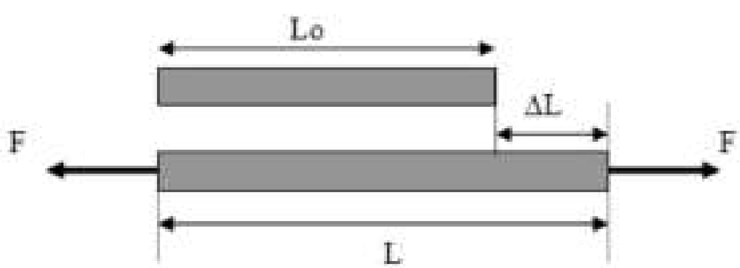
Strain

Strain is the deformation of a material from stress. It is simply a ratio of the change in length to the original length. Deformations that are applied perpendicular to the cross section are normal strains, Change in length at some instant of the material per unit original length

$$\varepsilon = \frac{\Delta L}{L_0}$$

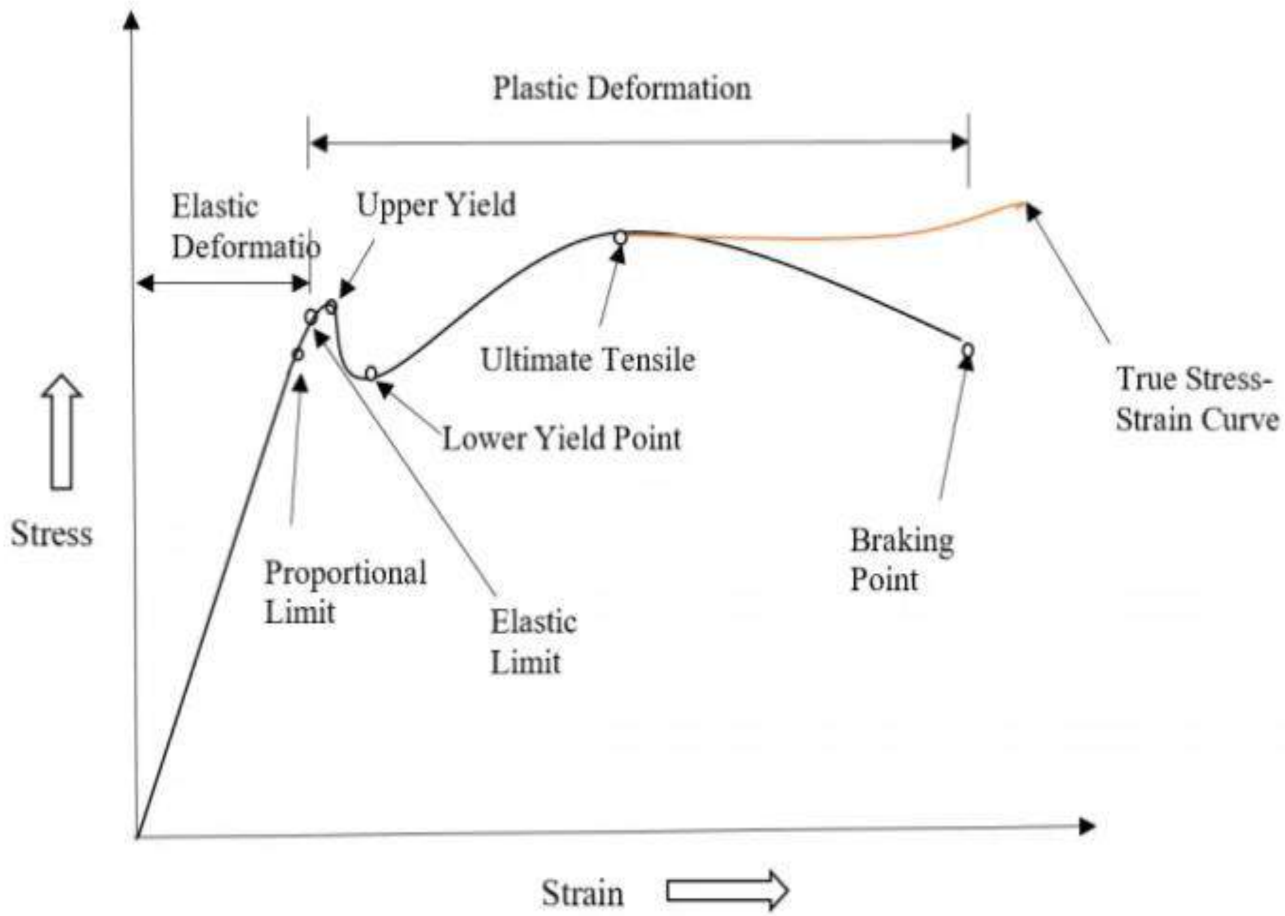
where

- ε strain
- ΔL total elongation [m]
- L_0 original length [m]



Stress- strain curve

- Plotting of stress to strain gives a considerable number of properties of the material in strength of materials study.
- The stress-strain curve is stress versus strain curve in which strain is on independent axis i.e., x-axis and stress is on dependent i.e. y-axis. It is an important characteristic of the material.
- On the load application, two types of deformation occur in the material depending upon the strain value, first is elastic deformation and second is plastic deformation.



Elastic Deformation

- Elastic deformation is the deformation in which material regains its original shape on the removal of the force.
- This region has a proportional limit, elastic limit, upper yield point and lower yield point.

Modulus of Elasticity | Hooke's Law

- When this type of deformation occurs, the strain in the metal piece is nearly proportional to the stress; therefore, this deformation occurs as a straight line in Stress versus strain plot except for some materials like grey cast iron, concrete and many polymers.
- Stress is proportional to the strain through this relationship.

$$\boldsymbol{\sigma} \propto \in$$

- This is known as Hooke's Law, where Y the proportionality constant is known as Young's Modulus or Modulus of Elasticity. It is also denoted by E. It is the slope of the stress-strain curve in the elastic limit. It is one of the most important law in the studies of strength of material.

Modulus of Elasticity Formula

$$E = \frac{\sigma}{\in}$$

Its value is slightly higher for ceramics than metals and value is slightly lower for polymers than metals. Or most structures are required to have deformation only in the elastic limit; therefore, this region is quite important.

Elastic Limit

- It is the point in the curve up to which material shows elastic behaviour.
- After this point, plastic deformation in the material begins.
- Beyond the elastic limit, Stress causes the material to flow or yield.

yield point phenomenon.

- **Upper Yield Point:** It is the point in the graph at which maximum load or Stress required to initiate the plastic deformation of the material.
- **Lower Yield Point**: It is a point at which minimum Stress or load is required to maintain the material's plastic behavior.

Ultimate Strength Definition | Ultimate Stress Definition

- After yielding, as plastic deformation continues, it reaches a maximum limit known as ultimate Stress or ultimate strength.
- It is also known as **Ultimate Tensile Strength (UTS)** or tensile strength. It is the maximum stress that can be sustained by material in tension.
- All deformation up to this point is uniform, but at this maximum stress, small narrowing of material begins to form, this phenomenon is termed as **'necking'.**

Rupture Point | Fracture Point | Breaking Point

- Stress necessary to continue plastic deformation starts to decrease after ultimate strength and eventually breaks the material at a point known as rupture point or fracture point.
- The stress of the material at rupture point is known as **'rupture strength'.**

Deformation of a body due to force acting on it: consider a body is subjected to a tensile stress

$$\sigma = \frac{P}{A} \qquad . \quad \epsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

$$deformation \qquad \delta L = \epsilon * L = \frac{\sigma L}{E} = \frac{PL}{AE}$$

Not

1- we can use this equation also for compression is the same as that for tension

2- same time in calculation the tensile stress and tensile strain are taken as positive where as compressive stress and compressive strain as negative

Plastic Deformation

- If the applied force is removed in this region, then the material does not regain its original shape.
- The deformation in the material is permanent.
- In this region, Hooke's law is not valid.
- This region has ultimate tensile strength of materials and breaking point.
- There are some points on the curve around which type of deformation changes. These points are very important as they tell us about the limitations and ranges of material which are ultimately useful in material's application.

EXAMPLE 3.2. *A hollow cylinder 2 m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa.*

SOLUTION. Given : Length (l) = 2 m = 2 × 10^3 mm ; Outside diameter (D) = 50 mm ; Inside diameter (d) = 30 mm ; Load (P) = 25 kN = 25 × 10^3 N and modulus of elasticity (E) = 100 GPa = 100 × 10^3 N/mm^2.

Stress in the cylinder

We know that cross-sectional area of the hollow cylinder.

$$A = \frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4} \times [(50)^2 - (30)^2] = 1257 \text{ mm}^2$$

and stress in the cylinder,

$$\sigma = \frac{P}{A} = \frac{25 \times 10^3}{1257} = 19.9 \text{ N/mm}^2 = 19.9 \text{ MPa} \qquad \textbf{Ans.}$$

Deformation of the cylinder

We also know that deformation of the cylinder,

$$\delta l = \frac{P.l}{A.E} = \frac{(25 \times 10^3) \times (2 \times 10^3)}{1257 \times (100 \times 10^3)} = 0.4 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 3.5. *A hollow steel tube 3.5 m long has external diameter of 120 mm. In order to determine the internal diameter, the tube was subjected to a tensile load of 400 kN and extension was measured to be 2 mm. If the modulus of elasticity for the tube material is 200 GPa, determine the internal diameter of the tube.*

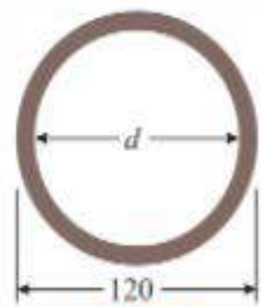


Fig. 3.3

SOLUTION. Given : Length (l) = 3.5 m = 3.5 × 10^3 mm ; External diameter (D) = 120 mm ; Load (P) = 400 kN = 400 × 10^3 N; Extension (δl) = 2 mm and modulus of elasticity E = 200 GPa = 200 × 10^3 N/mm^2.

Let d = Internal diameter of the tube in mm.

