Basic Laws

2.1 Introduction

Chapter 1 introduced basic concepts in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as **Ohm's law** and **Kirchhoff's laws**, form the foundation upon which electric circuit analysis is built. In addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis.

2.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1. Network: Any arrangement of the various, electrical energy source along with the different circuit elements is called an electrical network. Such a network is shown in the **Fig. 2.1**.

2. Network Element: Any individual circuit element with two terminals which can be connected to other circuit element is called a network element. Network elements can be either active elements or passive elements.

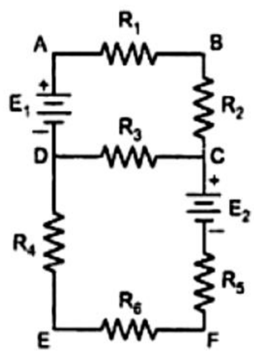
3. Branch: A part of the network which connects the various points of the network with one another is called a branch. In the **Fig. 2.1**, **AB, BC, CD, DA, DE, CF** and **EF** are the various branches. The branch may consist of more than one element.

4. Junction Point: A point where three or more branches meet is called a junction point. Points D and C are the junction points in the network shown in the **Fig. 2.1**.

5. Node: A point at which two or more elements are joined together is called node. The junction points are also the nodes of the network. In the network shown in the **Fig. 2.1**, **A, B, C, D, E** and **F** are the nodes of the network.

6. Mesh (or Loop): Mesh (or Loops) is a set of branches forming a closed path in a network in such way that if one branch is removed then remaining branches do not form a closed path. In the **Fig. 2.1** paths **A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D** etc are the loops of the network.

In this chapter, the analysis of **d.c.** circuits consisting of pure resistors and **d.c.** sources is included.



2.3 Classification of Electric Networks

The behavior of the entire network depends on the behavior and characteristics of its elements. Based on such characteristics electrical network can be classified as below,

<u>i) Linear Network</u>: A circuit or network whose parameter i.e. elements are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network**.

<u>ii) Nonlinear Network</u>: A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **nonlinear network**.

Figure 2.1 an electrical network.

<u>iii) Bilateral Network</u>: A circuit whose characteristics, behavior is same irrespective of the direction of current through various elements of it is called **bilateral network**.

<u>iv) Unilateral Network</u>: A circuit whose, operation, behavior is dependent on the direction of the current through various elements is called **unilateral network**.

<u>v) Active Network</u>: A circuit whose contain at least one source of energy is called active. An energy source may be a voltage or current source.

<u>vi) Passive Network</u>: A circuit which contains no energy source is called passive circuit. This is shown in the **Fig 2.2**.

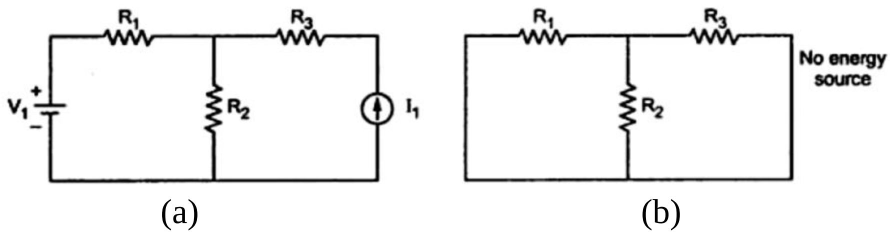


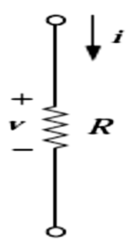
Figure 2.2 (a) active network, (b) passive network

<u>vii) Lumped Network</u>: A network in which all the network elements are physically separable is known as **lumped network**. Most of the electric networks are lumped in nature, which consists of element like **R**, **L**, **C**, and voltage source etc.

<u>viii) Distributed Network</u>: A network in which the network elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called distributed network. The best example of such a network is a transmission line, where resistances, inductance and. capacitance of a transmission line are distributed all along its length and cannot be shown as separate elements, anywhere in the circuit.

<u>2.4 OHM'S LAW</u>

As shows in chapter one, the materials in general have a characteristic behavior of resisting the flow of electric charge. The resistance ***R*** of any



material with a uniform cross-sectional area ***A*** depends on ***A*** and its length ***l***.

The circuit element used to model the current-resisting behavior of a material is the resistor. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit symbol for the resistor is shown in **Fig. 2.3**, where ***R*** stands for the resistance of the resistor. The resistor is the simplest passive element. Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

Figure 2.3 Circuit symbol for resistance.

> Key Point: *Ohm's law states that the voltage **v** across a resistor is directly proportional to the current **i** flowing through the resistor.*

Ohm defined the constant of proportionality for a resistor to be the resistance; **R**. (The resistance is material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus,

$$v = iR \qquad (2.1)$$

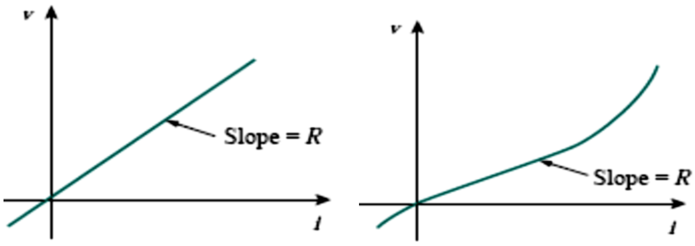
The resistance ***R*** of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

Then $$R = v/i \qquad (2.2)$$

so that $$1\ \Omega = 1\ \text{V/A}$$

It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a linear resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in **Fig. 2.4(a)**. A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in **Fig. 2.4 (b)**. Examples of devices with nonlinear resistance are the light bulb and the diode. A useful quantity in circuit analysis is the reciprocal of resistance **R**, known as conductance and denoted by **G**:

$$G = 1/R = i/v \qquad (2.3)$$



(a) **(b)**

Figure 2.4 The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the mho (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mhos, in this lectures we prefer to use the **Siemens** (**S**), the **SI** unit of conductance:

1 S = 1 ℧ = 1 A/V

Thus,

Conductance is the ability of an element to conduct electric current; it is measured in mhos *(℧)* or Siemens (S).

From **Eq. (2.3)**, we may write

$$i = Gv \qquad (2.4)$$

The power dissipated by a resistor can be expressed in terms of **R**. Using **Eqs. (1.23)** and **(2.1)**,

$$p = vi = i^2R = v^2/R \qquad (2.5)$$

The power dissipated by a resistor may also be expressed in terms of **G** as

$$p = vi = v^2G = i^2/G \qquad (2.6)$$

We should note two things from **Eqs. (2.5)** and **(2.6)**:

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since **R** and **G** are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit.

2.4.1 Limitations of Ohm's Law

The Limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diode, zener diode, voltage regulators.
2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by, $V = kI^m$ where **k, m** are constants.

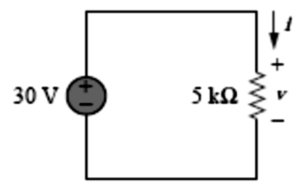
EXAMPLE 2.1: An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law, **R = v/i =120/2 = 60 Ω**

EXAMPLE 2.2: In the circuit shown below, calculate the current i, the conductance G, and the power **P**.

Solution:



The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

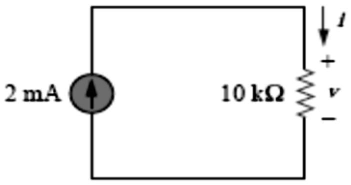
$$\mathbf{i = v / R = 30 /(5 \times 10^3) = 6\ mA}$$

The conductance is $\mathbf{G = 1 /R = 1/ 5 \times 10^3 = 0.2\ mS}$

We can calculate the power in various ways using either **Eqs. (1.29)**, **(2.5)**, or **(2.6)**.

$$\mathbf{p = vi = 30 \times (6 \times 10^{-3}) = 180\ mW}$$

PRACTICE PROBLEM 2.1: For the circuit shown below, calculate the voltage v, the conductance G, and the power p.



Answer: 20 V, 100 μS, 40 mW.

2.5 SERIES RESISTORS

A series circuit is one in which several resistances are connected one after the other. There is only one path for the flow of current. Consider the resistances shown in the **Fig. 2.5**. The resistance $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, said to be in series.

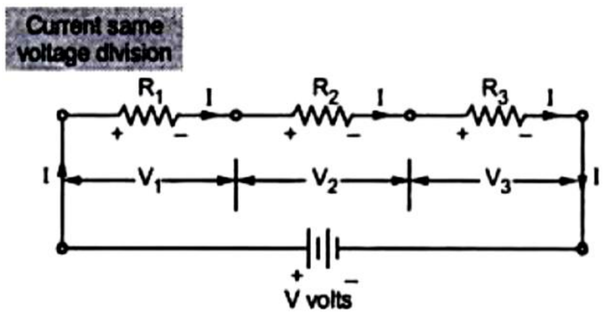


Fig. 2.5 series circuit

Req= Equivalent resistance of the circuit.

$$\mathbf{Req = R_1 + R_2 + R_3}$$

i.e. total or equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

For **N** resistances in series, $\mathbf{R = R_1 + R_2 + R_3 + \ldots + R_N}$ (2.7)

If $\mathbf{R_1 = R_2 = \cdots = R_N = R}$, then

$$\mathbf{R_{eq} = N \times R} \quad (2.8)$$

2.5.1 Characteristics of Series Circuits

1) The same current flows through each resistance.

2) The supply voltage **V** is the sum of the individual voltage drops across the resistances.

$$\mathbf{V = V_1 + V_2 + V_3 + \ldots + V_N} \quad (2.9)$$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances.

i.e. $\mathbf{R > R_1, R > R_2, \ldots R > R_N}$

2.6 PARALLEL RESISTORS

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the **Fig. 2.6**.

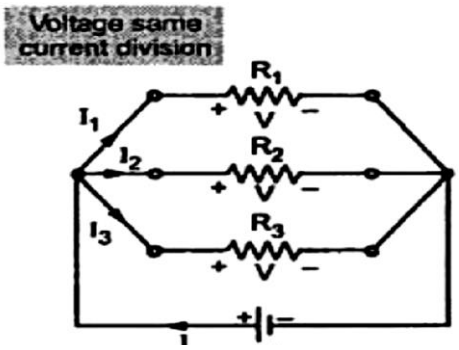


Fig. 2.6 A parallel circuit.

R_{eq}= Total or equivalent resistance of the circuit,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general if '**N**' resistances are in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \qquad (2.10)$$

Note that **Req** is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdot\ \cdot\ \cdot = R_N = R$, then

$$R_{eq} = R/N \qquad (2.11)$$

Conductance (G):

It b known that, **1/R = G** (conductance) hence,

$$G=G_1+G_2+G_3+\ldots+G_N \qquad (2.12)$$

Important result:

Now If **N** = 2, two resistance are in parallel then,.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R = \frac{R_1R_2}{R_1+R_2} \qquad (2.13)$$

2.6.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of the individual currents.
3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4) The equivalent resistance is the smallest of all the resistances $R < R_1, R<R_2, R<R_N$.
5) The equivalent conductance is the arithmetic addition of the individual conductances.

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single equivalent resistance **Req**.

Example 2.3: Find **Req** for the circuit shown in **Fig. 1**.

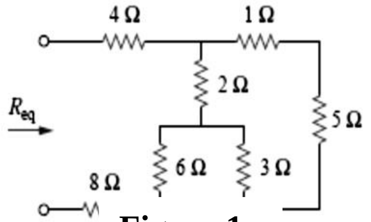


Figure 1

To get $\mathbf{R_{eq}}$, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$\mathbf{6\ \Omega \parallel 3\ \Omega = 6 \times 3/(6+3) = 2\ \Omega}$$

(The symbol || is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$\mathbf{1\ \Omega + 5\ \Omega = 6\ \Omega}$$

Thus the circuit in **Fig. 1** is reduced to that in **Fig. 2(a)**. **In Fig. 2(a)**, we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$\mathbf{2\ \Omega + 2\ \Omega = 4\ \Omega}$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in **Fig. 2 (a)**; their equivalent resistance is

$$\mathbf{4\ \Omega \parallel 6\ \Omega = 4 \times 6/(4+6) = 2.4\ \Omega}$$

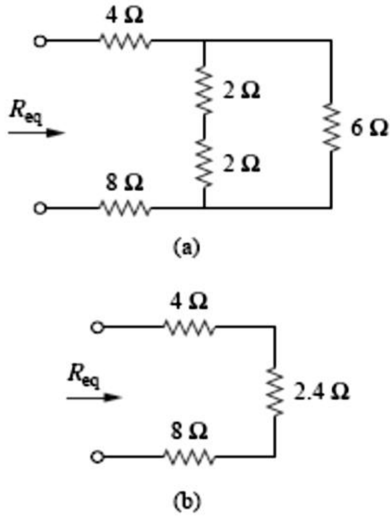


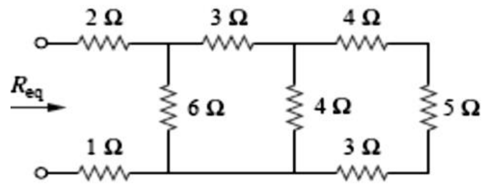
Figure 2

The circuit in **Fig. 2 (a)** is now replaced with that in **Fig. 2 (b)**. In **Fig. 2 (b)**, the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$\mathbf{R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega}$$

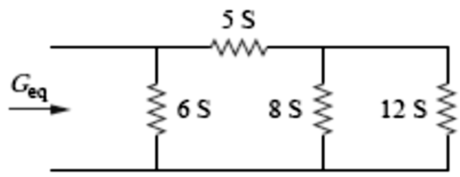
PRACTICE PROBLEM 2.2: By combining the resistors in Figure below, find **Req**.

Answer: 6 Ω.



PRACTICE PROBLEM 2.3: Find the conductance $\mathbf{G_{eq}}$ for the circuit in Figure below.

Answer: 10 S.

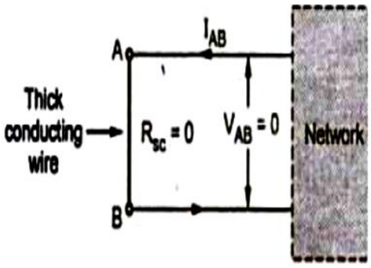


2.7 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the network plays an important role. Since the value of **R** can range from zero to infinity, it is important that we consider the two extreme possible values of **R**.

2.7.1 Short Circuit

When any two points in a network are joined directly to each other with a thick meta1ic conducting wire the two points are said to be short circuited. The resistance of such short circuit is zero.



The part of the network, which is short circuited, is shown in the **Fig. 2.7**. The points A and B are short circuited. The resistance of the branch **AB** is **Rsc**=0. The Current $\mathbf{I_{AB}}$ is flowing through the short circuited path. According to Ohm's law,

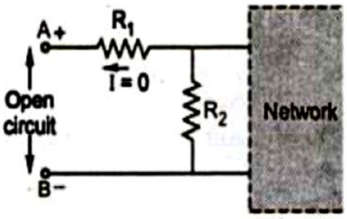
$$\mathbf{V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0\ V}$$

Figure 2.7 Short circuit (R_{sc} = 0)

Key Point: ***The voltage across short circuit is always zero though current flows through the short circuited path.***

2.7.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.



As there is no direct connection in an open circuit, the resistance of the open circuit is ∞. The part of the network which is open circuited is shown in the **Fig. 2.8**. The points A and B are said to be open circuited. The resistance of the branch AB is $\mathbf{R_{OC}} = \infty\ \Omega$.

Figure 2.8 Open circuit (R_{OC} =∞).