We know that area of the tube,

$$A = \frac{\pi}{4}\ [(120)^2 - d^2] = 0.7854\ [(120)^2 - d^2]$$

and extension of the tube (δl),

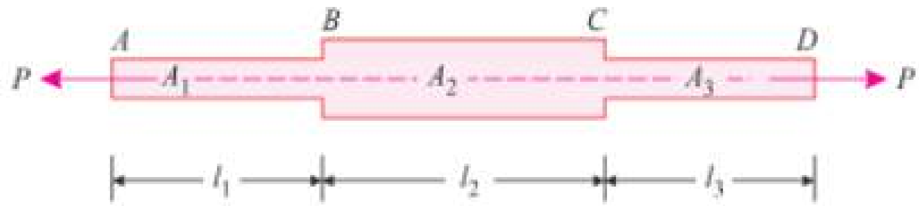
$$2 = \frac{P.l}{A.E} = \frac{(400 \times 10^3) \times (3.5 \times 10^3)}{0.7854\,[(120)^3 - d^2\,(200 \times 10^3)} = \frac{8913}{14400 - d^2}$$

$$\therefore \qquad 28800 - 2d^2 = 8913 \qquad \text{or} \qquad 2d^2 = 28800 - 8913 = 19887$$

$$\text{or} \qquad d^2 = \frac{19887}{2} = 9943.5 \qquad \text{or} \qquad d = 99.71 \text{ mm} \qquad \textbf{Ans.}$$

Stress in the bars of different section

Sometime a bar is made up of different lengths having different cross section area os shown in the figure



In such cases, the stresses, strains and hence changes in lengths for each section is worked out separately as usual. The total changes in length is equal to the sum of the changes of all the individual lengths. It may be noted that each section is subjected to the same external axial pull or push.

P = Force acting on the body,

E = Modulus of elasticity for the body,

l_1 = Length of section l

A_1 = Cross-sectional area of section 1,

I_2 A_2, = Corresponding values for section 2 and so on.

We know that the change in length of section 1.

$$\delta L_1 = \frac{PL_1}{A_1 E} \quad . \quad \delta L_2 = \frac{PL_2}{A_2 E} . \quad \delta L_3 = \frac{PL_3}{A_3 E}$$

Total deformation of the bar,

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3$$

$$\delta L = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} + \cdots\right]$$

Note: sometime the modulus of the elasticity is different for different section in such cases the total deformation

$$\delta L = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3} + \cdots\right]$$

EXAMPLE 4.3. *A 6 m long hollow bar of circular section has 140 mm diameter for a length of 4 m, while it has 120 mm diameter for a length of 2 m. The bore diameter is 80 mm throughout as shown in Fig. 4.4.*

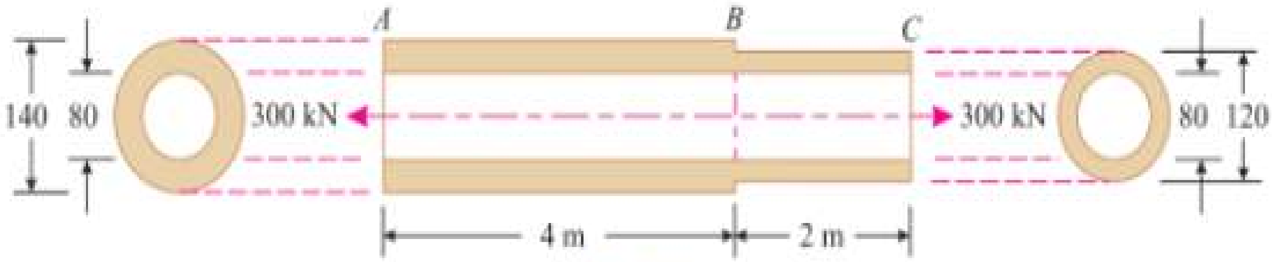


Fig. 4.4

Find the elongation of the bar, when it is subjected to an axial tensile force of 300 kN. Take modulus of elasticity for the bar material as 200 GPa.

SOLUTION. Given : Total length $(L) = 6 \text{ m} = 6 \times 10^3$ mm ; Diameter of section 1 (D_1)= 140 mm; Length of section 1 $(l_1) = 4 \text{ m} = 4 \times 10^3$ mm ; Diameter of section 2 (D_2) = 120 mm ; Length of section 2 $(l_2) = 2 \text{ m} = 2 \times 10^3$ mm ; Inner diameter $(d_1) = d_2$ = 80 mm ; Axial tensile force (P) = 300 kN = 300 × 10 N and modulus of elasticity $(E) = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$.

We know that area of portion *AB*,

$$A_1 = \frac{\pi}{4} \times [D_1^2 - d_1^2] = \frac{\pi}{4} \times [(140)^2 - (80)^2] = 3300\,\pi \text{ mm}^2$$

and area of portion *BC*.

$$A_2 = \frac{\pi}{4} \times [D_2^2 - d_2^2] = \frac{\pi}{4} \times [(120)^2 - (80)^2] = 2000\,\pi \text{ mm}^2$$

∴ Elongation of the bar,

$$\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right] = \frac{300 \times 10^3}{200 \times 10^3} \times \left[\frac{4 \times 10^3}{3300\pi} + \frac{2 \times 10^3}{2000\pi}\right] \text{mm}$$

$$= 1.5 \times (0.385 + 0.318) = 1.054 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 4.4. *A compound bar ABC 1.5 m long is made up of two parts of aluminium and steel and that cross-sectional area of aluminium bar is twice that of the steel bar. The rod is subjected to an axial tensile load of 200 kN. If the elongations of aluminium and steel parts are equal, find the lengths of the two parts of the compound bar. Take E for steel as 200 GPa and E for aluminium as one-third of E for steel.*

SOLUTION. Given: Total length $(L) = 1.5 \text{ m} = 1.5 \times 10^3 \text{ mm}$; Cross-sectional area of aluminium bar $(A_A) = 2\,A_S$; Axial tensile load $(P) = 200 \text{ kN} = 200 \times 10^3 \text{ N}$; Modulus of elasticity of steel $(E_S) = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$ and modulus of elasticity of aluminium $(E_A) = \frac{E_S}{3} = \frac{200 \times 10^3}{3} \text{ N/mm}^2$.

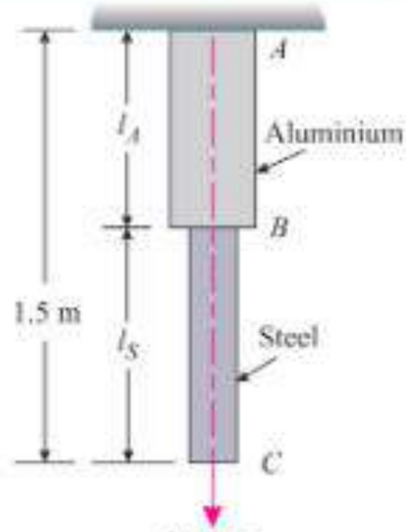


Fig. 4.5

Let, l_A = Length of the aluminium part,

and l_S = Length of the steel part.

We know that elongation of the aluminium part AB,

$$\delta l_A = \frac{P.l_A}{A_A.E_A} = \frac{(200 \times 10^3) \times l_A}{2A_S \times \left(\frac{200 \times 10^3}{3}\right)}$$

$$= \frac{1.5\,l_A}{A_S} \qquad ...(i)$$

and elongation of the steel part BC,

$$\delta l_S = \frac{P.l_S}{A_S.E_S} = \frac{(200 \times 10^3) \times l_S}{A_S \times (200 \times 10^3)} = \frac{l_S}{A_S} \qquad ...(ii)$$

Since elongations of aluminium and steel parts are equal, therefore equating equations (*i*) and (*ii*),

$$\frac{1.5 l_A}{A_S} = \frac{l_S}{A_S} \qquad \text{or} \qquad l_S = 1.5\,l_A$$

We also know that total length of the bar ABC (L)

$$1.5 \times 10^3 = l_A + l_S = l_A + 1.5\,l_A = 2.5\,l_A$$

$$\therefore \qquad l_A = \frac{1.5 \times 10^3}{2.5} = 600 \text{ mm} \qquad \textbf{Ans.}$$

and

$$l_S = (1.5 \times 10^3) - 600 = 900 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 4.2. *A member formed by connecting a steel bar to an aluminium bar is shown in Fig. 4.3.*

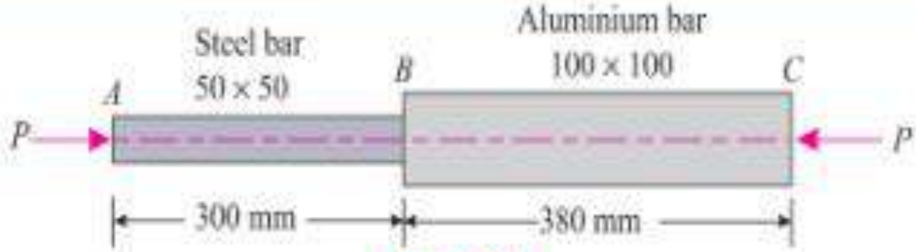


Fig. 4.3

Assuming that the bars are prevented from buckling sidewise, calculate the magnitude of force P, that will cause the total length of the member to decrease by 0.25 mm. The values of elastic modulus for steel and aluminium are 210 GPa and 70 GPa respectively.

SOLUTION. Given : Decrease in length (δl) = 0.25 mm ; Modulus of elasticity for steel (E_S) = 210 GPa = 210×10^3 N/mm^2 ; Modulus of elasticity for aluminium (E_A) = 70 GPa = 70×10^3 N/mm^2; Area of steel section (A_S) = 50 × 50 = 2 500 mm^2 ; Area of aluminium section (A_A) = 100 × 100 = 10000 mm^2 ; Length of steel section (l_S) = 300 mm and length of aluminium section (l_A) = 380 mm.

Let P = Magnitude of the force in kN.

We know that decrease in the length of the member (δl),

$$0.25 = P\left(\frac{l_S}{A_S E_S} + \frac{l_A}{A_A E_A}\right)$$

$$= P\left(\frac{300}{2500 \times (210 \times 10^3)} + \frac{380}{10000 \times (70 \times 10^3)}\right)$$

$$= \frac{780 P}{700 \times 10^6}$$

$$\therefore \quad P = \frac{0.25 \times (700 \times 10^6)}{780} = 224.4 \times 10^3 \text{ N} = 224.4 \text{ kN} \qquad \textbf{Ans.}$$

Engineering Mechanics

First Stage

Lecture-21

Shear Stress & Torsional Stress & Thermal Stress

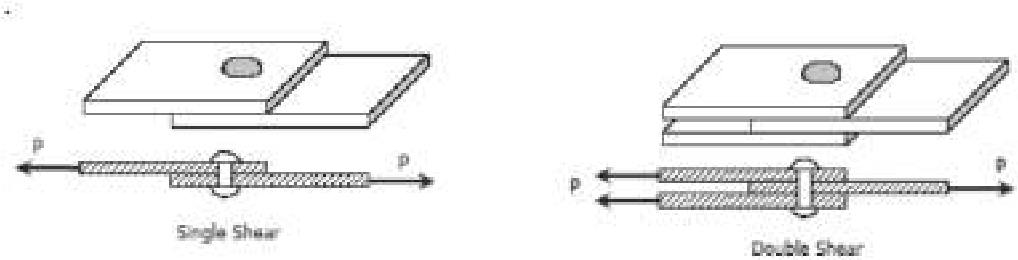
Asst Lect. Hayder Salim

Shearing Stress

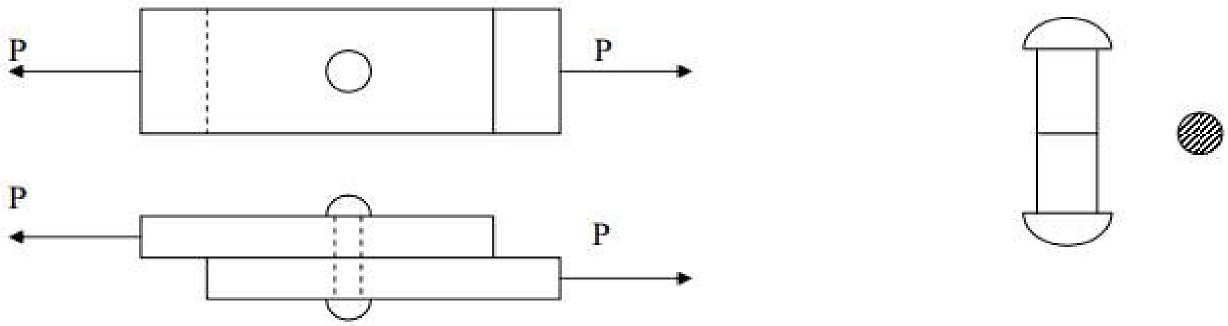
Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\boldsymbol{\tau = \frac{V}{A}}$$

where V is the resultant shearing force which passes which passes through the centroid of the area A being sheared.

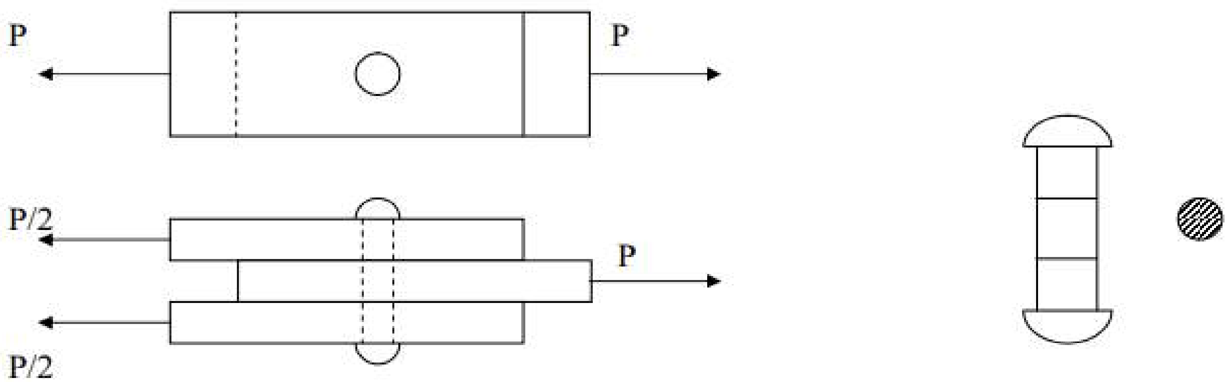


Area resisting shear is the shaded area as shown above.



Single shear stress

$$\tau = \frac{P}{A}$$

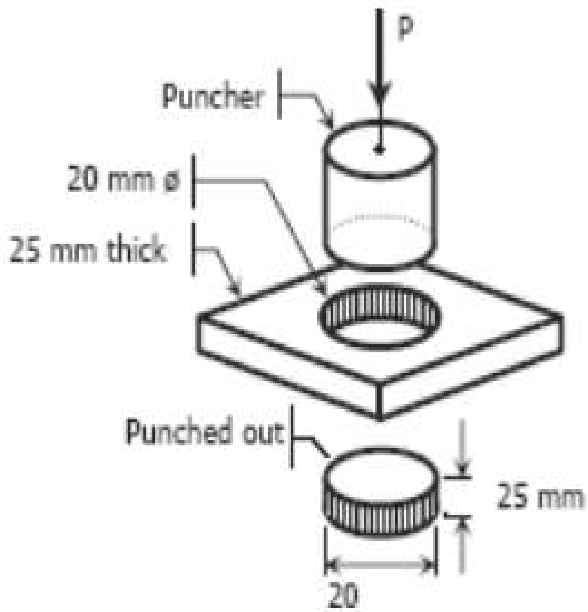


Double shear stress

$$\tau = \frac{P/2}{A}$$

Example1/ What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m^2.

Sol.



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

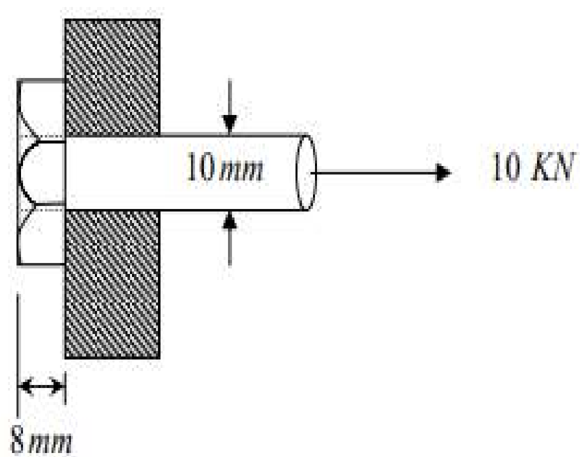
$$V = \tau A$$

$$P = 350[\pi(20)(25)]$$

$$= 549\,778.7 \text{ N}$$

$$= \mathbf{549.8\ kN}$$

Example 2: Consider a steel bolt 10 mm in diameter and subjected to an axial tensile load of 10 KN as shown. Determine the average shearing stress in the bolt head, assuming shearing on a cylindrical surface of the same diameter as the bolt.



$A=\pi dt$

$A=\pi\times10\times10^{-3}\times8\times10^{-3}=0.000251327 \text{ m}^2$

$$\tau = \frac{F}{A}$$

$$\tau=\frac{10\times10^3}{0.000251327}$$

$\tau=39.7888\times10^6 \text{ N/m}^2$

$\tau=39.7888 \text{ MN/m}^2$

Example 3/ Find the smallest diameter bolt that can be used in the clevis shown in Fig. if P = 400 kN. The shearing strength of the bolt is 300 MPa.

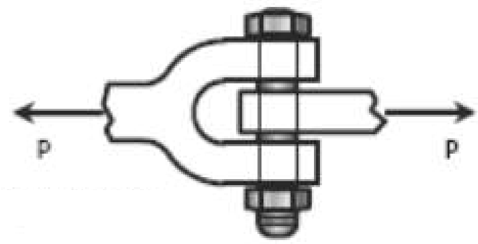
Sol.

The bolt is subject to double shear.

$V = \tau A$

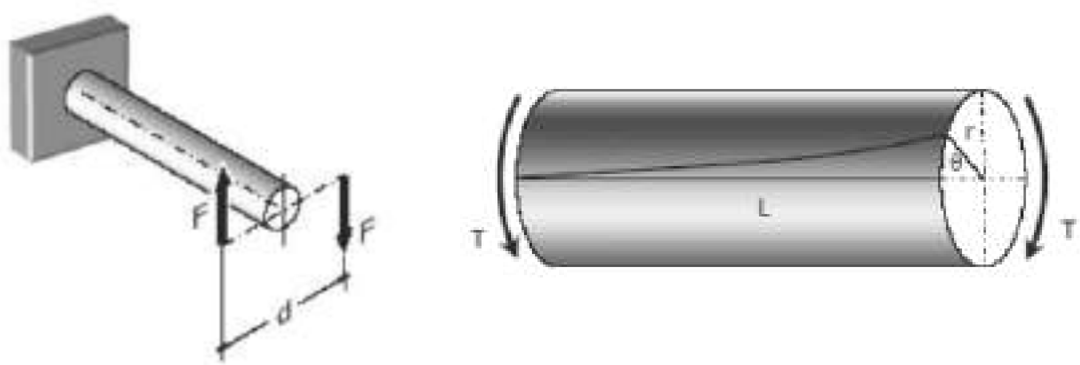
$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

$d = 29.13$ mm



Torsion Stress

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to F × d, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.



Torsional Shearing Stress, τ

For a solid or hollow circular shaft subject to a twisting moment T, the torsional shearing stress τ at a distance ρ from the center of the shaft is

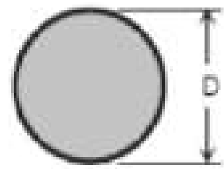
$$\tau \frac{T\,p}{J} \text{ and } \tau_{max} = \frac{T\,r}{J}$$

where J is the polar moment of inertia of the section and r is the outer radius.

For solid cylindrical shaft:

$$J = \frac{\pi}{32} D^4$$

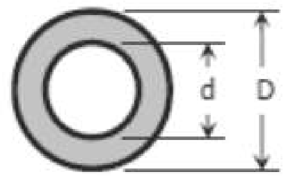
$$\tau_{max} = \frac{16\,T}{\pi\, D^3}$$



For hollow cylindrical shaft:

$$J = \frac{\pi}{32} (D^4 - d^4)$$

$$\tau_{max} = \frac{16\,TD}{\pi\,(D^4 - d^4)}$$



Angle of Twist

The angle θ through which the bar length L will twist is:

$$\theta = \frac{TL}{JG} \quad in\ radians$$

where **T** is the torque in N·mm, **L** is the length of shaft in mm, **G** is shear modulus in MPa, **J** is the polar moment of inertia in mm^4, **D** and **d** are diameter in mm, and **r** is the radius in mm.

Power Transmitted by The Shaft

A shaft rotating with a constant angular velocity ω (in radians per second) is being acted by a twisting moment T. The power transmitted by the shaft is

$$P = T \times \omega$$

In SI units power is expressed in (watts) when torque is measured in (N.m) and ω in (rad/s).