According to Ohm's law,

$$\mathbf{I_{OC} = V_{AB} / R_{OC} = V_{AB} / \infty = 0\ A}$$

Key Point: ***The current through open circuit is always zero though there exist voltage across open circuited terminals.***

2.8 The voltage-divider and current-divider circuits

2.8.1 The voltage-divider circuit

Voltage-divider circuit, shown in **Fig.2.9**. We analyze this circuit by directly applying Ohm's law and Kirchhoff's laws. To aid the analysis we introduce the current ***i*** as shown in **Fig.2.9 (b)**. From Kirchhoff's current law $\mathbf{R_1}$ and $\mathbf{R_2}$, carry the same current. Applying **Kirchhoff's voltage law** around the closed loop yields

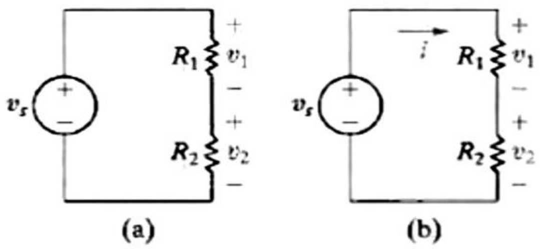


Figure 2.9 (a) A voltage-divider circuit and (b) The voltage-divider circuit with current i indicated

$v_s = i R_1 + i R_2$,

Now we can use Ohm's law to calculate v_1 and, v_2:

$$v_1 = \frac{R_1 v_s}{R_1 + R_2}, \quad v_2 = \frac{R_2 v_s}{R_1 + R_2} \tag{2.14}$$

In general, if a voltage divider has **N** resistors (**R_1, R_2, . . . , R_N**) in series with the source voltage ***v_s***, the Nth resistor (**R_N**) will have a voltage drop of

$$v_N = \frac{R_N v_s}{R_1 + R_2 + \cdots + R_N} = \frac{R_N v_s}{R_{eq}} \tag{2.15}$$

2.8.2 The current-divider circuit

The **current-divider circuit** shown in **Fig. 2.10.** The current divider is designed to divide the current ***i_s*** between **R_1** and **R_2**. We find the relationship between the current ***i_s***, and the current in each resistor (that is, ***i_1*** and ***i_2***) by directly applying **Ohm's law and Kirchhoff's current law**. The voltage across the parallel resistors is

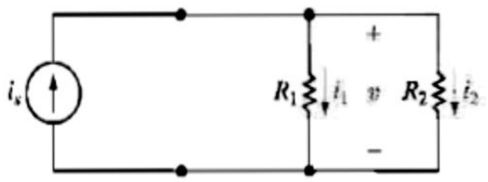


Figure2.10 the current-divider circuit.

$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2} i_s$$

$$i_1 = \frac{R_2 i_s}{R_1 + R_2}, \quad i_2 = \frac{R_1 i_s}{R_1 + R_2} \tag{2.16}$$

If we divide both the numerator and denominator by **R_1R_2**, **Eq. (2.16)** become

$$i_1 = \frac{G_1 i_s}{G_1 + G_2}, \quad i_2 = \frac{G_2 i_s}{G_1 + G_2} \tag{2.17}$$

Thus, in general, if a current divider has **N** conductors (**G_1, G2, . . . , G_N**) in parallel with the source current i, the nth conductor (**G_N**) will have current

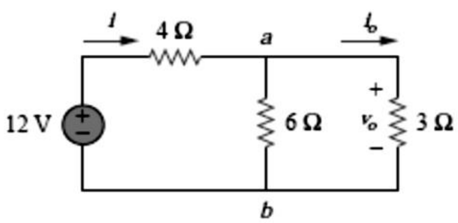
$$i_N = \frac{G_N i_s}{G_1 + G_2 + \cdots + G_N} = \frac{R_{eq} i_s}{R_N} \tag{2.18}$$

EXAMPLE 2.4: Find ***i_o*** and ***v_o*** in the circuit shown in **Fig. 1(a)**. Calculate the power dissipated in the 3-Ω resistor.

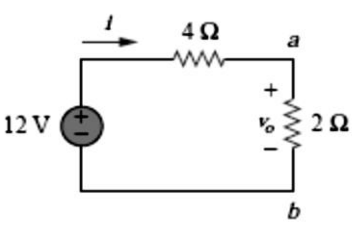
Solution: The 6-Ω and 3-Ω resistors are in parallel, so their combined resistance is

6 Ω || 3 Ω = 6 × 3/ (6 + 3) = 2 Ω

By apply voltage division, since the 12 V in **Fig. 1(b)** is divided between the 4-Ω and 2-Ω resistors. Hence,



(a)



(b)

v_o =2(12 V)/(2 + 4) = 4 V

Apply current division to the circuit in **Fig. 1(a)** now that we know ***i***, by writing

$$i = 12/\,4 + 2 = 2\text{ A}$$
$$i_o = 6\,i\,/(6 + 3) = 4/3\text{ A}$$

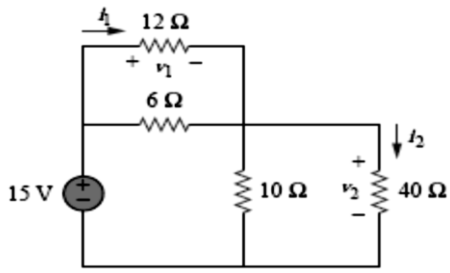
Figure 1(a) Original circuit, (b) Its equivalent circuit.

The power dissipated in the 3-Ω resistor is

$$p_o = v_o\, i_o = 4(4/3) = 5.333\text{ W}$$

PRACTICE PROBLEM 2.4: Find v_1 and v_2 in the circuit shown in Figure below. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: v_1 = 5 V, i_1 = 416.7 mA, p_1 = 2.083 W, v_2 = 10 V, i_2 = 250 mA, p_2 = 2.5 W.



2.9 WYE-DELTA TRANSFORMATIONS

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in **Fig. 2.11**. How do we combine resistors R_1 through R_6 when the resistors are neither in series nor in parallel? Many circuits of the type shown in **Fig. 2.11** can be simplified by using three-terminal equivalent networks. These are the wye (**Y**) or tee (**T**) network shown in **Fig. 2.12** and the delta (**Δ**) or pi (**π**) network shown in **Fig. 2.13**.

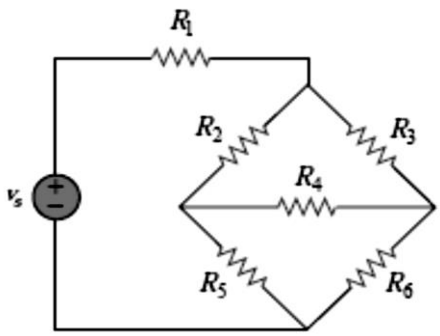


Figure 2.11The bridge network.

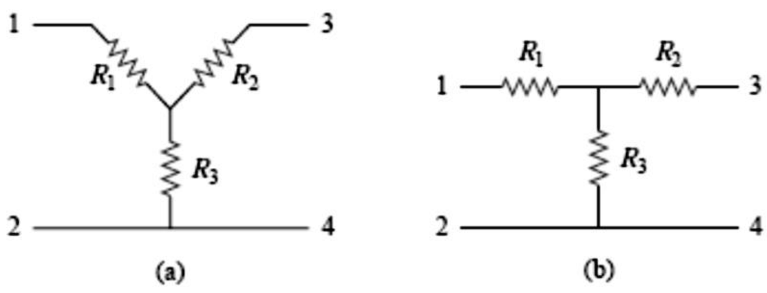


Figure 2.12 Two forms of the same network: (a) Y, (b) T.

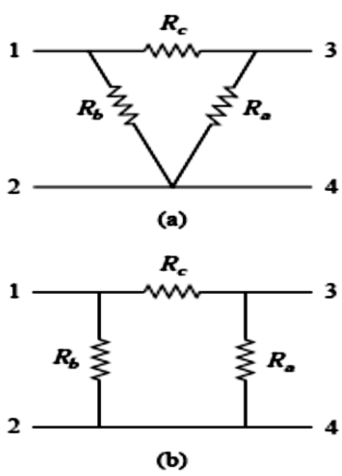


Figure 2.13 Two forms of the same network: (a) Δ, (b) π.

Delta to Wye Conversion

Suppose it is more convenient to work with a **wye** network in a place where the circuit contains a delta configuration. We superimpose a **wye** network on the existing **delta** network and find the equivalent resistances in the **wye** network. For terminals 1 and 2 in **Figs. 2.12** and **2.13**, for example, $\mathbf{R_{12}(Y) = R1 + R3}$, $\mathbf{R_{12}(\Delta) = R_b \,||\, (R_a + R_c)}$ (2.19)

Setting $\mathbf{R_{12}(Y) = R_{12}(\Delta)}$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \qquad R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.20}$$

By solving previous equations, we get

$$R_1 = \frac{R_b\, R_c}{R_a + R_b + R_c} \tag{2.21}$$

$$R_2 = \frac{R_c\, R_a}{R_a + R_b + R_c} \tag{2.22}$$

$$R_3 = \frac{R_a\, R_b}{R_a + R_b + R_c} \tag{2.23}$$

Wye to Delta Conversion

Reversing the **Δ-to-Y** transformation also is possible. That is, we can start with the **Y** structure and replace it with an equivalent **Δ** structure. The expressions for the three **Δ**-connected resistors as functions of the three **Y**-connected resistors are

$$Ra = \frac{R1R2 + R2R3 + R3R1}{R1} \tag{2.24}$$

$$Rb = \frac{R1R2 + R2R3 + R3R1}{R2} \tag{2.25}$$

$$Rc = \frac{R1R2 + R2R3 + R3R1}{R3} \tag{2.26}$$

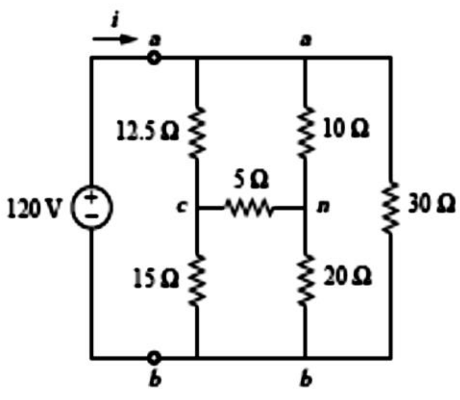
The Y and Δ networks are said to be balanced when

$$R_1 = R_2 = R_3 = R_Y,\ R_a = R_b = R_c = R_\Delta \tag{2.27}$$

Under these conditions, conversion formulas become

$$R_Y = R_\Delta / 3 \text{ or } R_\Delta = 3R_Y \tag{2.28}$$

EXAMPLE 2.5: Obtain the equivalent resistance $\mathbf{R_{ab}}$ for the circuit in **Fig. 1** and use it to find current **i**.



Solution:

In this circuit, there are two **Y**-networks and one **Δ**-network. Transforming just one of these will simplify the circuit. If we convert the **Y**-network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 5\ \Omega$

Thus, from Eqs. (2.24) to (2.26) we have

Figure 1.

$$Ra = \frac{R1R2 + R2R3 + R3R1}{R1} = \frac{10\times20+20\times5+5\times10}{10} = \frac{350}{10} = 35\ \Omega$$

$$Rb = \frac{R1R2 + R2R3 + R3R1}{R2} = \frac{350}{20} = 17.5\Omega$$

$$Rc = \frac{R1R2 + R2R3 + R3R1}{R3} = \frac{350}{5} = 70\Omega$$

With the **Y** converted to **Δ**, the equivalent circuit (with the voltage source removed for now) is shown in **Fig. 2 (a)**. Combining the three pairs of resistors in parallel, we obtain

70 || 30 =70 × 30/ (70 + 30) = 21 Ω

12.5 || 17.5 =12.5 × 17.5/ (12.5 + 17.5) = 7.2917 Ω

15 || 35 =15 × 35/ (15 + 35) = 10.5 Ω

so that the equivalent circuit is shown in **Fig. 2 (b)**. Hence, we find

R_{ab} = (7.292 + 10.5) || 21 =17.792 × 21/ (17.792 + 21) = 9.632 Ω

Then

i = vs/ Rab =120/ 9.632 = 12.458 A

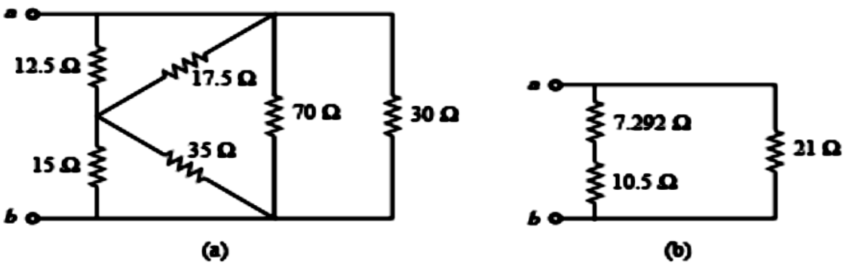
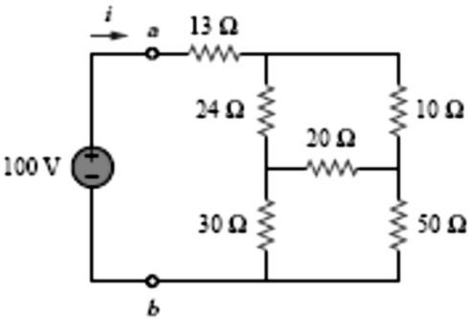


Figure 2 Equivalent circuits to Fig. 1, with the voltage removed.

<u>PRACTICE PROBLEM 2.5:</u> For the bridge network in Figure below, find **R_{ab}** and **i**.

Answer: 40 Ω, 2.5 A.



<u>2.10 Energy Sources</u>

There are basically two types of energy sources; voltage source and current source. These sources are classified as i) Ideal source and ii) Practical source. Let us see the difference between Ideal and practical sources.

2.10.1 Voltage Source

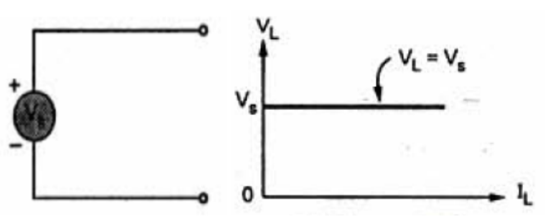
*Ideal voltage source:

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. This is indicated by **V- I** characteristics shown in the **Fig. 2.14 (b)**.

*Practical voltage source:

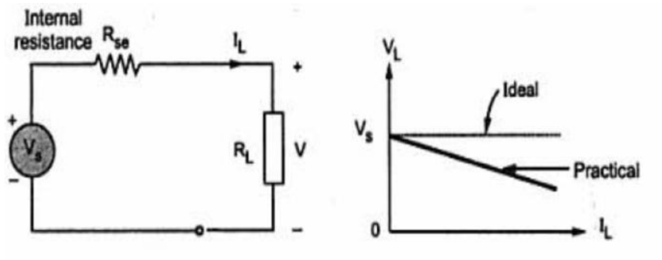
But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by $\mathbf{R_{se}}$ as shown in the **Fig. 2.15**. Because of the $\mathbf{R_{se}}$, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$V_L = V_S - I_L R_L$$



(a) symbol (b) characteristics

Figure 2.14 Ideal voltage source.



(a) circuit (b) characteristics

Figure 2.15 Practical voltage source.

Voltage sources are further classified as follows,

i) Time invariant Sources:

The sources in which voltage is not varying with time are known as time invariant voltage source or **D.C.** sources. These are denoted by capital letters. Such a source is represented in the **Fig. 2.16 (a)**.

ii) Time Variant Source:

The sources in which voltage is varying with time are known as time variant voltage sources or **A.C.** sources. These are denoted by small letters. This is shown in the **Fig. 2.16 (b)**.

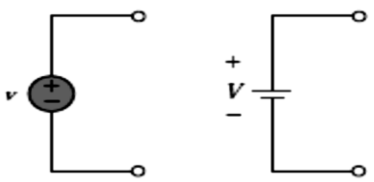


Figure 2.16 (a) D.C. sources.

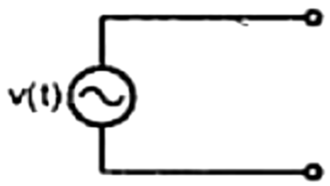


Figure 2.16(b) A.C. source.

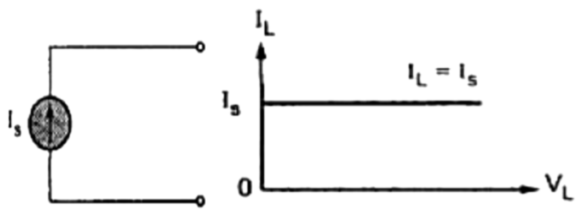
2.10.2 Current Source

***Ideal current source:**

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminal. This is explained by **V-I** characteristics shown in the **Fig. 2.17 (b)**.

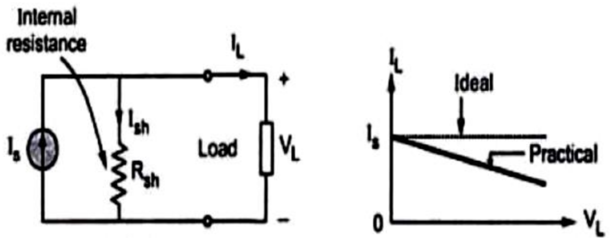
***Practical current source:**

But practically, every current source has high internal resistance, shown in parallel with current source and It is represented by $\mathbf{R_{sh}}$. This is shown in the **Fig. 2.18**. Because of $\mathbf{R_{sh}}$, current through its terminals decreases slightly with voltage at its terminals.



(a) symbol **(b) characteristics**

Figure 2.17 ideal current source.



(a) circuit **(b) characteristics**

Figure 2.18 ideal current source.

Similar to voltage sources, current sources are classified as follows,

i) **Time Invariant Sources:**

The sources in which current is not varying with time are known as time invariant current sources or D.C. sources. These are denoted by capital letters. Such a current source is represented in the **Fig. 2.19 (a)**.

ii) **Time Variant Sources:**

The sources in which current is varying with time are known as time variant current sources or **A.C. sources**. These are denoted by small letters. Such source is represented in the **Fig. 2.19 (b)**.

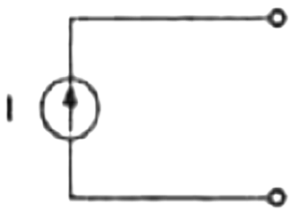


Figure 2.19 (a) D.C. source.

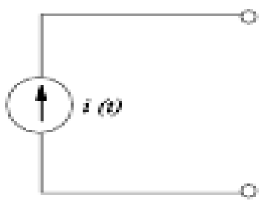


Fig. 2.19 (b) A.C. source.

The sources, which are discussed above are independent sources because these sources does not depend on other voltage or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside

2.10.3 Dependent Sources

Dependent source are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the **Fig. 2.20** and further classified as,

i) **Voltage-Controlled Voltage Source (VCVS)**: It produces a voltage as a function of voltage elsewhere in the given circuit. It is shown in the **Fig. 2.20 (a)**. The controlling voltage is named v_x the equation that determines the supplied voltage v_s is

$v_s = \mu\ v_x$, and the reference polarity for v_s is as indicated. Note that μ is a multiplying constant that is dimensionless.

ii) **Current-Controlled Voltage Source (CCVS)**: It produces voltage as a function of current elsewhere in the given circuit. It is shown In the **Fig. 2.20(b)**. the controlling current is i_x the equation for the supplied voltage v_s is $v_s = \rho\ i_x$,

the reference polarity is as shown and the multiplying constant **ρ** has the dimension volts per ampere

iii) **Voltage-Controlled Current Source (VCCS)**: It produces current as a function of voltage elsewhere in the given circuit. It is shown in the **Fig. 2.20(c)**. The controlling voltage is v_x, the equation for the supplied current i_s is $i_s = \alpha\ v_x$,

the reference direction is as shown and the multiplying constant **α** has the dimension amperes per volt

iv) **Current-Controlled Current Source (CCCS)**: It produces current as a function of current elsewhere in the given circuit. It is shown in the **Fig. 2.20 (d)**. the controlling current is i_x the equation for the supplied current i_s is $i_s = \beta\ i_x$,

the reference direction is as shown, and the multiplying constant β is dimensionless.

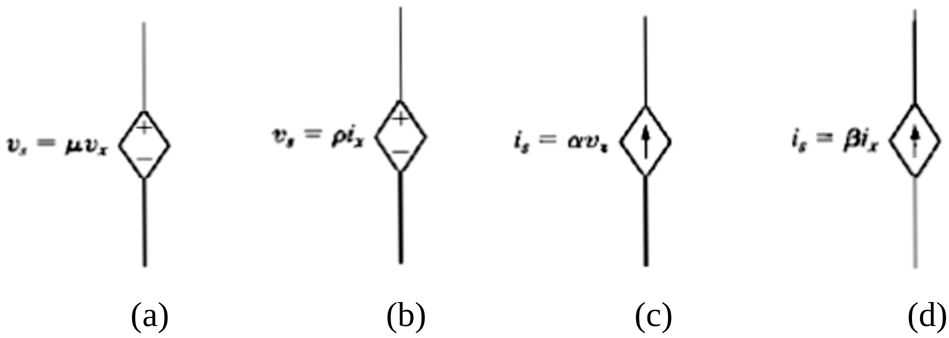


Figure 2.20 The circuit symbols a) an ideal dependent voltage-controlled voltage source, (b) an ideal dependent current-controlled voltages source, (c) an ideal dependent voltage-controlled current source (d) an ideal dependent current-controlled current source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits. An example of a current controlled voltage source is shown on the right-hand side of **Fig. 2.21**, where the voltage 10i of the voltage source depends on the current ***i*** through element C.

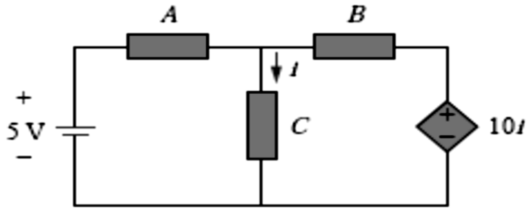


Figure 2.21 the source on the right-hand side is a current-controlled voltage source.

2.11 Combinations of Sources

In a network consisting of many sources, series and parallel combinations of sources exist. If such combinations are replaced by the equivalent source then the network simplification becomes much easier. Let us consider such series and parallel combinations of energy sources.

2.11.1 Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources. Consider the two sources as shown in the **Fig. 2.22**.

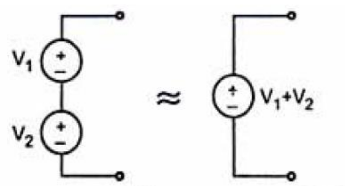


Figure 2.22

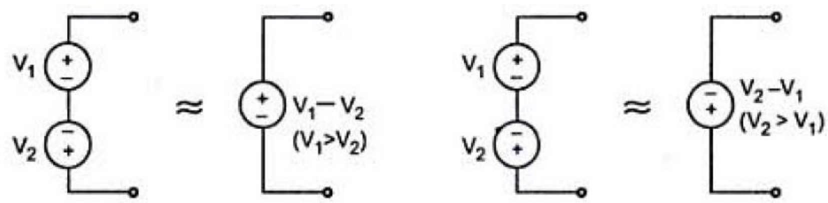


Figure 2.23

If the polarities of the two sources are same then the equivalent single source is the addition of the two sources with polarities same as that of the two sources.

Consider the two sources as shown in the **Fig. 2.23**. If the polarities of the two sources are different then the equivalent single source is the difference between the two voltage sources. The polarity of such source is same as that of the greater of the two sources.

> **Key Point:** *the voltage sources to be connected in series must have same current rating through their voltage ratings may be same or different.*

2.11.2 Voltage Sources in Parallel

Consider the two voltage source in parallel as shown in the **Fig. 2.24**. The equivalent single source has a value same as $\mathbf{V_1}$ and$\mathbf{V_2}$. It must be noted that all the open circuit voltage provided by each source must be equal as the sources are in parallel.

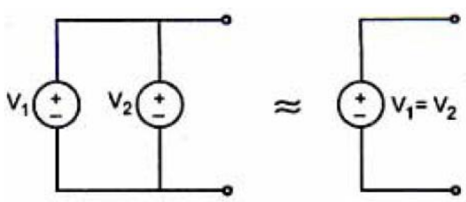


Figure 2.24

> **Key Point:** *the voltage sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.*

2.11.3 Current Sources in Series

Consider the two current sources in series is shown in the **Fig. 2.25**, the equivalent single source has a value same as $\mathbf{I_1}$ and $\mathbf{I_2}$.

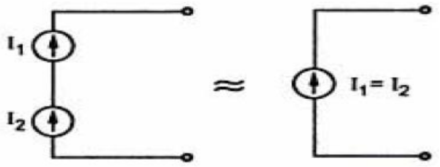


Figure 2.25

Key Point: *the current sources to be connected in series must have same current rating through their voltage ratings may be same or different.*

2.11.4 Current Sources in Parallel

Consider the two current sources in parallel as shown in the **Fig. 2.26**.

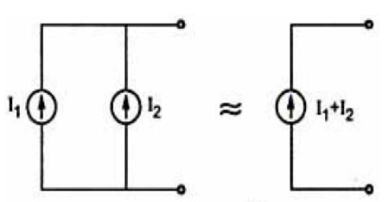


Figure 2.26

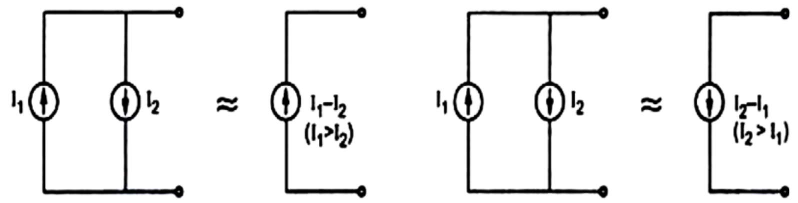


Figure 2.27

if the directions of the currents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources.

Consider the two current sources with opposite directions connected in parallel as shown in the **Fig. 2.27**. If the directions of the two sources are different then the equivalent single source has a direction same as greater of the two sources with value equal to the difference between the two voltage sources.

Key Point: *the current sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.*

2.12 NOTATION

Notation will play an increasingly important role in the analysis to follow.

i) Double-Subscript Notation

The fact that voltage is an across variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In **Fig. 2.28(a)**, the two points that define the voltage across the resistor **R** are denoted by **a** and **b**. Since a is the first subscript for $\mathbf{V_{ab}}$, point a must have a higher potential than point **b** if $\mathbf{V_{ab}}$

is to have a positive value. If, in fact, point b is at a higher potential than point a, $\mathbf{V_{ab}}$ will have a negative value, as indicated in **Fig. 2.28(b)**.

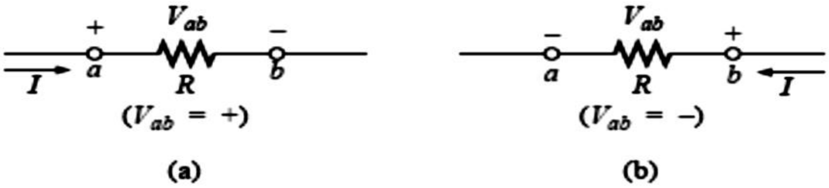


Figure 2.28 defining the sign for double-subscript notation.

In summary:

The voltage V_{ab} is the voltage at point a with respect to (w.r.t.) point b.

ii) Single-Subscript Notation

If point **b** of the notation $\mathbf{V_{ab}}$ is specified as ground potential (zero volts), then a single subscript notation can be employed that provides the voltage at a point with respect to ground.

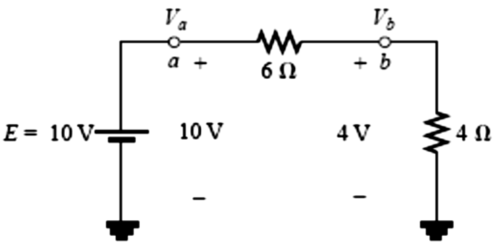


Figure 2.29 defining the use of single-subscript notation for voltage levels.

In **Fig. 2.29**, $\mathbf{V_a}$ is the voltage from point a to ground. In this case it is obviously **10 V** since it is right across the source voltage **E**. The voltage $\mathbf{V_b}$ is the voltage from point **b** to ground. Because it is directly across the **4-Ω** resistor, $\mathbf{V_b}$ = 4 V.

In summary:

The single-subscript notation Va specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of Va .

General Comments

A particularly useful relationship can now be established that will have extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$V_{ab} = V_a - V_b \qquad (2.29)$$

In other words, if the voltage at points **a** and **b** is known with respect to ground, then the voltage $\mathbf{V_{ab}}$ can be determined using **Eq. (2.29)**. In **Fig. 2.29**, for example,

$$V_{ab} = V_a - V_b = 10\text{ V} - 4\text{ V} = 6\text{ V}$$

2.13 KIRCHHOFF'S LAWS

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (**KCL**) and Kirchhoff's voltage law (**KVL**).

2.13.1 Kirchhoff's current law

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero or the sum of the currents entering a node is equal to the sum of the currents leaving the node.

Mathematically, **KCL** implies that

$$\sum_{n=1}^{N} i_n = 0 \qquad (2.30)$$

where **N** is the number of branches connected to the node and in is the nth current entering (or leaving) the node.

Consider the node in **Fig. 2.30**. Applying **KCL** gives

$$\mathbf{i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0} \qquad (2.31)$$

since currents $\mathbf{i_1}$, $\mathbf{i_3}$, and $\mathbf{i_4}$ are entering the node, while currents $\mathbf{i_2}$ and $\mathbf{i_5}$ are leaving it. By rearranging the terms, we get

$$\mathbf{i_1 + i_3 + i_4 = i_2 + i_5} \qquad (2.32)$$

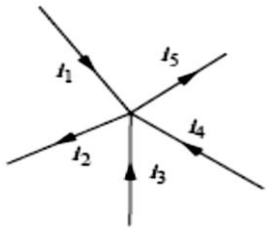
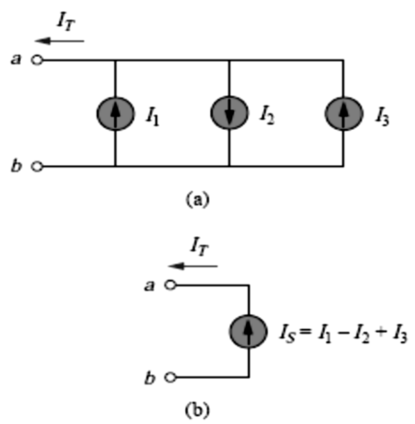


Figure 2.30 Currents at a node illustrating KCL.

A simple application of **KCL** is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in **Fig. 2.31(a)** can be combined as in **Fig. 2.31(b)**. The combined or equivalent current source can be found by applying **KCL** to node **a**.



$$\mathbf{I_T + I_2 = I_1 + I_3}$$

or

$\mathbf{I_T = I_1 - I_2 + I_3}$

Figure 2.31 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

A circuit cannot contain two different currents, $\mathbf{I_1}$ and $\mathbf{I_2}$, in series, unless $\mathbf{I_1} = \mathbf{I_2}$; otherwise **KCL** will be violated.

2.13.2 Kirchhoff's voltage law

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, **KVL** states that

$$\sum_{m=1}^{M} v_m = 0 \qquad (2.33)$$

Where **M** is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

To illustrate **KVL**, consider the circuit in **Fig. 2.32**. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, **KVL** yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \qquad (2.34)$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \qquad (2.35)$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises (2.36)

This is an alternative form of **KVL**. Notice that if we had traveled counterclockwise, the result would have been $+v_1$, $-v_5$, $+v_4$, $-v_3$, and $-v_2$, which is the same as before, except that the signs are reversed. Hence, **Eqs. (2.34) and (2.35)** remain the same.