1 W=1 N.m/s

In the foot-pound-second or FPS system the units of power are (ft.lb/s); however horsepower (hp) is often used in engineering practice where:

1 hp=550 ft.lb/s

For machinery the frequency of a shaft's rotation f is often reported. This is a measured of the number of revolutions or cycles the shaft per second and is expressed in hertz (1 Hz=1 cycle/s), 1 cycle=2π rad, then ω=2πf

P=2πfT

Example 4: If a twisting moment of 1 KN.m is impressed upon a 50 mm diameter shaft, what is the maximum shearing stress developed? Also what is the angle of twist in a 1 m length of the shaft? The material is steel, for which G=85 GPa.

$$\tau_{max} = \frac{Tr}{J}$$

$$J = \frac{\pi}{32} D^4 = \frac{\pi}{32} (50 \times 10^{-3})^4 = 0.6135 \times 10^{-6} \text{ m}^4$$

$$\tau_{max} = \frac{1 \times 10^3 \times 25 \times 10^{-3}}{0.6135 \times 10^{-6}}$$

τ_{max}=40.74979 MPa

Example 5/A steel shaft 3 ft long that has a diameter of 4 in. is subjected to a torque of 15 kip·ft. Determine the maximum shearing stress and the angle of twist. Use G = 12 × 10^6 psi.

Sol.

$$\tau_{max} = \frac{Tr}{J}$$

$$J = \frac{\pi}{32} D^4 = \frac{\pi}{32} (50 \times 10^{-3})^4 = 0.6135 \times 10^{-6} \text{ m}^4$$

$$\tau_{max} = \frac{1 \times 10^3 \times 25 \times 10^{-3}}{0.6135 \times 10^{-6}}$$

τ_{max}=40.74979 MPa

$$\theta = \frac{TL}{GJ}$$

$$\theta = \frac{1 \times 10^3 \times 1}{85 \times 10^9 \times 0.6135 \times 10^{-6}}$$

θ=0.01917 rad.

Example 6/A steel propeller shaft is to transmit 4.5 MW at 3 Hz without exceeding a shearing stress of 50 MPa or twisting through more than 1° in a length of 26 diameters. Compute the proper diameter if G = 83 GPa.

$$T = \frac{P}{2\pi f} = \frac{4.5(1\,000\,000)}{2\pi(3)}$$

$$T = 238\,732.41 \text{ N·m}$$

Based on maximum allowable shearing stress:

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$50 = \frac{16(238732.41)(1000)}{\pi d^3}$$

$$d = 289.71 \text{ mm}$$

Based on maximum allowable angle of twist:

$$\theta = \frac{TL}{JG}$$

$$1°\left(\frac{\pi}{180°}\right) = \frac{238732.41(26d)(1000)}{\frac{1}{32}\pi d^4(83000)}$$

$$d = 352.08 \text{ mm}$$

Use the bigger diameter, d = **352 mm**

Example 7:

The gear motor can have developed 0.1 hp when it turns at 80 rev/min. If the allowable shear stress for the shaft is τallow=4 ksi, determine the smallest diameter of the shaft that can be used.

$$\tau_{allow} = \frac{Tr}{J}$$

$$J = \frac{\pi}{2} r^4$$

$$\tau_{allow} = \frac{Tr}{\frac{\pi}{2} r^4}$$

$$\tau_{allow} = \frac{2T}{\pi r^3}$$

$$r = \sqrt[3]{\frac{2T}{\pi \tau_{allow}}}$$

$$P = T.\omega \quad \Longrightarrow \quad T = \frac{P}{\omega}$$

$P = 0.1 \times 550 = 55$ lb/s

$\omega = 80 \times 2\pi/60 = 8.377$ rad/s

$$T = \frac{55}{8.377} = 6.5655 \text{ lb.ft}$$

$T = 6.5655 \times 12 = 78.786$ lb.in

$$r = \sqrt[3]{\frac{2 \times 78.786}{\pi \times 4 \times 10^3}} = 0.2323 \text{ in}$$

$d = 0.4646$ in

Thermal Stresses:

A change in temperature can cause a material to change its dimensions. If the temperature increases, generally a material expands, whereas if the temperature decreases the material will contract. The deformation of a member having a length L can be calculated using the formula:

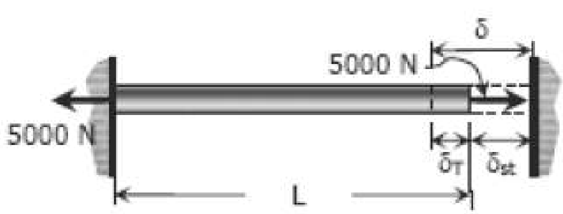
$$\delta_T = \alpha \times \Delta T \times L$$

$$\delta_T = \frac{FL}{AE} = \alpha \times \Delta T \times L$$

$$\sigma_T = E \times \alpha \times \Delta T$$

α: Linear coefficient of thermal expansion. The units measure strain per degree of temperature. They are (1/°F) in the foot-pound-second system and (1/°C) or (1/°K) in SI system.
ΔT: Change in temperature of the member.
L: The original length of the member.
δ_T: The change in length of the member.

Example 8/A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$

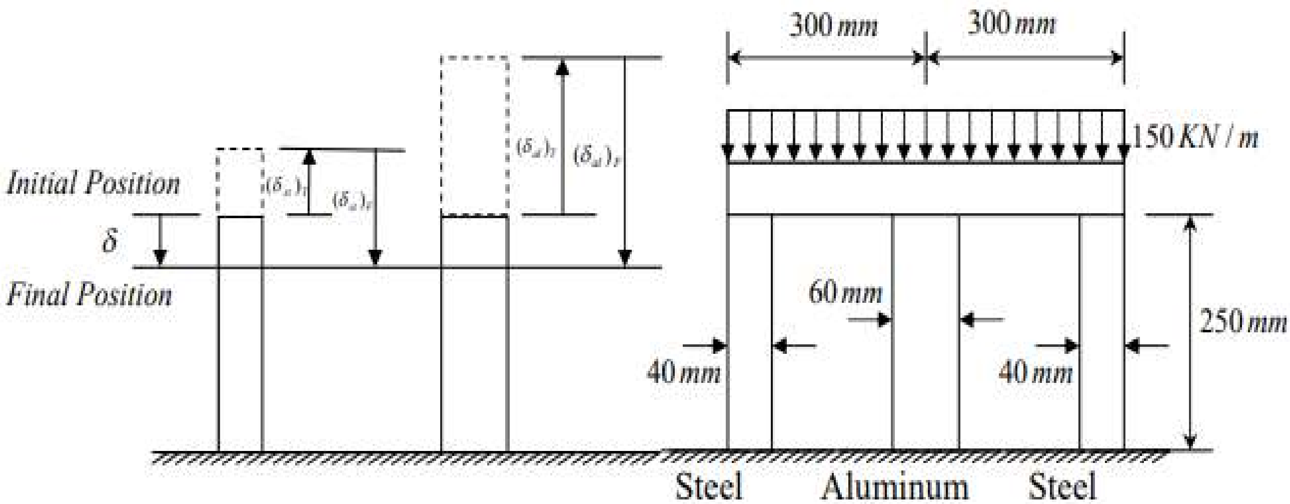
$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

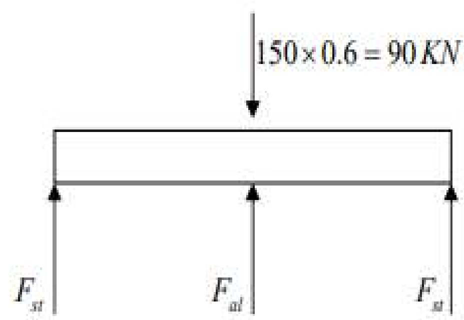
$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

$$\tfrac{1}{4}\pi d^2 = 137.36; \quad d = 13.22 \text{ mm}$$

Example 9/: The rigid bar shown is fixed to the top of the three posts made of A-36 steel and 2014-T6 aluminum. The posts each have a length of 250 mm when no load is applied to the bar, and the temperature is T1=20ºC. Determine the force supported by each posts if the bar is subjected to a uniform distributed load of 150 KN/m and the temperature is raised to T2=80ºC. For steel α=12×10-6 1/ºC, E=200 GPa , for aluminum α=23×10-6 1/ºC, E=73.1 G



$\sum F_y = 0$



$2F_{st}+F_{al}=90000$ ....(1)

$\delta=(\delta_{st})_T-(\delta_{st})_F=(\delta_{al})_T-(\delta_{al})_F$

$$[\alpha\times\Delta T\times L-\frac{F_{st}L}{AE}]_{st}=[\alpha\times\Delta T\times L-\frac{F_{al}L}{AE}]_{al}$$

$$12\times10^{-6}\times0.25\times(80\text{-}20)-\frac{F_{st}\times0.25}{\frac{\pi}{4}(40\times10^{-3})^2\times200\times10^9}=23\times10^{-6}\times0.25\times(80\text{-}20)-\frac{F_{al}\times0.25}{\frac{\pi}{4}(60\times10^{-3})^2\times73.1\times10^9}$$

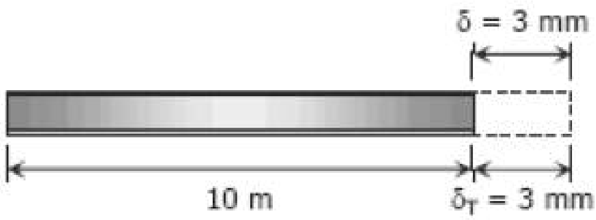
$1.20956\times10^{-9}F_{al}-0.994718\times10^{-9}F_{st}=0.000165$ ....(2)

From equations (1) and (2)

F_{st}=-16444.7 N

F_{al}=122888.8 N

Example 10/: Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume $\alpha = 11.7$ µm/(m·°C) and E = 200 GPa.



Temperature at which $\delta_T = 3$ mm:

$$\delta_T = \alpha L(\Delta T)$$

$$\delta_T = \alpha L(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$$T_f = \mathbf{40.64°C}$$

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T)$$

$$\sigma = \alpha E(T_f - T_i)$$

$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$

$$\sigma = \mathbf{60\ MPa}$$

Engineering Mechanics

First Stage

Lecture -22-

Beams, types of beams, types of loads

Shear force (S.F) &bending moment (B.M) of simple support beam under an axial load

Asst Lect. Hayder Salim

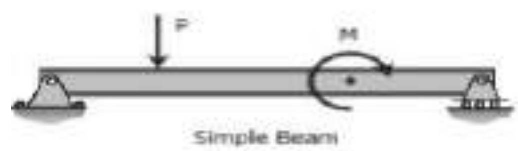
Definition of A Beam

A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal of the bar. beams. In regards to beams, if the reaction forces can be calculated using equilibrium equations alone, they are statically determinate. On the other hand, if the reaction force can't be determined using equilibrium equations only, other methods have to be used, and the structure is said to be statically indeterminate.