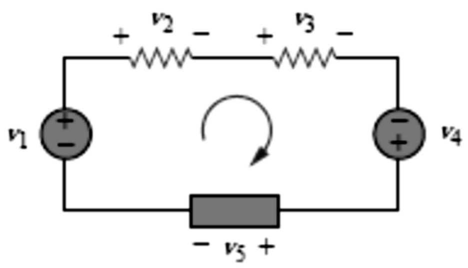


Figure 2.32 A single-loop circuit illustrating KVL.

When voltage sources are connected in series, **KVL** can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources.

2.13.3 Steps to Apply Kirchhoff. Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1: Draw the circuit diagram from the given information and insert all the value of sources with appropriate polarities and all the resistances.

Step 2: Mark all the branch currents with assumed directions using **KCL** at various nodes and junction points. Kept the number of unknown currents as minimum as far as possible to limit the mathematical calculations required to solve them later on. Assumed directions may be wrong; in such case answer of such current will be mathematically negative which indicates the correct direction of the current.

Step 3: Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistance of the network. This is necessary for application of **KVL** to various closed loops.

Step 4: Apply **KVL** to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any preview equation.

2.14 Solving Simultaneous Equations and Cramer's Rule

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error. Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equations happens to be very large, use of a digital computer can make the task easy. Let us assume that set of simultaneous equations obtained is, as follows,

$$a_{11} x_1 + a_{12} x_2 + \ldots\ldots\ldots\ldots + a_{1n} x_n = C_1$$
$$a_{21} x_1 + a_{22} x_2 + \ldots\ldots\ldots\ldots + a_{2n} x_n = C_2$$
$$\vdots$$
$$a_{n1} x_1 + a_{n2} x2 + \ldots\ldots\ldots\ldots + a_{nn} x_n = C_n$$

where C_1, C_2, ………, C_n constants. Then Cramer's rule says that form a system determinant **Δ** or **D** as,

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = D$$

Then obtain the subdeterminant **Dj** by replacing **j^th^** column of **Δ** by the column of constants existing on right hand side of equations i.e. $C_1, C_2, \ldots C_n$;

$$D_1 = \begin{bmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_n & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \qquad D_2 = \begin{bmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{bmatrix}$$

and

$$D_n = \begin{bmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & C_n \end{bmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D}, \quad X_2 = \frac{D_2}{D}, \cdots, X_n = \frac{D_n}{D}$$

Where D_1, D_2, …, D_n and **D** are values of the respective determents

__Example 2.6:__ Apply Kirchhoff's laws to the circuit shown in figure 1 below Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative?

If yes, explain the significance of the negative sign.

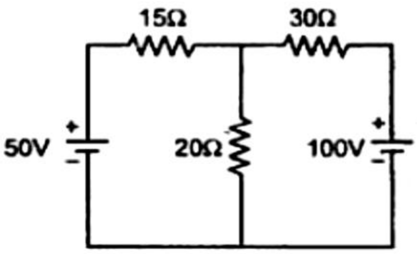


Figure 1

__Solution:__ Application Kirchhoff's laws:

Step 1and 2: Draw the circuit with all the values which are same as the given network.

Mark all the branch currents starting from **+ve** of any of the source, say +ve of 50 V source

Step 3: Mark all the polarities for different voltages across the resistance. This is combined with step 2 shown in the network below in Fig. 1 (a).

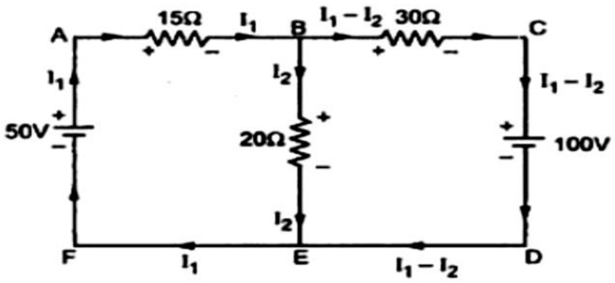


Figure 1 (a)

Step 4: Apply **KVL** to different loops.

Loop 1: **A-B-E-F-A**, $-15\,I_1 - 20\,I_2 + 50 = 0$

Loop 2: **B-C-D-E-D**, $-30\,(I_1 - I_2) - 100 + 20\,I_2 = 0$

Rewriting all the equations, taking constants on one side,

$$15\,I_1 + 20\,I_2 = 50, \qquad -30\,I_1 + 50\,I_2 = 100$$

Apply Cramer's rule, $D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$

Calculating **D_1**, $D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500$

$$I_1 = \frac{D_1}{D} = \frac{500}{1350} = 0.37\ A$$

Calculating **D_2**, $D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000$

$$I_2 = \frac{D_2}{D} = \frac{3000}{1350} = 2.22\ A$$

For **I_1** and **I_2** as answer is positive, assumed direction is correct.

For **I_1** answer is **0.37** A. For **I_2** answer is **2.22** A

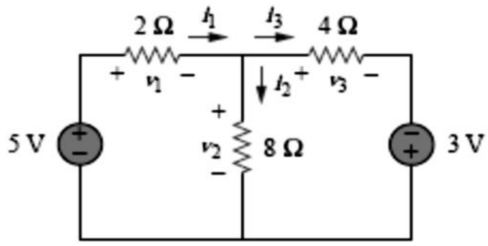
$$I_1 - I_2 = 0.37 - 2.22 = -1.85\ A$$

Negative sign indicates assumed direction is wrong.

i.e. **$I_1 - I_2$** = 1.85 A flowing in opposite direction to that of the assumed direction.

<u>Practice problem 2 .6:</u> Find the currents and voltages in the circuit shown below.

Answer: **$v_1 = 3$ V, $v_2 = 2$ V, $v_3 = 5$ V, $i_1 = 1.5$ A, $i_2 = 0.25$ A, $i_3 = 1.25$ A.**



METHODS OF ANALYSIS

3.1 INTRODUCTION

Having understood the fundamental laws of circuit theory (**Ohm's law** and **Kirchhoff's laws**), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: nodal analysis, which is based on a systematic application of Kirchhoff's current law (**KCL**), and mesh analysis, which is based on a systematic application of Kirchhoff's voltage law (**KVL**). The two techniques are so important that this chapter should be regarded as the most important in the lectures.

3.2 NODAL ANALYSIS

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously. To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $\mathbf{v_1}$, $\mathbf{v_2}$,. . $\mathbf{v_{n-1}}$ to the remaining **n-1** nodes. The voltages are referenced with respect to the reference node.
2. Apply **KCL** to each of the **n-1** nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

We shall now explain and apply these three steps.

The first step in nodal analysis is selecting a node as the reference or datum node. The reference node is commonly called the ground since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in **Fig. 3.1**. We shall always use the symbol in **Fig. 3.1(b)**. Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit in **Fig. 3.2(a)**. Node 0 is the reference node (**v = 0**), while nodes 1 and 2 are assigned voltages $\mathbf{v_1}$ and $\mathbf{v_2}$, respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in **Fig. 3.2(a)**, each node voltage is the voltage with respect to the reference node.

The number of nonreference nodes is equal to the number of independent equations that we will derive.

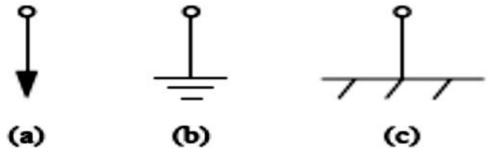


Figure 3.1 Common symbols for indicating a reference node.

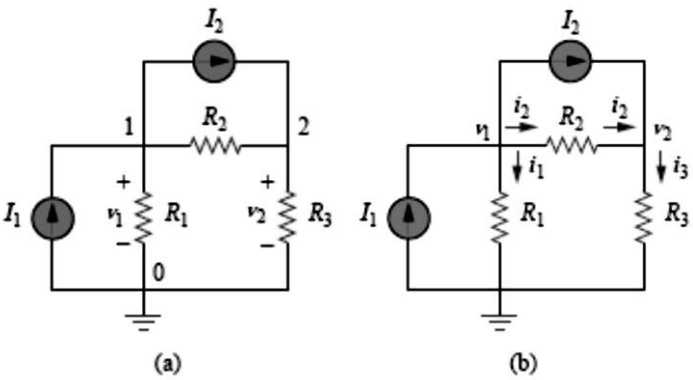


Figure 3.2 Typical circuits for nodal analysis.

As the second step, we apply **KCL** to each nonreference node in the circuit. To avoid putting too much information on the same circuit, the circuit in **Fig. 3.2(a)** is redrawn in **Fig. 3.2(b)**, where we now add $\mathbf{i_1}$, $\mathbf{i_2}$, and $\mathbf{i_3}$ as the currents through resistors $\mathbf{R_1}$, $\mathbf{R_2}$, and $\mathbf{R_3}$, respectively. At node 1, applying **KCL** gives

$$\mathbf{I_1 = I_2 + i_1 + i_2} \tag{3.1}$$

At node 2,

$$\mathbf{I_2 + i_2 = i_3} \tag{3.2}$$

We now apply Ohm's law to express the unknown currents $\mathbf{i_1}$, $\mathbf{i_2}$, and $\mathbf{i_3}$ in terms of node voltages.

Current flows from a higher potential to a lower potential in a resistor.

We can express this principle as

$$i = \frac{v_{higher} - v_{lower}}{R} \tag{3.3}$$

Note that this principle is in agreement with the way we defined resistance in Chapter 2 (see **Fig. 2.3**). With this in mind, we obtain from **Fig. 3.2(b)**,

$$i_1 = \frac{v_1 - 0}{R_1}, \quad \text{or } i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2}, \quad \text{or } i_2 = G_2 (v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3}, \quad \text{or } i_3 = G_3 v_2 \tag{3.4}$$

Substituting **Eq. (3.4) in Eqs. (3.1)** and **(3.2)** results, respectively, in

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \tag{3.5}$$

$$I_2 + v_1 - \frac{v_2}{R_2} = \frac{v_2}{R_3} \tag{3.6}$$

In terms of the conductances, Eqs. (3.5) and (3.6) become

$$I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2) \tag{3.7}$$

$$I_2 + G_2 (v_1 - v_2) = G_3 v_2 \tag{3.8}$$

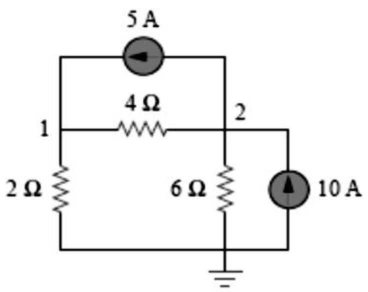
The third step in nodal analysis is to solve for the node voltages. If we apply **KCL** to **n−1** nonreference nodes, we obtain **n−1** simultaneous equations such as **Eqs. (3.5)** and **(3.6)** or **(3.7)** and **(3.8)**. For the circuit of **Fig. 3.2**, we solve **Eqs. (3.5)** and **(3.6)** or **(3.7)** and **(3.8)** to obtain the node voltages v_1 and v_2 using any standard method, such as the substitution method, the elimination method, Cramer's rule, or matrix inversion. To use either of the last two methods, one must cast the simultaneous equations in matrix form. For example, **Eqs. (3.7)** and **(3.8)** can be cast in matrix form as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \tag{3.9}$$

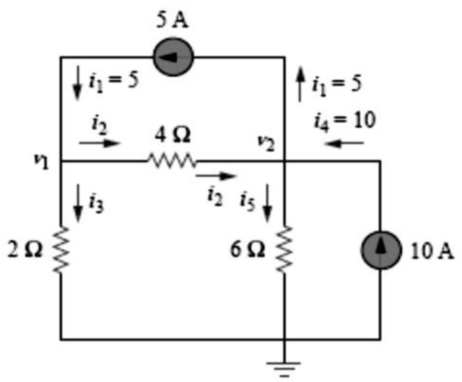
which can be solved to get v_1 and v_2.

Example 3.1: Calculate the node voltages in the circuit shown in **Fig. 3.3(a)**.

Solution:



(a)



(b)

Consider **Fig. 3.3(b)**, where the circuit in **Fig. 3.3(a)** has been prepared for nodal analysis. Notice how the currents are selected for the application of **KCL**. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that i2 enters the 4_resistor from the left-hand side, i2 must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages v1 and v2 are now to be determined.

At node 1, applying **KCL** and **Ohm's law** gives

$\mathbf{i_1 = i_2 + i_3 \Rightarrow\ 5\ = \frac{v1 - v2}{4} + \frac{v1 - 0}{2}}$

Multiplying each term in the last equation by 4, we obtain

$$\mathbf{20 = v_1 - v_2 + 2v_1}$$

or

$$\mathbf{3v_1 - v_2 = 20} \qquad (3.1.1)$$

At node 2, we do the same thing and get

$$\mathbf{i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v1 - v2}{4} + 10 = 5 + \frac{v2 - 0}{6}}$$

Multiplying each term by 12 results in

$$3v_1 - 3v2 = + 120 = 60 + 2v_2$$

or

$$-3v_1 + 5v_2 = 60 \qquad (3.1.2)$$

: (a) original
sis

Now we have two simultaneous **Eqs. (3.1.1)** and **(3.1.2)**. We can solve the equations using any method and obtain the values of v1 and v2.

<u>METHOD 1:</u> Using the elimination technique, we add **Eqs. (3.1.1)** and **(3.1.2)**.

$$4v_2 = 80 \Rightarrow v_2 = 20\text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \Rightarrow v_1 = 40/3 = 13.33\text{ V}$$

<u>METHOD 2:</u> To use **Cramer's rule**, we need to put **Eqs. (3.1.1)** and **(3.1.2)** in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \qquad (3.1.3)$$

The determinant of the matrix is

$$\boldsymbol{\Delta = D = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12}$$

We now obtain $\mathbf{v_1}$ and $\mathbf{v_2}$ as

$$\boldsymbol{v_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{D} = \frac{100+60}{12} = 13.33V}$$

$$\boldsymbol{v_2 = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{D} = \frac{180+60}{12} = 20V}$$

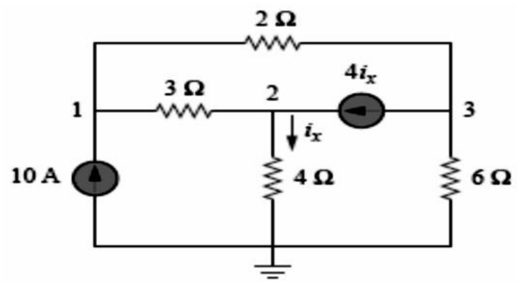
If we need the currents, we can easily calculate them from the values of the nodal voltages.

$\mathbf{i_1 = 5\ A,\ } \boldsymbol{i_2 = \frac{v_1 - v_2}{4} = -1.6667\ A,\ i_3 = \frac{v_1}{2} = 6.666A,}\ \mathbf{i_4 = 10\ A,\ } \boldsymbol{i_5 = \frac{v_2}{6} = 3.333A}$

The fact that $\mathbf{i_2}$ is negative shows that the current flows in the direction opposite to the one assumed.

Practice problem 3.1: Find the voltages at the three nonreference nodes in the circuit of Figure below.

Answer: v_1 = 80 V, v_2 = −64 V, v_3 = 156 V.



3.2.1 NODAL ANALYSIS WITH VOLTAGE SOURCES

We now consider how voltage sources affect nodal analysis. We use the circuit in **Fig. 3.4** for illustration. Consider the following two possibilities.

CASE 1: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In **Fig. 3.4**, for example,

$$\mathbf{v_1 = 10\ V} \qquad (3.10)$$

Thus our analysis is somewhat simplified by this knowledge of the voltage at this node.

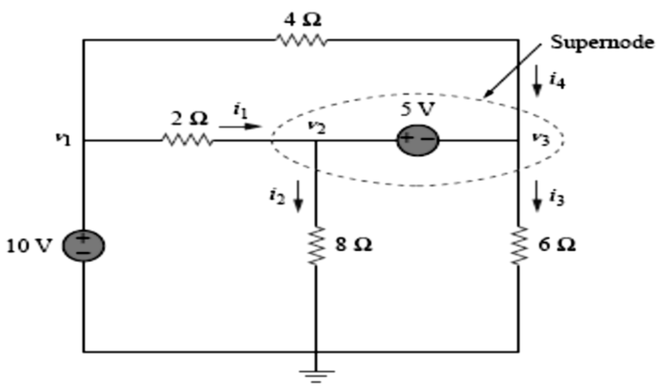


Figure 3.4 A circuit with a supernode.

CASE 2: If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both **KCL** and **KVL** to determine the node voltages.

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

In **Fig. 3.4**, nodes 2 and 3 form a supernode. (We could have more than two nodes forming a single supernode. For example, see the circuit in the **Practice problem 3.4**). We analyze a circuit with supernodes using the same three steps mentioned in the previous section except that the supernodes are treated differently. Why? Because an essential component of nodal analysis is applying **KCL**, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, **KCL** must be satisfied at a supernode like any other node. Hence, at the supernode in **Fig. 3.5**,

$$\mathbf{i_1 + i_4 = i_2 + i_3} \tag{3.11a}$$

or

$$\frac{v1 - v2}{2} + \frac{v1 - v3}{4} = \frac{v2 - 0}{8} + \frac{v3 - 0}{6} \tag{3.11b}$$

To apply Kirchhoff's voltage law to the supernode in **Fig. 3.4**, we redraw the circuit as shown in **Fig. 3.5**. Going around the loop in the clockwise direction gives

$$\mathbf{-v_2 + 5 + v_3 = 0 \Rightarrow v_2 - v_3 = 5} \tag{3.12}$$

From **Eqs. (3.10), (3.11b),** and **(3.12)**, we obtain the node voltages.

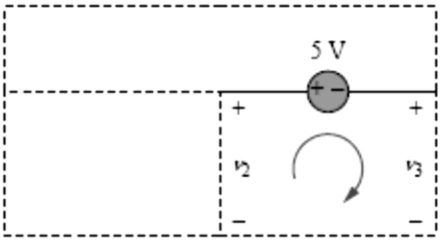


Figure 3.5 Applying KVL to a supernode.

<u>Example 3.2:</u> For the circuit shown in **Fig. 3.6**, find the node voltages.

<u>Solution:</u>

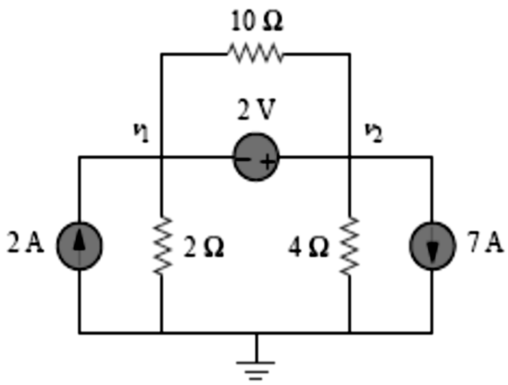


Figure 3.6 For Example 3.2.

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying **KCL** to the supernode as shown in **Fig. 3.7(a)** gives

$$\mathbf{2 = i_1 + i_2 + 7}$$

Expressing $\mathbf{i_1}$ and $\mathbf{i_2}$ in terms of the node voltages

$$2 = \frac{v1 - 0}{2} + \frac{v2 - 0}{4} + 7$$

or

$$\mathbf{v_2 = -20 - 2v_1} \tag{3.2.1}$$

To get the relationship between $\mathbf{v_1}$ and $\mathbf{v_2}$, we apply **KVL** to the circuit in **Fig. 3.7(b)**. Going around the loop, we obtain

$$\mathbf{-v_1 - 2 + v_2 = 0 \ \Rightarrow v_2 = v_1 + 2} \tag{3.3.2}$$

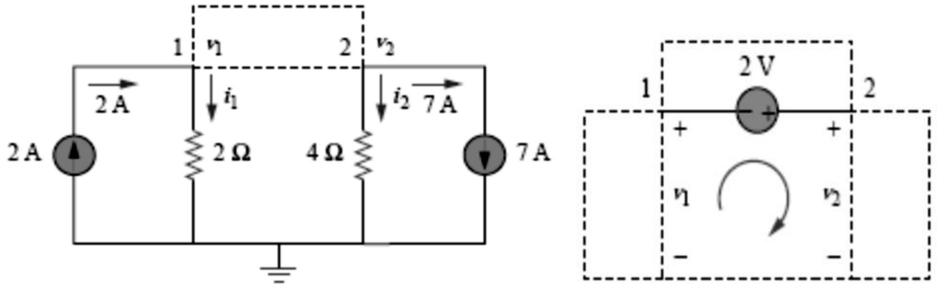
From **Eqs. (3.2.1)** and **(3.2.2)**, we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Rightarrow v_1 = -7.333\ V$$

and $v_2 = v_1 + 2 = -5.333$ **V**. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

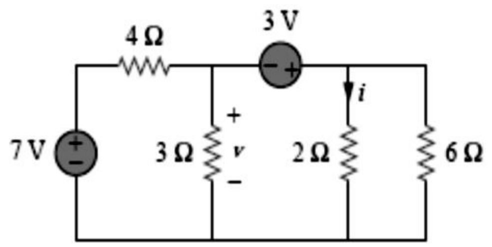


(a) (b)

Figure 3.7 Applying: (a) KCL to the supernode, (b) KVL to the loop.

Practice problem 3.2: Find **v** and **i** in the circuit in Figure below.

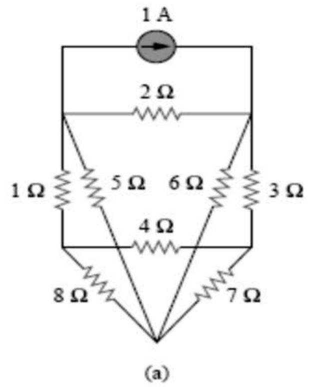
Answer: **−0.2 V, 1.4 A.**



3.3 MESH ANALYSIS

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it.

Nodal analysis applies **KCL** to find unknown voltages in a given circuit, while mesh analysis applies **KVL** to find unknown currents. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. A **planar** circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**. A circuit may have crossing branches and still be **planar** if it can be redrawn such that it has no crossing branches. For example, the circuit in **Fig. 3.8 (a)** has two crossing branches, but it can be redrawn as in **Fig. 3.8 (b)**. Hence, the circuit in **Fig. 3.8 (a)** is planar. However, the circuit in **Fig. 3.9** is **nonplanar**, because there is no way to redraw it and avoid the branches crossing. **Nonplanar** circuits can be handled using nodal analysis, but they will not be considered in this text.



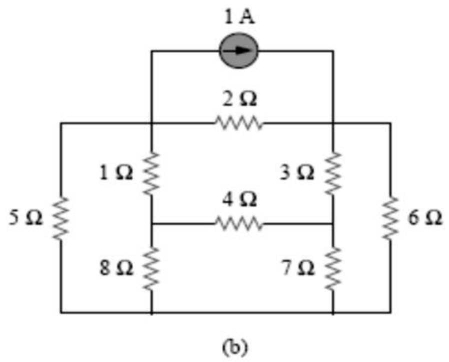


Figure 3.8 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

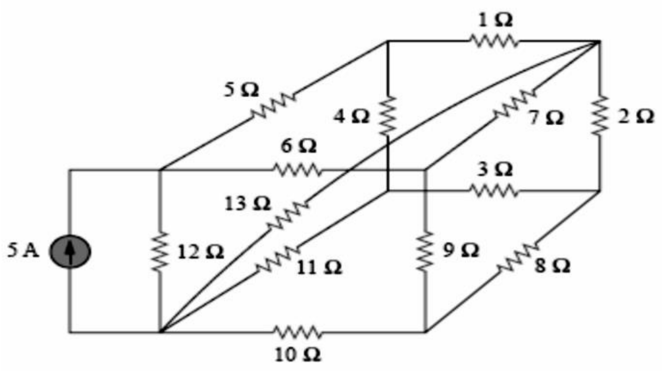


Figure 3.9 A nonplanar circuit.

To understand mesh analysis, we should first explain more about what we mean by a mesh. A mesh is a loop which does not contain any other loops within it.

In **Fig. 3.10**, for example, paths **abefa** and **bcdeb** are meshes, but path **abcdefa** is not a mesh. The current through a mesh is known as mesh current. In mesh analysis, we are interested in applying **KVL** to find the mesh currents in a given circuit.

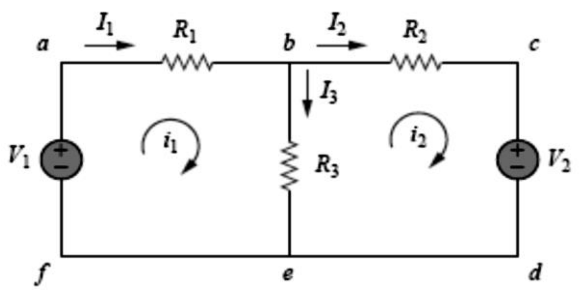


Figure 3.10 circuit with two meshes.

In this section, we will apply mesh analysis to planar circuits that do not contain current sources. In the next sections, we will consider circuits with current sources. In the mesh analysis of a circuit with **n** meshes, we take the following three steps.

Steps to Determine mesh currents:

1. Assign mesh currents $\mathbf{i_1}$, $\mathbf{i_2}$, . . . , $\mathbf{i_n}$ to the n meshes.
2. Apply **KVL** to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

To illustrate the steps, consider the circuit in **Fig. 3.10**. The first step requires that mesh currents $\mathbf{i_1}$ and $\mathbf{i_2}$ are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

As the second step, we apply **KVL** to each mesh. Applying **KVL** to mesh 1, we obtain

$$-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0$$

or

$$(R_1 + R_3)\, i1 - R_3 i_2 = V_1 \qquad (3.13)$$

For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3)\, i_2 = -V_2 \qquad (3.14)$$

Note in **Eq. (3.13)** that the coefficient of $\mathbf{i_1}$ is the sum of the resistances in the first mesh, while the coefficient of $\mathbf{i_2}$ is the negative of the resistance common to meshes 1 and 2. Now observe that the same is true in **Eq. (3.14)**. This can serve as a shortcut way of writing the mesh equations.

The third step is to solve for the mesh currents. Putting **Eqs. (3.13)**. and **(3.14)** in matrix form yields

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} \qquad (3.15)$$

which can be solved to obtain the mesh currents $\mathbf{i_1}$ and $\mathbf{i_2}$. We are at liberty to use any technique for solving the simultaneous equations. If a circuit has **n** nodes, **b** branches, and ***l*** independent loops or meshes, then $l = b - n + 1$. Hence, ***l*** independent simultaneous equations are required to solve the circuit using mesh analysis.

Notice that the branch currents are different from the mesh currents unless the mesh is isolated. To distinguish between the two types of currents, we use **i** for a mesh current and **I** for a branch current. The current elements I_1, I_2, and I_3 are algebraic sums of the mesh currents. It is evident from **Fig. 3.13** that

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2 \qquad (3.16)$$

<u>Example 3.3:</u> For the circuit in **Fig. 3.11**, find the branch currents I_1, I_2, and I_3 using mesh analysis.

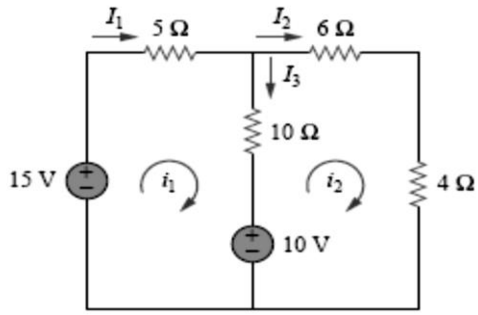


Figure 3.11 For Example 3.3.

<u>Solution:</u>

We first obtain the mesh currents using **KVL**. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or 15i1 -10i2 = 5

$$3i_1 - 2i_2 = 1 \qquad (3.5.1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

Or 20I2 + 10I1 = 10

$$i_1 = 2i_2 - 1 \qquad (3.5.2)$$

Using the substitution method, we substitute **Eq. (3.3.2)** into **Eq. (3.3.1)**, and write

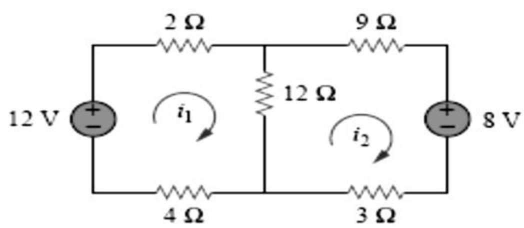
$$6i_2 - 3 - 2i_2 = 1 \Rightarrow \quad i_2 = 1 \text{ A}$$

From **Eq. (3.5.2)**, $i_1 = 2i_2 - 1 = 2 - 1 = 1$ **A**. Thus,

$$I_1 = i_1 = 1 \text{ A}, I_2 = i_2 = 1 \text{ A}, I_3 = i_1 - i_2 = 0$$

<u>Practice problem 3.3:</u> Calculate the mesh currents i_1 and i_2 in the circuit of Figure below.

<u>Answer:</u> $i_1 = 23$ **A**, $i_2 = 0$ **A.**



CHAPTER FOUR

CIRCUIT THEOREMS

4.1 INTRODUCTION

The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to ***linear* circuits**, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of **superposition**, **maximum power transfer**, **Millman's theorem**, **Substitution theorem**, and **Reciprocity theorem** in this chapter.

4.2 LINEARITY PROPERTY

Linearity is the property of an element describing a linear relationship between cause and effect. The property is a combination of both the homogeneity property and the additivity property.

The homogeneity property requires that if the input (also called the excitation) is multiplied by a constant, then the output (also called the response) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input **i** to the output **v**,

$$v = iR \quad (4.1)$$

If the current is increased by a constant **k**, then the voltage increases correspondingly by **k**, that is,

$$kiR = kv \quad (4.2)$$

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the relationship of a resistor, if

$$v_1 = i_1R \quad \text{and} \quad v_2 = i_2R \quad (4.3)$$

then applying $(i_1 + i_2)$ gives

$$v = (i_1 + i_2)\,R = i_1R + i_2R = v_1 + v_2 \quad (4.4)$$

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

4.3 SUPERPOSITION

The idea of superposition rests on the linearity property.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables. With these in mind, we apply the superposition principle in three steps:

Steps to Apply Super position Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. Keep in mind that superposition is based on linearity.

Example 4.2: Use the superposition theorem to find **v** in the circuit in **Fig. 4.2**.

Solution:

Since there are two sources, let

$$\mathbf{v = v_1 + v_2}$$

Where $\mathbf{v_1}$ and $\mathbf{v_2}$ are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain $\mathbf{v_1}$, we set the current source to zero, as shown in **Fig. 4.3(a)**. Applying **KVL** to the loop in **Fig. 4.3(a)** gives

$$\mathbf{12i_1 - 6 = 0 \Rightarrow i_1 = 0.5\ A}$$

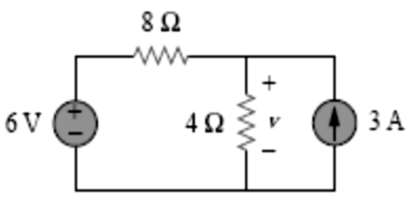
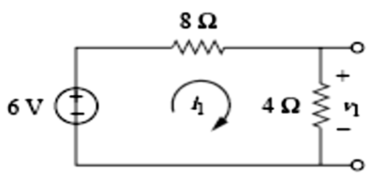


Figure 4.2 for Example 4.2.