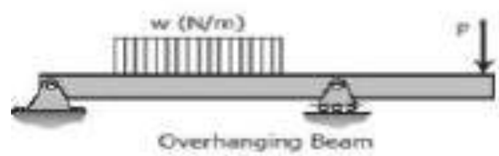
Type of beams

Cantilever beam

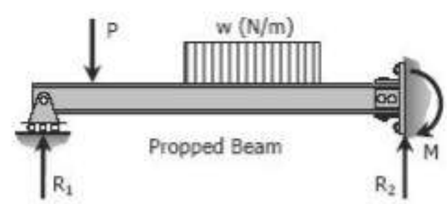
Simple supported beam



Overhanging beam

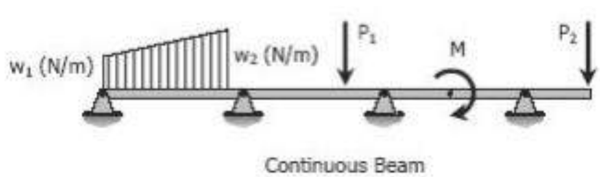


Propped beam

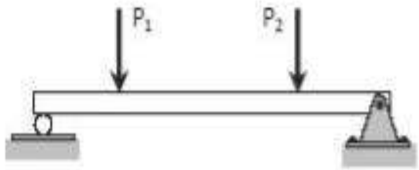


Fixed or restrained beam

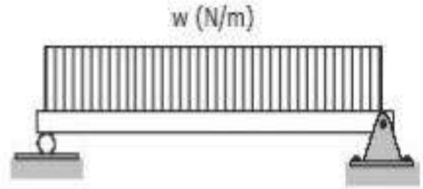
Continuous beam



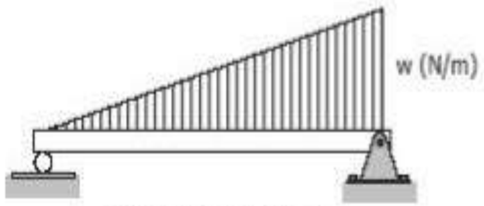
Types of Loading Loads applied to the beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



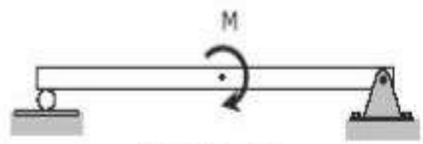
Concentrated Loads



Uniform Load



Uniformly Varying Load



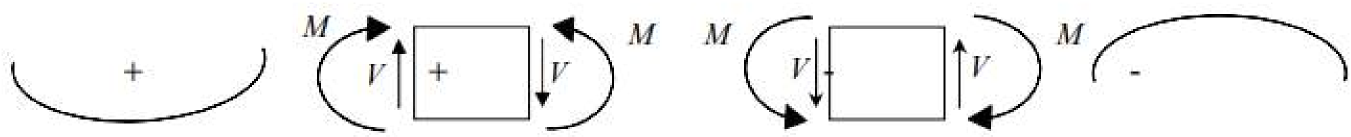
Applied Couple

Shear force (S.F) &bending moment (B.M) of simple support beam under an axial load

Shearing force and bending moment diagrams show the variation of these quantities along the length of a beam for any fixed loading condition. At every section in a beam carrying transverse loads there will be resultant forces on either side of the section which, for equilibrium, must be equal and opposite. Shearing force at the section is defined as the algebraic sum of the forces taken on one side of the section. The bending moment is defined as the algebraic sum of the moments of the forces about the section, taken on either side of the section.

Sign Convention:

Forces upwards to the left of a section or downwards to the right of a section are positive. Clockwise moments to the left and counter clockwise to the right are positive.

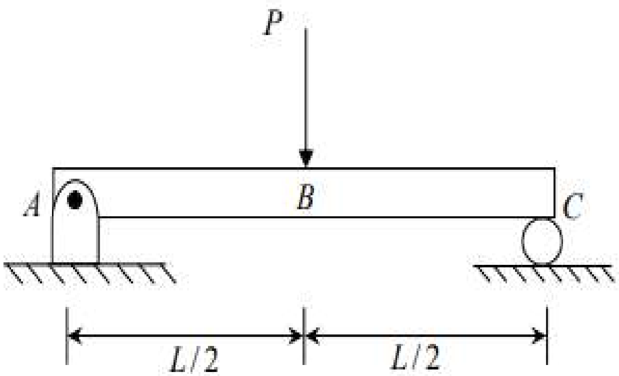


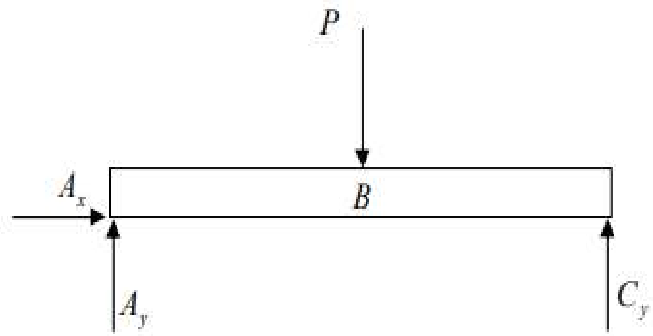
Procedure of Analysis:

The shear and moment diagrams for a beam can be constructed using the following procedure: -

1. Determine all the reactive forces and couple moments acting on the beam, and resolve all the forces into components acting perpendicular and parallel to the beam's axis.
2. Specify separate coordinates x having an origin at the beam's left end extending to regions of the beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading.
3. Section the beam perpendicular to its axis at each distance x, and draw the free body diagram of one of the segments. Be sure V and M are shown acting in their positive sense, in accordance with the sign convention given as above.
4. The shear is obtained by summing forces perpendicular to the beam's axis.
5. The moment is obtained by summing moment about the sectioned end of the segment.
6. Plot the shear diagram (V versus x) and the moment diagram (M versus x). If numerical values of the functions describing V and M are positive, the values are plotted above the x-axis, whereas negative values are plotted below the axis.