(a)

Thus,

$$\mathbf{v_1 = 4i_1 = 2\ V}$$

We may also use voltage division to get $\mathbf{v_1}$ by writing

$$v_1 = \frac{4}{4+8}(6) = 2\ V$$

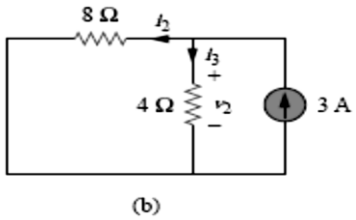


Figure 4.3 for Example 4.2:
(a) Calculating v_1, (b) calculating v_2.

To get $\mathbf{v_2}$, we set the voltage source to zero, as in **Fig. 4.3(b)**. Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2\ A$$

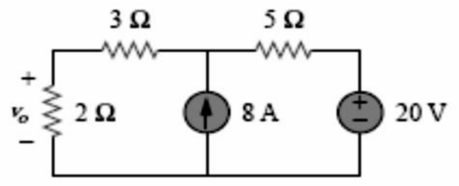
Hence, $\mathbf{v_2 = 4i_3 = 8\ V}$

And we find $\mathbf{v = v_1 + v_2 = 2 + 8 = 10\ V}$

<u>Practice problems:</u>

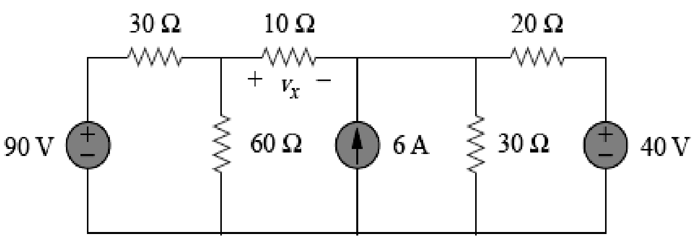
1-Using the superposition theorem, find $\mathbf{v_o}$ in the circuit in Figure below.

<u>Answer:</u> 12 V.



2- Use superposition to obtain v_x in the circuit of Figure below.

<u>Answer:</u> v_x = -7.3V.



<u>4.4 SOURCE TRANSFORMATION</u>

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. Source transformation is another tool for simplifying circuits. We can substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in **Fig. 4.4**. Either substitution is known as a ***source transformation***.

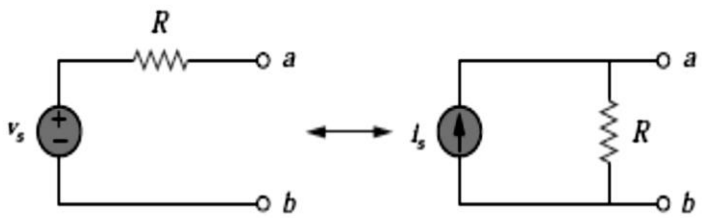


Figure 4.4 Transformation of independent sources.

Key Point: *A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.*

We need to find the relationship between $\boldsymbol{v_s}$ and $\boldsymbol{i_s}$ that guarantees the two configurations in **Fig. 4.4** are equivalent with respect to nodes **a, b**.

Suppose $\mathbf{R_L}$, is connected between nodes **a, b** in **Fig. 4.4(a)**. Using Ohms law, the current in $\mathbf{R_L}$ is.

$$i_L = \frac{v_s}{(R+R_L)} \qquad \text{R and } R_L \text{ in series} \qquad (4.5)$$

If it is to be replaced by a current source then load current must be $\frac{V}{(R+R_L)}$

Now suppose the same resistor $\mathbf{R_L}$, is connected between nodes **a**, **b** in **Fig. 4.4 (b)**. Using current division, the current in $\mathbf{R_L}$, is

$$i_L = i_s \frac{R}{(R+R_L)} \qquad (4.6)$$

If the two circuits in **Fig. 4.4** are equivalent, these resistor currents must be the same. Equating the right-hand sides of **Eqs.4.5** and **4.6** and simplifying

$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R \qquad (4.7)$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in **Fig. 4.5**, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.

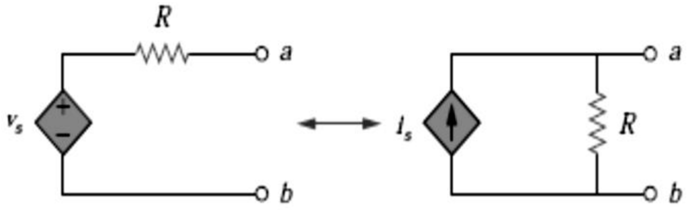


Figure 4.5 Transformation of dependent sources.

However, we should keep the following points in mind when dealing with source transformation.

1. Note from **Fig. 4.4** (or **Fig. 4.5**) that the arrow of the current source is directed toward the positive terminal of the voltage source.

2. Note from **Eq. (4.7)** that source transformation is not possible when **R = 0**, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, **R ≠ 0**. Similarly, an ideal current source with **R** =∞cannot be replaced by a finite voltage source.

__**Example 4.3:**__ Use source transformation to find v_o in the circuit in **Fig. 4.6**.

__**Solution:**__

We first transform the current and voltage sources to obtain the circuit in **Fig. 4.7(a)**. Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us **Fig. 4. 7(b)**. We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in **Fig. 4.7 (c)**.

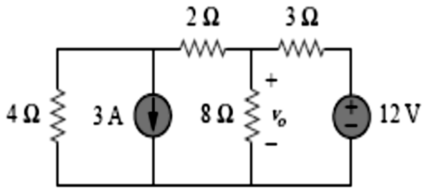


Figure 4.6

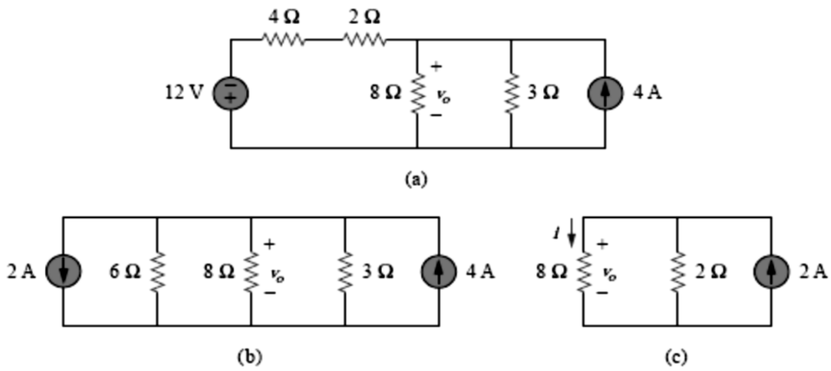


Figure 4.7

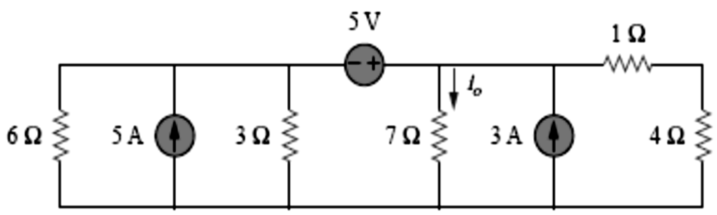
Alternatively, since the 8-Ω and 2-Ω resistors in **Fig. 4.7(c)** are in parallel, they have the same voltage ***vo*** across them. Hence,

$$v_o = (8||2)(2\ \text{A}) = \frac{8\times 2}{10}(2) = 3.2\ \text{V}$$

__**Practice problems:**__

1-Find i_o in the circuit shown below using source transformation.

Answer: 1.78 A.



__4.5 THEVENIN'S THEOREM__

It often occurs in practice that a particular element in a circuit is variable (usually called the load) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in **Fig. 4.8(a)** can be replaced by that in **Fig. 4.8(b)** is known as the Thevenin equivalent circuit; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

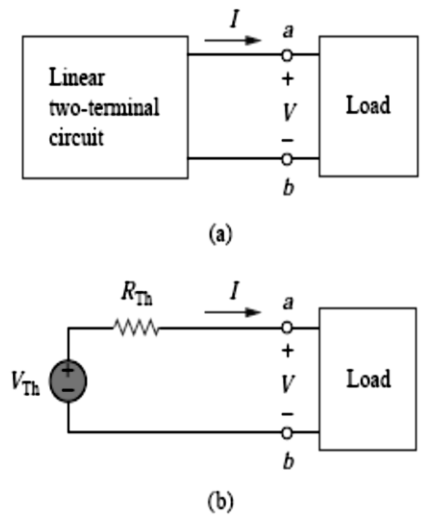


Figure 4.8 Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

To find the Thevenin equivalent voltage $\mathbf{V_{Th}}$ and resistance $\mathbf{R_{Th}}$, suppose the two circuits in **Fig. 4.8** are equivalent. The open-circuit voltage across the terminals a-b in **Fig. 4.8(a)** must be equal to the voltage source $\mathbf{V_{Th}}$ in **Fig. 4.8(b)**, since the two circuits are equivalent. Thus $\mathbf{V_{Th}}$ is the open-circuit voltage across the terminals as shown in **Fig. 4.9(a)**; that is,

$$\mathbf{V_{Th} = v_{oc}} \qquad (4.8)$$

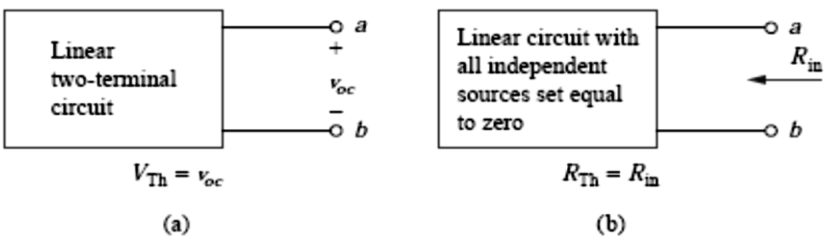


Figure 4.9 Finding V_{Th} and R_{Th}.

$\mathbf{R_{Th}}$ is the input resistance at the terminals when the independent sources are turned off, as shown in **Fig. 4.9(b)**; that is,

$$R_{Th} = R_{in} \quad (4.9)$$

To apply this idea in finding the Thevenin resistance $\mathbf{R_{Th}}$, we need to consider two cases.

CASE 1: If the network has no dependent sources, we turn off all independent sources. $\mathbf{R_{Th}}$ is the input resistance of the network looking between terminals **a** and **b**, as shown in **Fig. 4.9(b)**.

CASE 2: If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source $\mathbf{v_o}$ at terminals **a** and **b** and determine the resulting current $\mathbf{i_o}$. Then $\mathbf{R_{Th}} = \mathbf{v_o/i_o}$, as shown in **Fig. 4.10(a)**. Alternatively, we may insert a current source $\mathbf{i_o}$ at terminals **a-b** as shown in **Fig. 4.10(b)** and find the terminal voltage $\mathbf{v_o}$. Again $\mathbf{R_{Th}} = \mathbf{v_o/i_o}$. Either of the two approaches will give the same result. In either approach we may assume any value of $\mathbf{v_o}$ and $\mathbf{i_o}$. For example, we may use $\mathbf{v_o}$ = 1 V or $\mathbf{i_o}$ = 1 A, or even use unspecified values of $\mathbf{v_o}$ or $\mathbf{i_o}$.

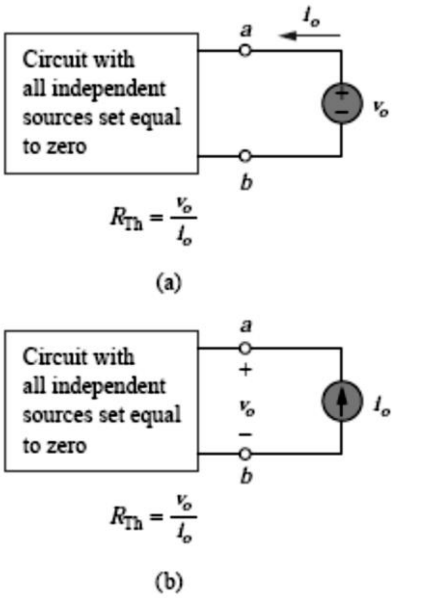


Figure 4.10 Finding RTh when circuit has dependent sources.

It often occurs that $\mathbf{R_{Th}}$ takes a negative value. In this case, the negative resistance ($\mathbf{v = -iR}$) implies that the circuit is supplying power. This is possible in a circuit with dependent sources.

The current $\mathbf{I_L}$ through the load and the voltage $\mathbf{V_L}$ across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in **Fig. 4.11(b)**. From **Fig. 4.11(b)**, we obtain

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \quad (4.10a)$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th} \quad (4.10b)$$

Note from **Fig. 4.11(b)** that the Thevenin equivalent is a simple voltage divider, yielding $\mathbf{V_L}$ by mere inspection.

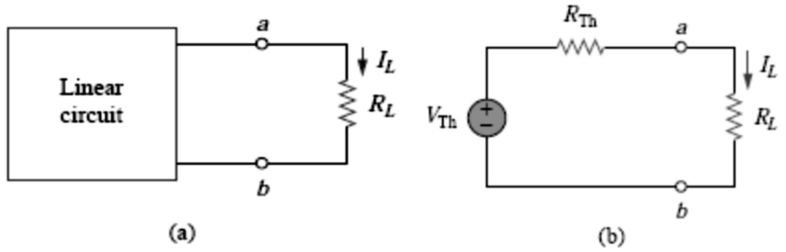


Figure 4.11 A circuit with a load :(a) original circuit, (b) Thevenin equivalent.

<u>Example 4.4:</u> Find the Thevenin equivalent circuit of the circuit shown in **Fig. 4.12**, to the left of the terminals a-b. Then find the current through R_L = 6, 16, and 36 Ω.

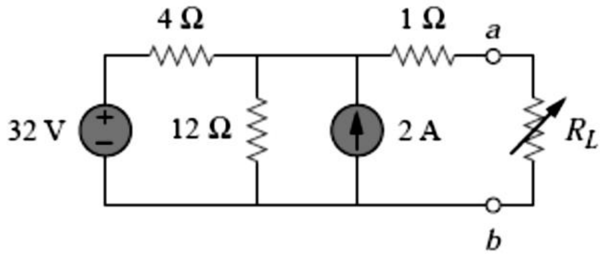


Figure 4.12 For Example 4.4.

<u>Solution:</u>

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in **Fig. 4.13(a)**. Thus,

$$\boldsymbol{R_{Th} = 4||12 + 1 = \frac{4\times 12}{16} + 1 = 4\,\Omega}$$

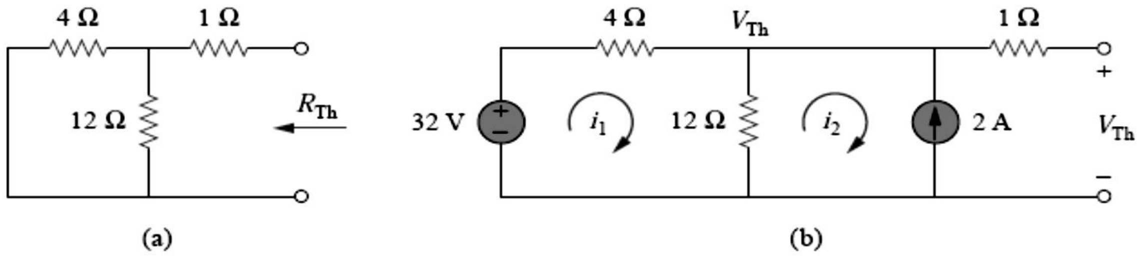


Figure 4.13 For Example 4.4: (a) finding R_{Th}, (b) finding V_{Th}.

To find $\mathbf{V_{Th}}$, consider the circuit in **Fig. 4.13(b)**. Applying mesh analysis to the two loops, we obtain

$$\mathbf{-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\ A}$$

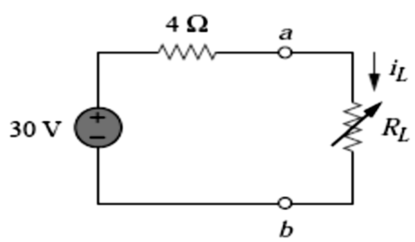
Solving for $\mathbf{i_1}$, we get $\mathbf{i_1 = 0.5\ A}$. Thus,

$$\mathbf{V_{Th} = 12(i1 - i2) = 12(0.5 + 2.0) = 30\ V}$$

The Thevenin equivalent circuit is shown in **Fig. 4.14**. The current through $\mathbf{R_L}$ is

$$\boldsymbol{I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}}$$

When $\mathbf{R_L}$ = 6, $\quad \boldsymbol{I_L} = \frac{\mathbf{30}}{\mathbf{10}} = 3\ A$



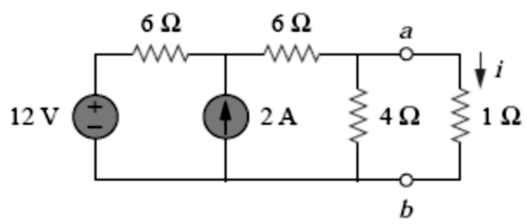
When $R_L = 16$, $\quad I_L = \frac{30}{20} = 1.5\ A$

When RL = 36, $\quad I_L = \frac{30}{40} = 0.75\ A$

Figure 4.14 The Thevenin equivalent circuit

Practice problem: Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in Figure below. Then find **i**.

Answer: $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, i = 1.5 A.



4.6 NORTON'S THEOREM

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in **Fig. 4.15(a)** can be replaced by the one in **Fig. 4.15(b)**.

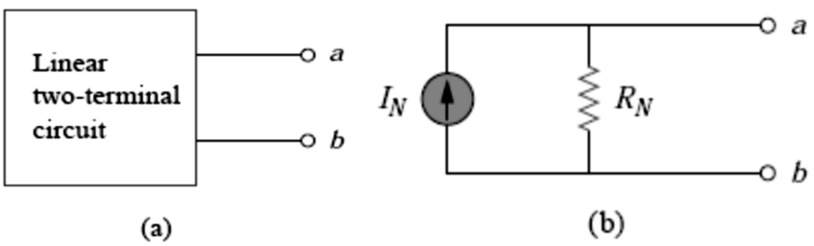


Figure 4.15 (a) Original circuit, (b) Norton equivalent circuit.

We are mainly concerned with how to get R_N and I_N. We find R_N in the same way we find R_{Th}. In fact, the Thevenin and Norton resistances are equal; that is,

$$R_N = R_{Th} \tag{4.11}$$

To find the Norton current I_N, we determine the short-circuit current flowing from terminal a to b in both circuits in **Fig. 4.15**. It is evident that the short-circuit current in **Fig. 4.15(b)** is I_N. This must be the same short-circuit current from terminal a to b in **Fig. 4.15(a)**, since the two circuits are equivalent. Thus,

$$I_N = i_{sc} \tag{4.12}$$

Dependent and independent sources are treated the same way as in Thevenin's theorem. Observe the close relationship between Norton's and Thevenin's theorems: $\mathbf{R_N = R_{Th}}$ as in **Eq. (4.11)**, and

$$I_N = \frac{V_{Th}}{R_{Th}}$$

(4.13)

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. **Example 4.10** will illustrate this. Also, since

$$V_{Th} = v_{oc}$$

(4.14a)

$$I_N = i_{sc}$$

(4.14b)

$$R_{Th} = \frac{voc}{isc} = R_N$$

(4.14c)

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent.

<u>Example 4.5</u> Find the Norton equivalent circuit of the circuit in **Fig. 4.16**.

<u>Solution:</u>

We find $\mathbf{R_N}$ in the same way we find $\mathbf{R_{Th}}$ in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in **Fig. 4.17(a)**, from which we find $\mathbf{R_N}$. Thus,

$$R_N = 5\|(8+4+8) = 5\|20 = \frac{20\times 5}{25} = 4\,\Omega$$

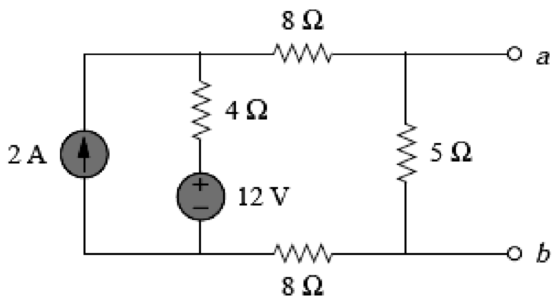


Figure 4.16 For Example 4.5.

To find $\mathbf{I_N}$, we short-circuit terminals a and b, as shown in **Fig. 4.17(b)**. We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine $\mathbf{I_N}$ from $\mathbf{V_{Th}/R_{Th}}$. We obtain $\mathbf{V_{Th}}$ as the open-circuit voltage across terminals a and b in **Fig. 4.17(c)**. Using mesh analysis, we obtain

$$\mathbf{i_3 = 2\ A}$$

$$\mathbf{25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow i_4 = 0.8\ A}$$

and

$$\mathbf{v_{oc} = V_{Th} = 5i_4 = 4\ V}$$

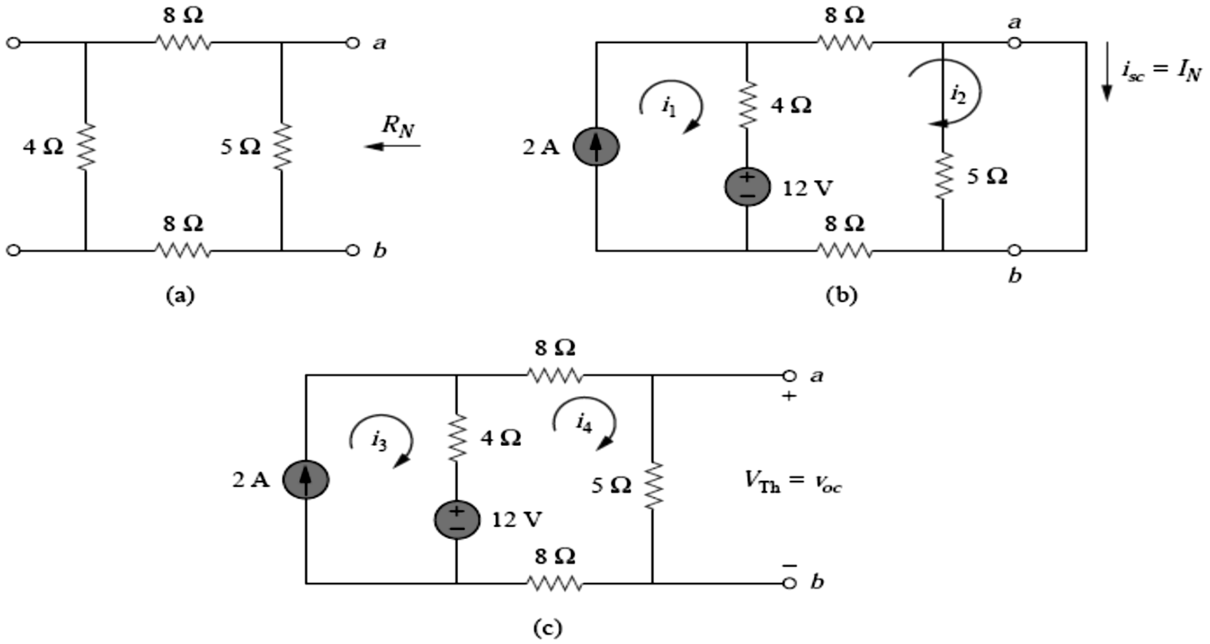


Figure 4.17 For Example 4.5; finding: (a) RN, (b) IN = isc, (c) V_{Th} = voc.

Hence,

$$I_N = \frac{VTh}{RTh} = \frac{4}{4} = 1\ A$$

as obtained previously. This also serves to confirm Eq. that $\mathbf{R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega}$. Thus, the Norton equivalent circuit is as shown in **Fig. 4.18**.

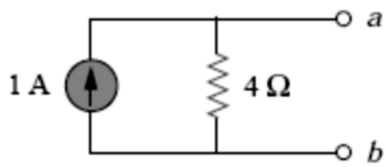
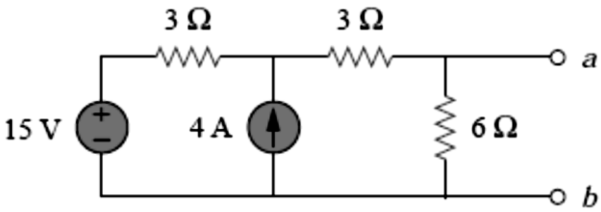


Figure 4.18 Norton equivalent of the circuit in Fig. 4.16.

Practice problem: Find the Norton equivalent circuit for the circuit in Figure below.

Answer: $\mathbf{R_N = 3\ \Omega,\ I_N = 4.5\ A.}$



4.6 MAXIMUM POWER TRANSFER

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses.

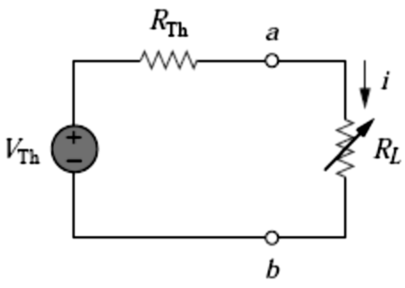


Figure 4.19 The circuit used for maximum power transfer.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance $\mathbf{R_L}$. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in **Fig. 4.19**, the power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

(4.15)

For a given circuit, $\mathbf{V_{Th}}$ and $\mathbf{R_{Th}}$ are fixed. By varying the load resistance $\mathbf{R_L}$, the power delivered to the load varies as sketched in **Fig. 4.20**. We notice from **Fig. 4.20** that the power is small for small or large values of $\mathbf{R_L}$ but maximum for some value of $\mathbf{R_L}$ between **0** and **∞**. We now want to show that this maximum power occurs when $\mathbf{R_L}$ is equal to $\mathbf{R_{Th}}$. This is known as the maximum power theorem.

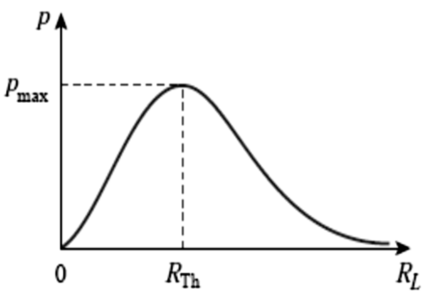


Figure 4.20 Power delivered to the load as a function of RL

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in **Eq. (4.15)** with respect to $\mathbf{R_L}$ and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(RTh + RL)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

This implies that

$$0 = (R_{Th} + R_L - 2R_L) = (R_{Th} - R_L)$$

(4.16)

which yields

$$R_L = R_{Th} \quad (4.17)$$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that **Eq. (4.17)** gives the maximum power by showing that $d^2p/dR^2_L < 0$.

The maximum power transferred is obtained by substituting **Eq. (4.17)** into **Eq. (4.15)**, for

$$p_{max} = \frac{V^2_{Th}}{4R_{Th}} \quad (4.18)$$

Equation (4.18) applies only when $R_L = R_{Th}$. When $R_L \neq R_{Th}$, we compute the power delivered to the load using **Eq. (4.15)**.

Example 4.6: Find the value of R_L for maximum power transfer in the circuit of **Fig. 4.21**. Find the maximum power.

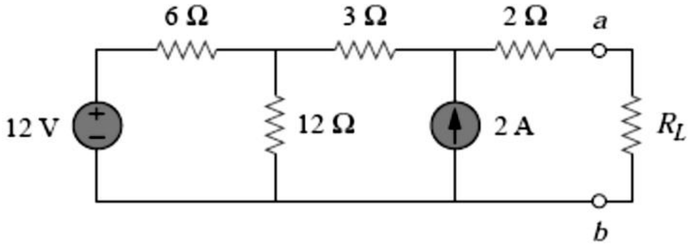


Figure 4.21 For Example 4.6.

Solution:

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th}, we use the circuit in **Fig. 4.22(a)** and obtain

$$R_{Th} = 2 + 3 + 6\|12 = 5 + \frac{6 \times 12}{18} = 9\,\Omega$$

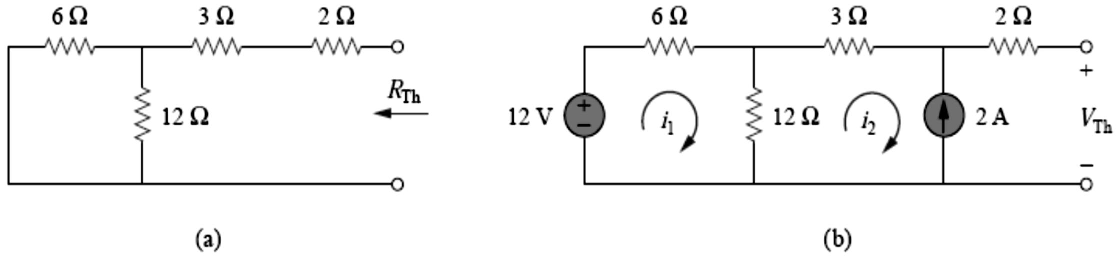


Figure 4.22 For Example 4.6: (a) finding R_{Th}, (b) finding V_{Th}.

To get V_{Th}, we consider the circuit in **Fig. 4.22(b)**. Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying **KVL** around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \Rightarrow V_{Th} = 22\text{ V}$$

For maximum power transfer,

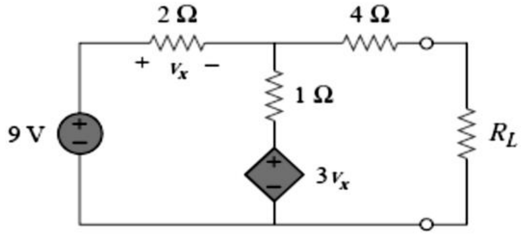
$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4RL} = \frac{22^2}{4 \times 9} = 13.44\ W$$

Practice problem: Determine the value of R_L that will draw the maximum power from the rest of the circuit in Figure below. Calculate the maximum power.

Answer: **4.22 Ω, 2.901 W.**



4.7 MILLMAN'S THEOREM

Through the application of Millman's theorem, any number of parallel voltage sources can be reduced to one. In **Fig. 4.23**, for example, the three voltage sources can be reduced to one. This would permit finding the current through or voltage across RL without having to apply a method such as mesh analysis, nodal analysis, superposition, and so on.

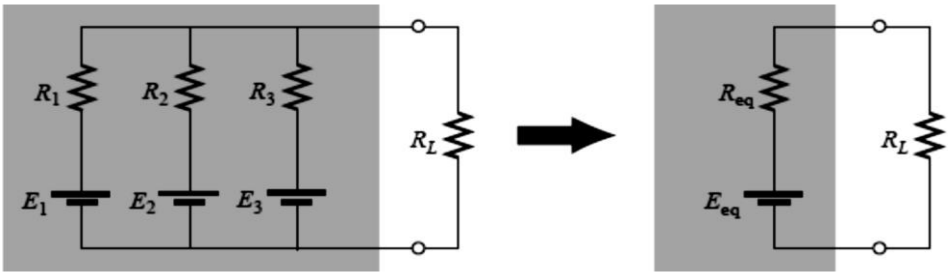


FIG. 4.23 Demonstrating the effect of applying Millman's theorem.