Example 1: Draw the shear and moment diagrams for the beam shown below





$\sum Fx = 0$ Ax=0

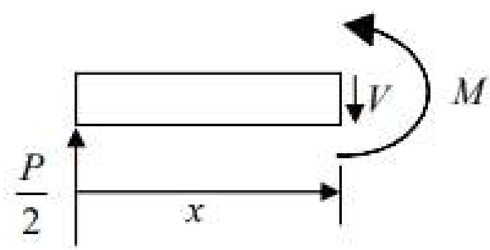
$\sum MC = 0$

P×L/2-Ay×L=0 $\Rightarrow$ Ay=P/2

$\sum Fy = 0$

Cy+Ay-P=0 $\Rightarrow$ Cy=P/2

- Segment AB



$$\sum F_y = 0$$

$$\frac{P}{2}\text{-V=0}$$

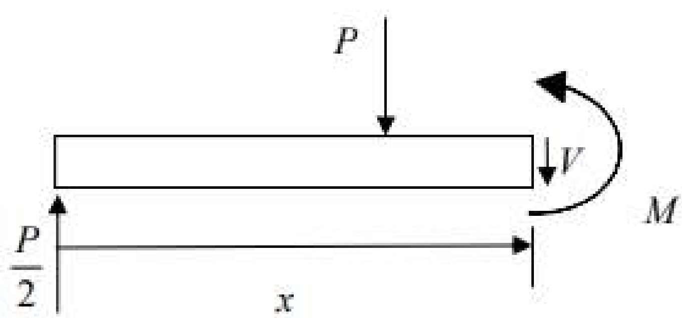
$$V=\frac{P}{2}$$

$$\sum M = 0$$

$$M-\frac{P}{2}\times x=0$$

$$M=\frac{P}{2}x$$

- Segment BC



$$\sum F_y = 0$$

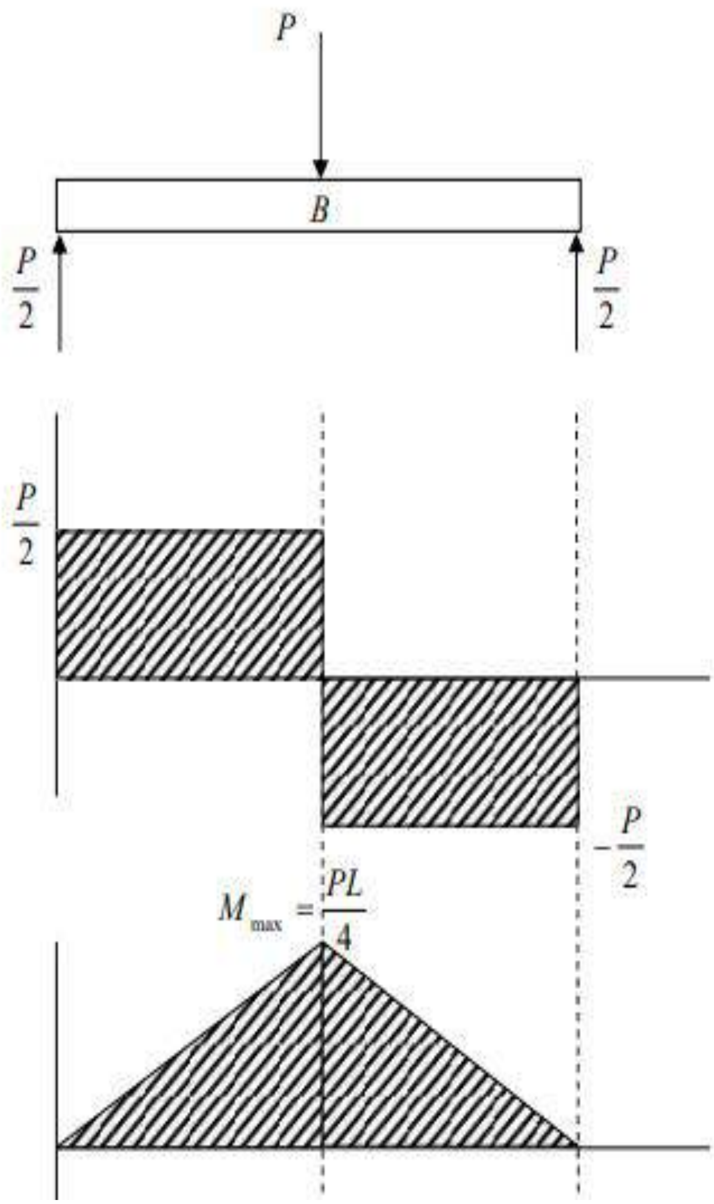
$$\frac{P}{2}-P-V=0$$

$$V=-\frac{P}{2}$$

$$\sum M=0$$

$$M-\frac{P}{2}\times x+P(x-\frac{L}{2})=0$$

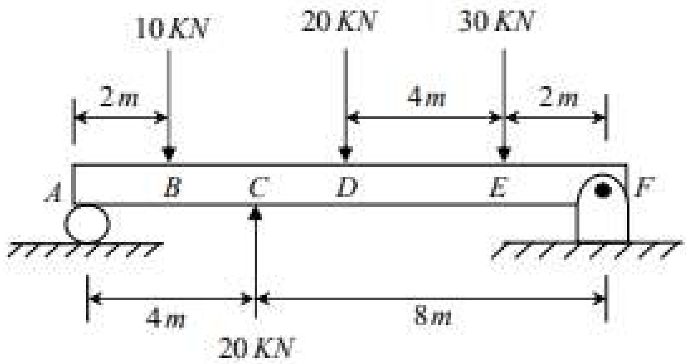
$$M=\frac{P}{2}(L-x)$$

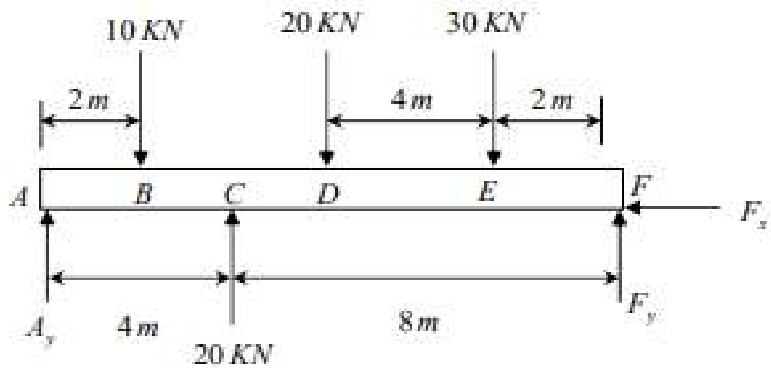


S.F. diagram

B.M. diagram

Example 2: Draw the shear and moment diagrams for the beam shown below.





$\sum F_x = 0$

$F_x=0$

$\sum M_F = 0$

$-A_y \times 12+10\times 10-20\times 8+20\times 6$
$+30\times 2=0$

$A_y=10$ KN

$\sum F_y = 0$

$10-10+20-20-30+F_y=0$

$F_y=30$ KN

- Segment AB $\quad 0 \le x \le 2$

$\sum F_y = 0$

$10-V=0$

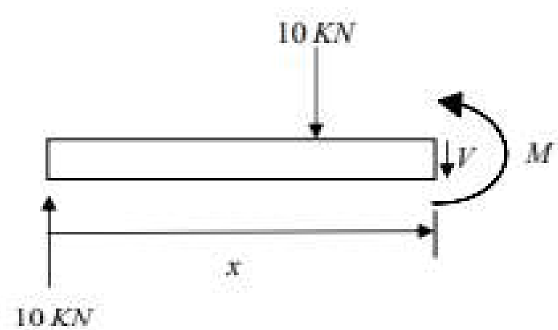
$V=10$ KN

$\sum M = 0$

$M-10\times x=0$

$M=10x$

- Segment BC $\quad 2 \le x \le 4$



$\sum F_y = 0$

$10-10-V=0$

$V=0$

$\sum M = 0$

$M-10x+10(x-2)=0$

$M=20$ KN.m

- Segment CD $\quad 4 \le x \le 6$

$\sum F_y = 0$

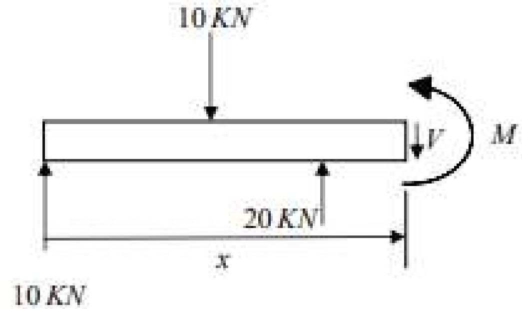
10-10+20-V=0

V=20 KN

$\sum M = 0$

M-10x+10(x-2)-20(x-4)=0

M=20(x-3)



- Segment DE $6 \le x \le 10$

$\sum F_y = 0$

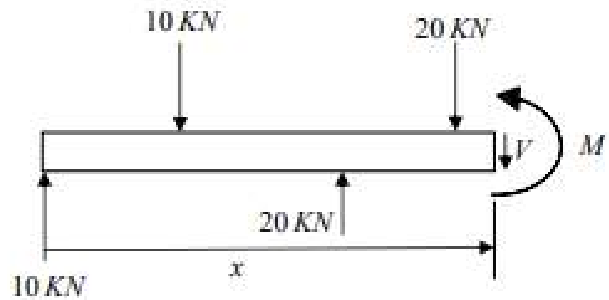
10-10+20-20-V=0

V=0

$\sum M = 0$

M-10x+10(x-2)-20(x-4)+20(x-6)=0

M=60 KN.m



- Segment EF $10 \le x \le 12$

$\sum F_y = 0$

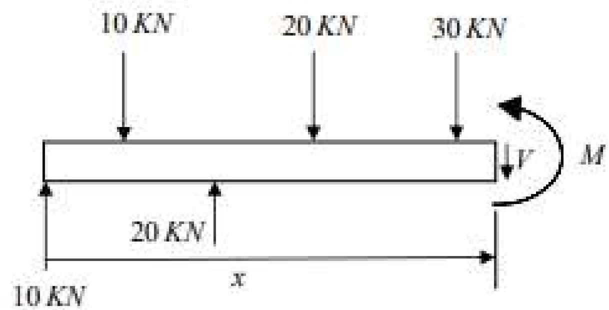
10-10+20-20-30-V=0

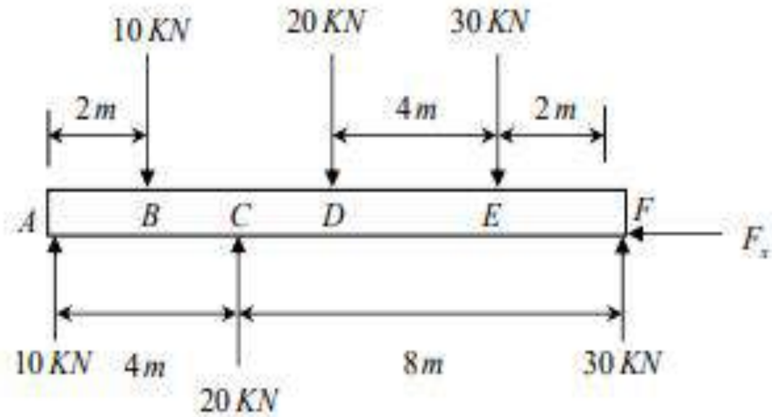
V=-30 KN

$\sum M = 0$

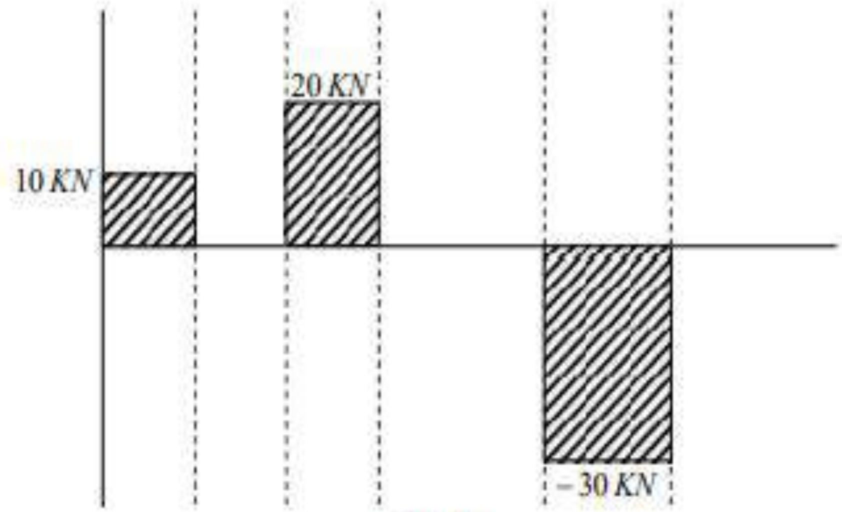
M-10x+10(x-2)-20(x-4)+20(x-6)
+30(x-10)=0

M=30(12-x)

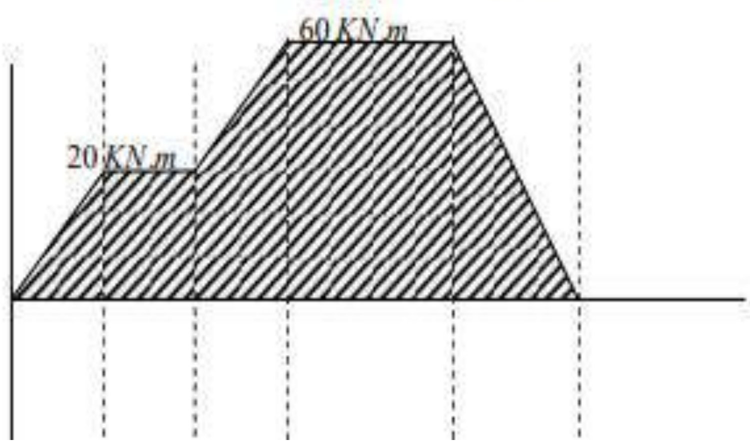




S.F Diagram



B.M. Diagram



Engineering Mechanics

First Stage

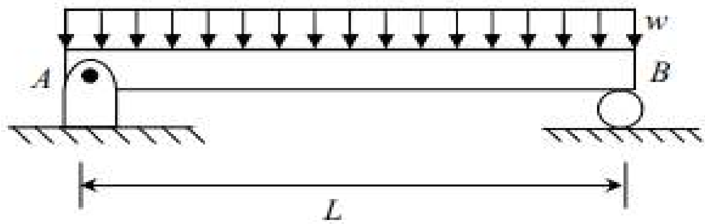
Lecture -23-

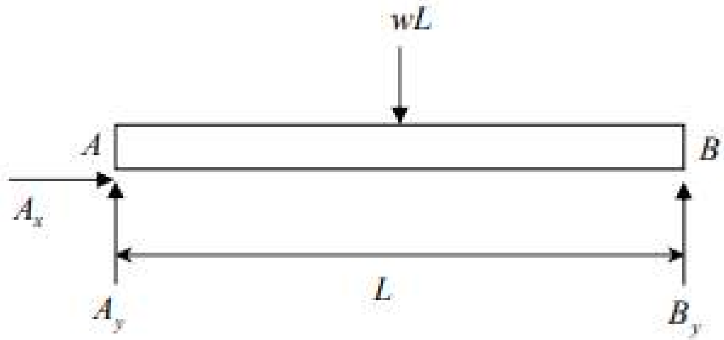
Shear force (S.F) &bending moment (B.M) of simple support beam under uniform distributed load

Shear force (S.F) &bending moment (B.M) of cantilever beam under an axial load& uniform distributed load

Asst Lect. Hayder Salim

Example 1: Draw the shear and moment diagrams for the beam shown below





$$\sum F_x = 0$$

$$A_x = 0$$

$$\sum M_B = 0$$

$$wL\frac{L}{2} - A_y L = 0$$

$$A_y = \frac{wL}{2}$$

$$\sum F_y = 0$$

$$\frac{wL}{2} + B_y - wL = 0$$

$$B_y = \frac{wL}{2}$$

$$\sum F_y = 0$$

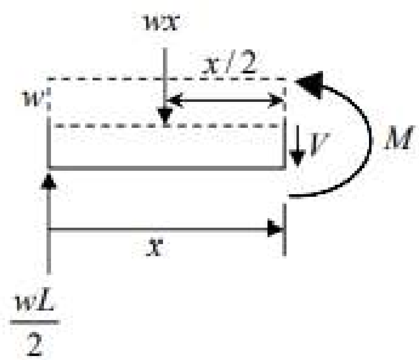
$$\frac{wL}{2} - wx - V = 0$$

$$V = -w(x - \frac{L}{2})$$

$$\sum M = 0$$

$$M - \frac{wL}{2}x + wx(\frac{x}{2}) = 0$$

$$M = \frac{w}{2}(xL - x^2)$$



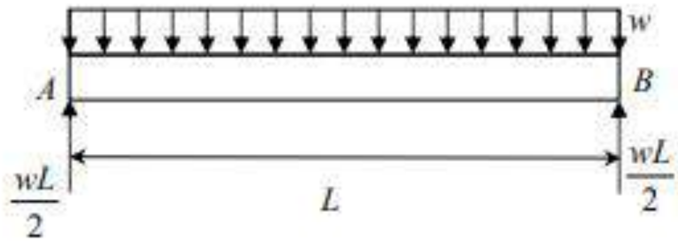
Maximum moment occur when $\frac{dM}{dx} = 0$

$$\frac{dM}{dx} = \frac{w}{2}(L - 2x) = 0$$

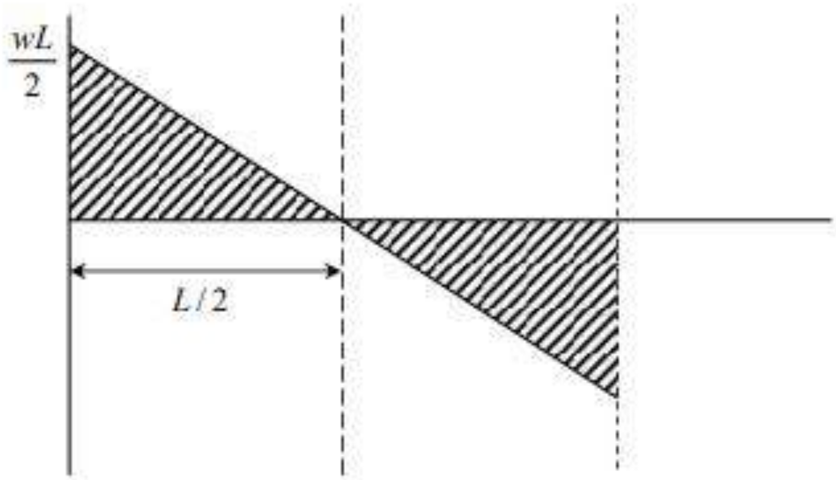
$$L - 2x = 0$$

$$x = \frac{L}{2}$$

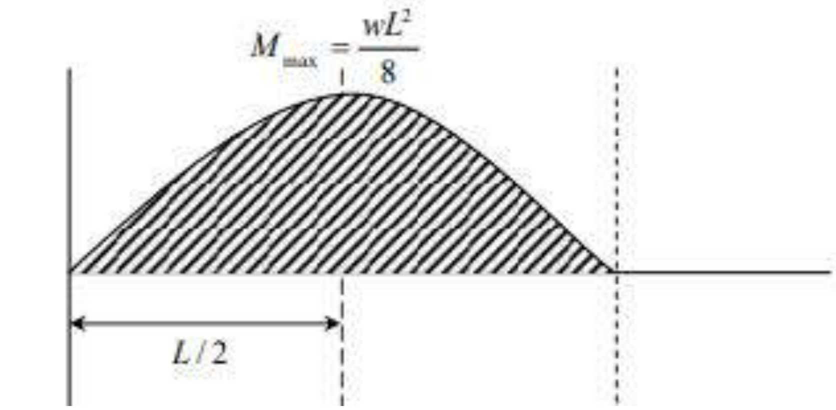
Location of maximum moment



S.F Diagram

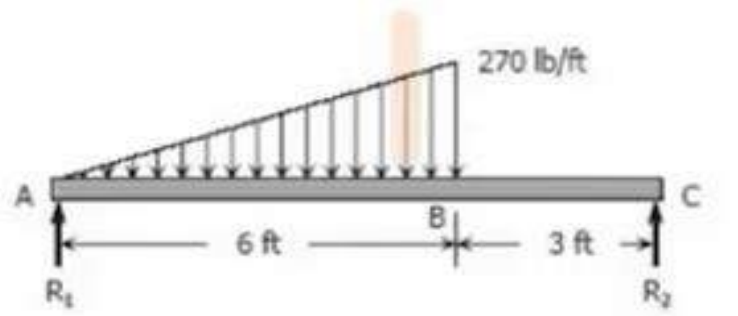


B.M Diagram



Example 2: Draw the shear and moment diagrams for the beam shown below

$F = \frac{1}{2}$*6*270= 810 lb



$$\sum M_C = 0$$

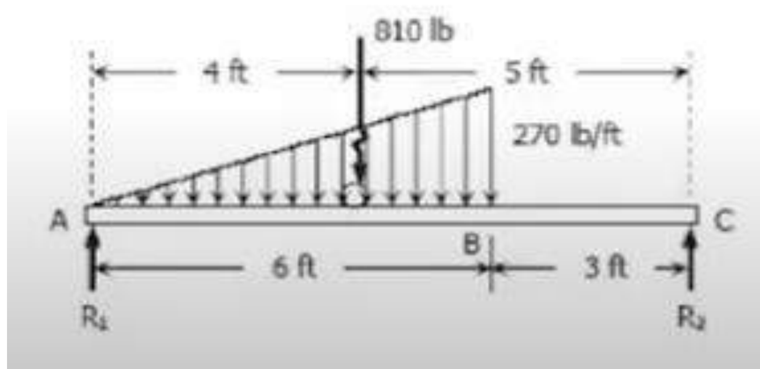
- R_A (9) +810(5) =0

R_A=450 lb

$$\sum M_A = 0$$

R_C (9) +810(4) =0

R_C=360 lb



Segment AB:

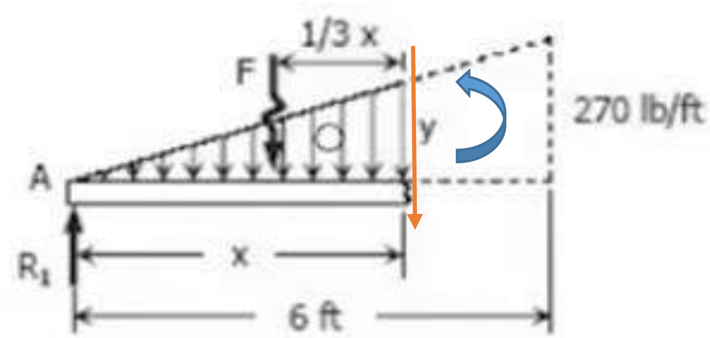
$$\frac{y}{x} = \frac{270}{6}$$

$$y = 45x$$

$$f = \frac{1}{2}(x)(y)$$

$$f = \frac{1}{2}(x)(45x)$$

$$f = 22.5x^2$$



$$\sum y = 0$$

$$-V_{Ab} - 22.5\,x^2 + 450 = 0$$

$$V_{Ab} = 450 - 22.5\,x^2$$

$$\sum M_{AB} = 0$$

$$M_{AB} + 22.5\,x^2\left(\frac{1}{3}x\right) - 450(x) = 0$$

$M_{AB} = 450(x) - 7.5\,x^3$ lb. ft

Segment BC:

$$-V_{BC} - 810 + 450 = 0$$

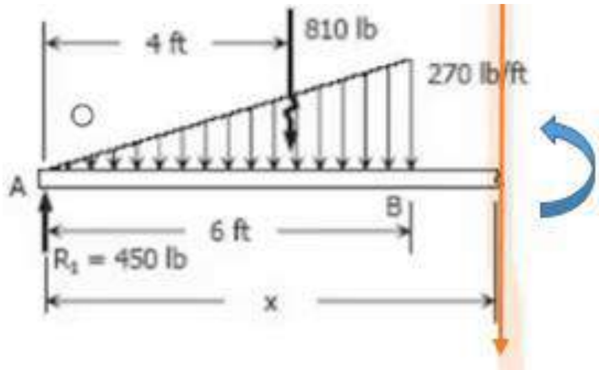
$$V_{BC} = -360\ lb$$

$$\sum M_{bc} = 0$$

$$M_{Bc} + 810(X - 4) - 450(x) = 0$$

$$M_{Bc} = 450(x) - 810(x - 4)$$

$$= 3240 - 360x \text{ lb. ft}$$



To draw the Shear Diagram:

(1) $V_{AB} = 450 - 22.5x^2$ is a second degree curve; at x = 0, V_{AB} = 450 lb; at x = 6 ft, V_{AB} = −360 lb.

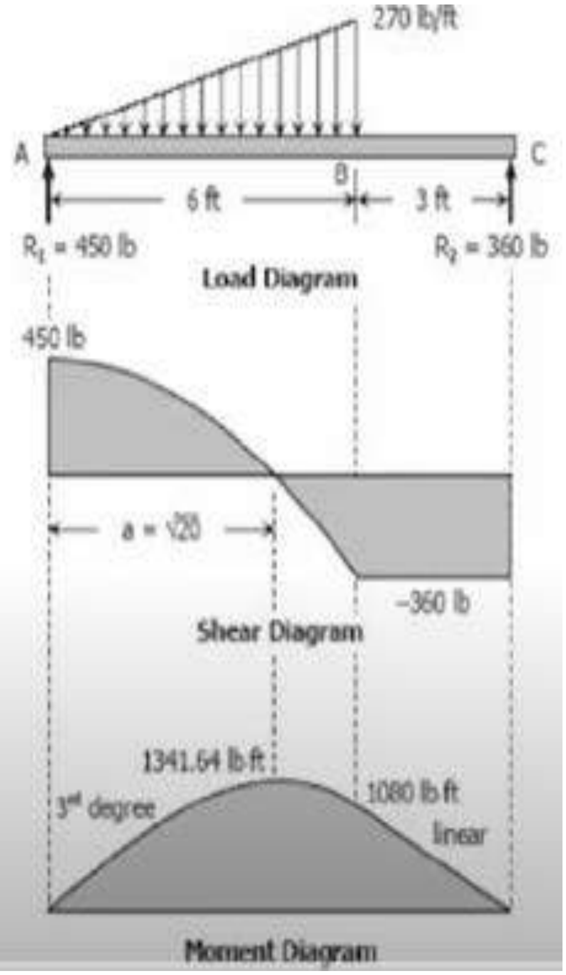
(2) At x = a, $V_{AB} = 0$,

$$450 - 22.5x^2 = 0$$
$$22.5x^2 = 450$$
$$x^2 = 20$$
$$x = \sqrt{20}$$

To draw the Moment Diagram:

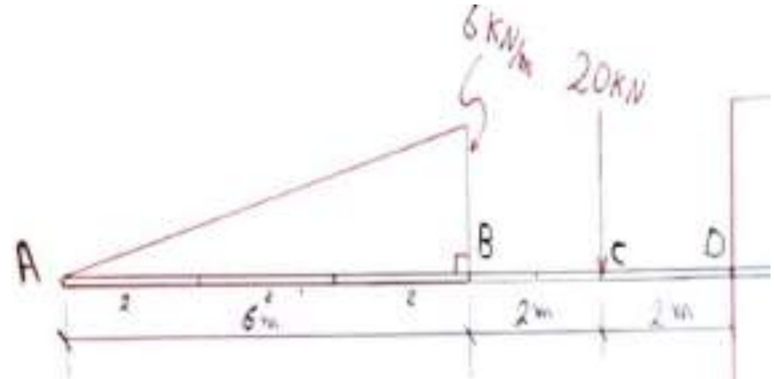
(1) $M_{AB} = 450x - 7.5x^3$ for segment AB is third degree curve; at x = 0, M_{AB} = 0; at x = $\sqrt{20}$, M_{AB} = 1341.64 lb·ft; at x = 6 ft, M_{AB} = 1080 lb·ft.

(2) $M_{BC} = 3240 - 360x$ for segment BC is linear; at x = 6 ft, M_{BC} = 1080 lb·ft; at x = 9 ft, M_{BC} = 0.



Example 2: Draw the shear and moment diagrams for the beam shown belowe

$F = \frac{1}{2}$*6*6= 18 KN



$\sum y = 0$

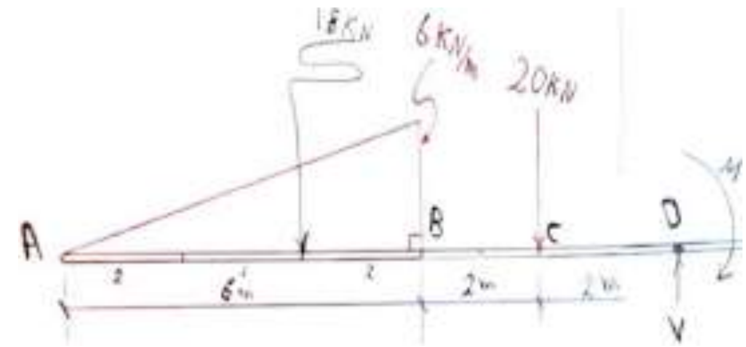
$V_D - 18 - 20 = 0$

$V_D = 18 + 20 = 38$ KN

$\sum M_D = 0$

M_D-20(2)-18(6) =0

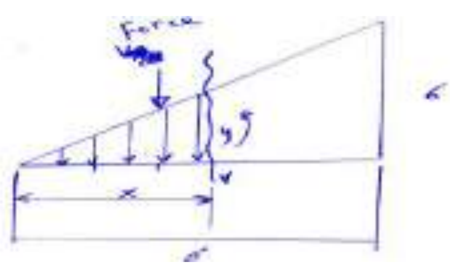
M_D=148 KN.m



Segment AB:

$\frac{y}{x} = \frac{6}{6}$

$y = x$



$f = \frac{1}{2}(x)(y)$

$f = \frac{1}{2}x^2$

$\sum y = 0$

$-V_{Ab} - f = 0$

$V_{Ab} = -\frac{1}{2}x^2$

$\sum M_{AB} = 0$

$M_{AB} + \frac{1}{2}x^2\left(\frac{1}{3}x\right) = 0$

$M_{AB} = -\frac{1}{6}x^3$

Segment BC:

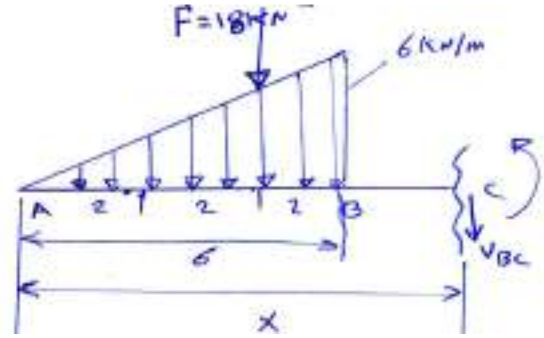
$-V_{BC} - 18 = 0$

$V_{BC} = -18\ KN$

$\sum M_{Bc} = 0$

$M_{Bc} + 18(x-4) = 0$

$M_{Bc} = -18x + 72$



Segment CD:

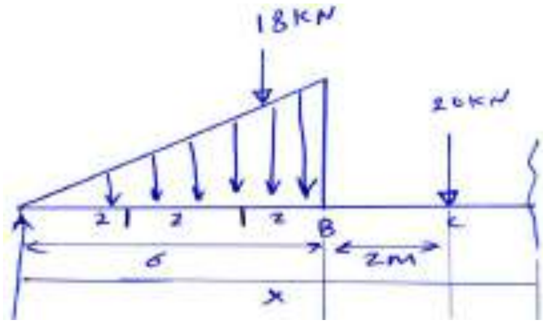
$-V_{CD} - 18 - 20 = 0$

$V_{CD} = -38\ KN$

$\sum M_{CD} = 0$

$M_{CD} + 18(x-4) + 20(x-8) = 0$

$M_{Bc} = -38X + 232$ KN



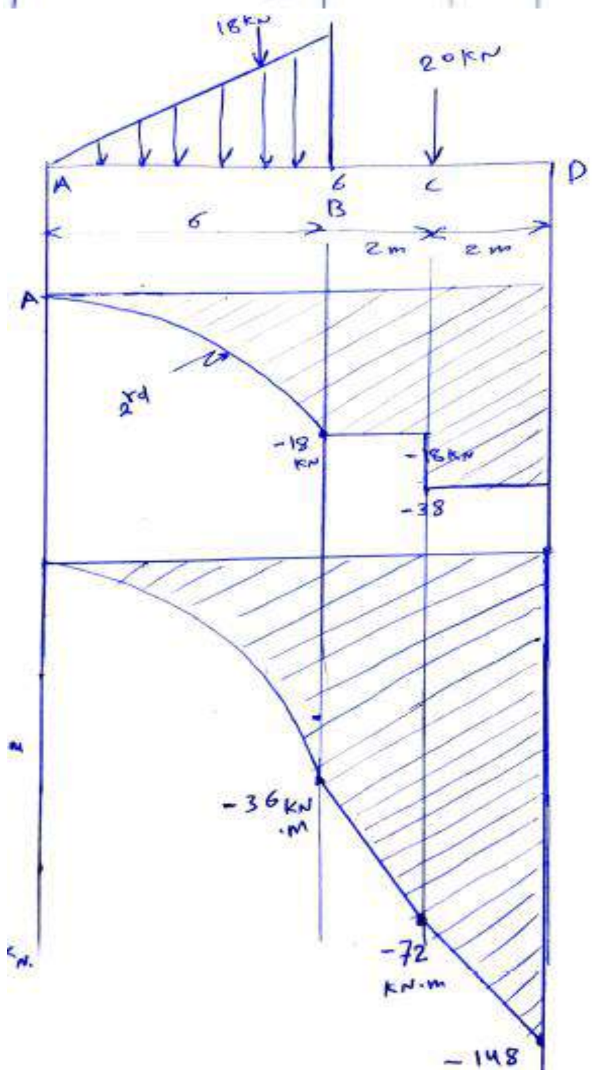
TO draw the shear diagram

1- $V_{Ab} = -\frac{1}{2}x^2$, At x=0 $V_{Ab} = 0$

At x=6 $V_{Ab} = -36$

2- $V_{BC} = -18\ KN$

3- $V_{CD} = -38\ KN$



TO draw the moment diagram

1- $M_{AB} = -\frac{1}{6}x^3$,at x=0 $M_{AB} = 0$

at x=6 $M_{AB} = -36KN$

2- $M_{Bc} = -18x + 72$, at x=6 $M_{Bc} = -36KN$

at x=8 $M_{AB} =$ $-72KN$

3- $M_{CD} = -38X + 232$ KN at x=8 $M_{CD} = -72KN$

at x=8 $M_{CD} = -148K$