In general, Millman's theorem states that for any number of parallel voltage sources,

$$E_{eq} = \frac{\pm \frac{E_1}{R_1} \pm \frac{E_2}{R_2} \pm \frac{E_3}{R_3} \pm \cdots \pm \frac{E_N}{R_N}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \qquad (4.19)$$

and

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \qquad (4.20)$$

Example 4.7: Using Millman's theorem, find the current through and voltage across the resistor **RL** of **Fig. 4.24**.

Solution: By Eq. (4.19),

$$E_{eq} = \frac{+\frac{E_1}{R_1} - \frac{E_2}{R_2} + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

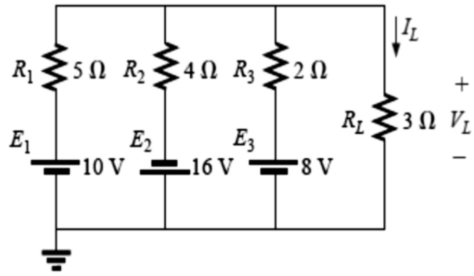


FIG. 4.24 Example 4.7.

The minus sign is used for **E_2 /R_2** because that supply has the opposite polarity of the other two. The chosen reference direction is therefore that of **E_1** and **E_3**. The total conductance is unaffected by the direction,

And

$$E_{eq} = \frac{+\frac{10V}{5\Omega} - \frac{16V}{4\Omega} + \frac{8V}{2\Omega}}{\frac{1}{5\Omega} + \frac{1}{4\Omega} + \frac{1}{2\Omega}} = \frac{2A - 4A + 4A}{0.2\,S + 0.25\,S + 0.5\,S}$$

$$= \frac{2\,A}{0.95\,S} = 2.105\,V$$

with $\boldsymbol{R_{eq} = \frac{1}{\frac{1}{5\Omega}+\frac{1}{4\Omega}+\frac{1}{2\Omega}} = \frac{1}{0.95\,S} = 1.053\,\Omega}$

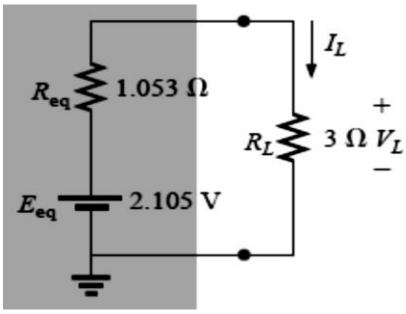


FIG. 4.25 The result of applying Millman's theorem *to* the network of Fig. 4.24.

The resultant source is shown in **Fig. 4.25**, and

$$\boldsymbol{I_L = \frac{2.105V}{1.053\Omega+3\Omega} = \frac{2.105\,V}{4.053\,\Omega} = 0.519\,A}$$

with **$V_L = I_L R_L$ = (0.519 A)(3 Ω) = 1.557 V**

*The dual of Millman's theorem appears in **Fig. 4.26**. It can be shown that Ieq and Req, as in **Fig. 4.26**, are given by

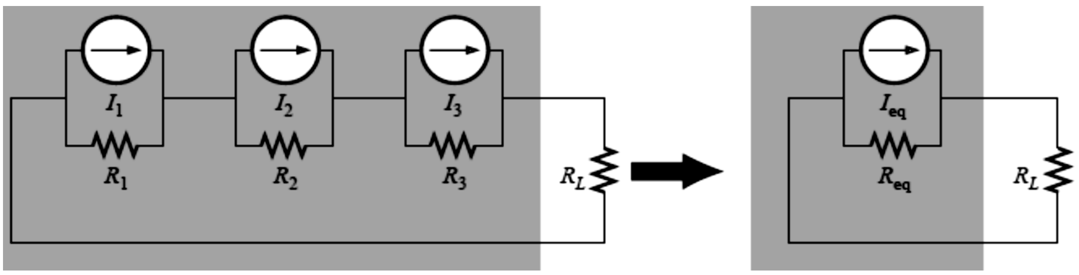


FIG. 4.26 The dual effect of Millman's theorem.

$$I_{eq} = \frac{\pm I_1R_1 \pm I_2R_2 \pm I_3R_3}{R_1 + R_2 + R_3} \qquad (4.21)$$

And $$R_{eq} = R_1 + R_2 + R_3 \qquad (4.22)$$

4.8 SUBSTITUTION THEOREM

The **substitution theorem** states the following:

If the voltage across and the current through any branch of a dc bilateral network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

More simply, the theorem states that for branch equivalence, the terminal voltage and current must be the same. Consider the circuit of **Fig. 9.27**, in which the voltage across and current through the branch a-b are determined. Through the use of the substitution theorem, a number of equivalent a-b branches are shown in **Fig. 4.28**.

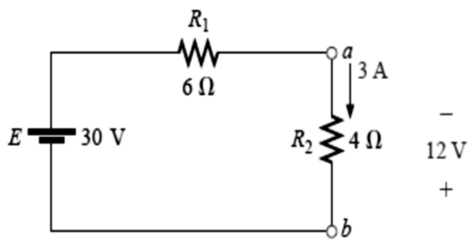


FIG. 4.27 Demonstrating the effect of the substitution theorem.

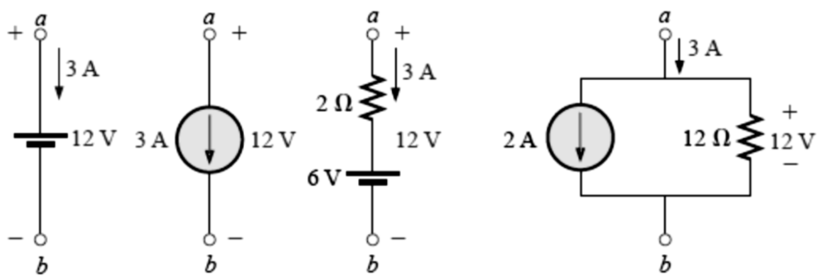


FIG. 4.28 Equivalent branches for the branch a-b of Fig. 4.27.

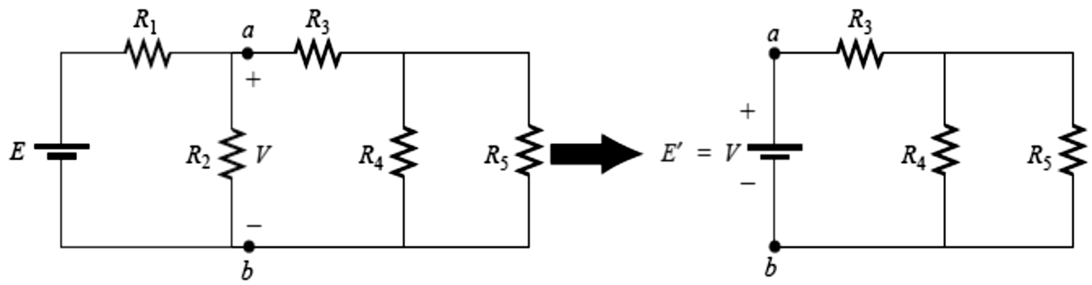


FIG. 4.29 Demonstrating the effect of knowing a voltage at some point in a complex network.

As demonstrated by the single-source equivalents of Fig. 4.28, a known potential difference and current in a network can be replaced by an ideal voltage source and current source, respectively. Understand that this theorem cannot be used to solve networks with two or more sources that are not in series or parallel. You will also recall from the discussion of bridge networks that V = 0 and I = 0 were replaced by a short circuit and an open circuit, respectively. This substitution is a very specific application of the substitution theorem.

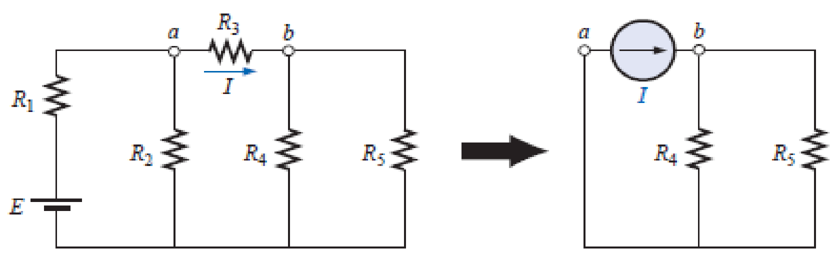


FIG. 4.30 ***Demonstrating the effect of knowing a current at some point in a complex network.***

4.9 RECIPROCITY THEOREM

The reciprocity theorem is applicable only to single-source networks. It is, therefore, not a theorem employed in the analysis of multisource networks described thus far. The theorem states the following:

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

In other words, the location of the voltage source and the resulting current may be interchanged without a change in current. The theorem requires that the polarity of the voltage source have the same correspondence with the direction of the branch current in each position. In the representative network of **Fig. 4.31(a)**, the current I due to the voltage source E was determined. If the position of each is interchanged as shown in **Fig.4.31 (b)**, the current I will be the same value as indicated. To demonstrate the validity of this statement and the theorem, consider the network of **Fig. 4.32**, in which values for the elements of **Fig. 4.31(a)** have been assigned.

The total resistance is

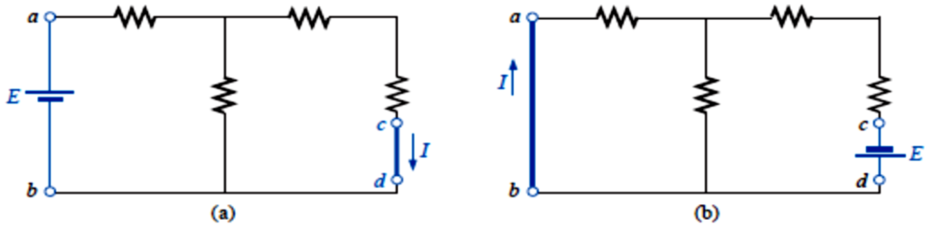


FIG. 4.31 Demonstrating the impact of the reciprocity theorem.

$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12\ \Omega + 6\ \Omega \parallel (2\ \Omega + 4\ \Omega) = 15\ \Omega$

and $I_S = E/R_T = 45/15 = 3$ A

with $I = 3\text{ A}/2 = 1.5$ A

For the network of **Fig. 4.33**, which corresponds to that of **Fig. 4.31(b)**, we find

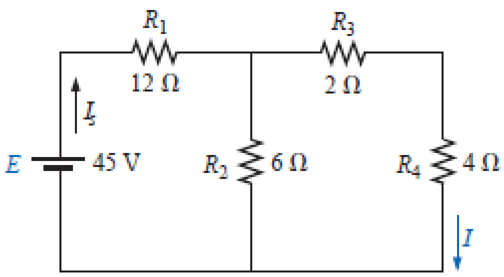


FIG. 4.32 Finding the current I due to a source E.

$$RT = R4 + R3 + (R1 \parallel R2)$$

$$= 4 + 2 + (12 \parallel 60) = 10\ \Omega$$

and $\quad Is = \frac{E}{RT} = \frac{45}{10} = 4.5\ A$

so that $\quad I = \frac{4.5 \times 6}{12+6} = \frac{4.5}{3} = 1.5\ A$

which agrees with the above.

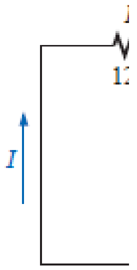


FIG. 4.33 Interchanging the location of E and I of Fig. 4.31 to demonstrate the validity of the reciprocity theorem.

Sinusoidal Alternating Waveforms

8.1 Introduction

The analysis thus far has been limited to dc networks, networks in which the currents or voltages are fixed in magnitude except for transient effects. We will now turn our attention to the analysis of networks in which the magnitude of the source varies in a set manner. Of particular interest is the time-varying voltage that is commercially available in large quantities and is commonly called the ac voltage, (The letters ac are an abbreviation for alternating current.)

Each waveform of Fig. 8.1 is an **alternating waveform** available from commercial supplies. The term alternating indicates only that the waveform alternates between two prescribed levels in a set time sequence (Fig. 8.1).

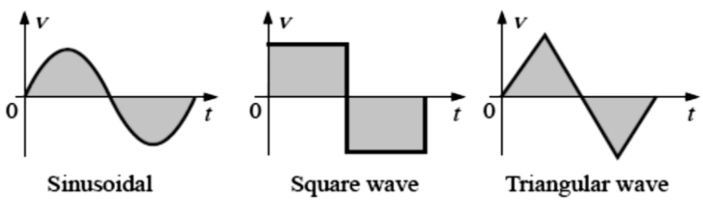


Fig. 8.1 Alternating waveforms.

To beabsolutely correct, the term sinusoidal, square wave, or triangular must also be applied. The pattern of particular interest is the **sinusoidal ac waveform** for voltage of Fig. 8.1. Since this type of signal is encountered in the vast majority of instances, the abbreviated phrases ac voltage and ac current are commonly applied without confusion.

8.2 Sinusoidal ac Voltage characteristics and definitions

Generation Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant; such a power plant is most commonly fueled by water power, oil, gas, or nuclear fusion. In each case an ac generator (also called an alternator).

The power to the shaft developed by one of the energy sources listed will turn a rotor (constructed of alternating magnetic poles) inside a set of windings housed in the stator (the stationary part of the dynamo) and will induce a voltage across the windings of the stator, as defined by Faraday's law,

$$e = N\frac{d\Phi}{dt}$$

Definitions

The sinusoidal waveform of Fig.8.2 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember as you proceed through the various definitions that the vertical scaling is in volts or amperes and the horizontal scaling is always in units of time.

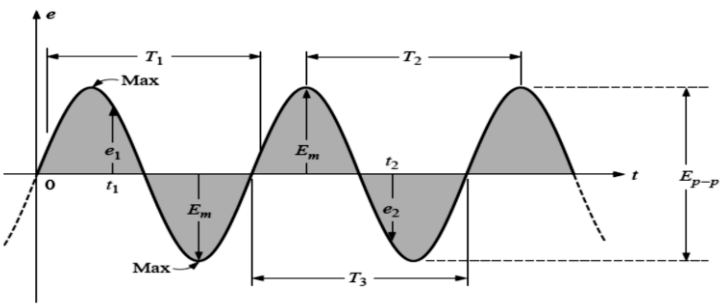


FIG. 8.2 Important parameters for a sinusoidal voltage.

Waveform: The path traced by a quantity, such as the voltage in Fig. 8.2, plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters ($\mathbf{e_1}$, $\mathbf{e_2}$).

Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters (such as $\mathbf{E_m}$ for sources of voltage and $\mathbf{V_m}$ for the voltage drop across a load).

Peak value: The maximum instantaneous value of a function as measured from the zero-volt level. For the waveform of Fig. 8.2, the peak amplitude and peak value are the same, since the average value of the function is zero volts.

Peak-to-peak value: Denoted by $E_{p\text{-}p}$or $V_{p\text{-}p}$, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform of Fig. 8.2 is a periodic waveform.

Period (T): The time interval between successive repetitions of a periodic waveform (the period T1 = T2 = T3 in Fig. 8.2).

Cycle: The portion of a waveform contained in one period of time. The cycles within T 1 , T 2 , and T 3 of Fig. 8.2 may appear different in Fig. 8.3.

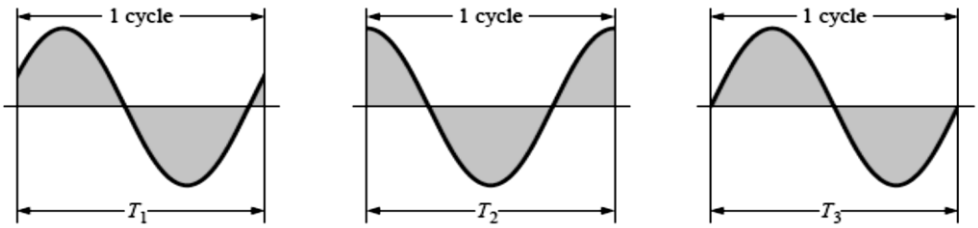


FIG. 8.3 Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform of Fig. 8.4(a) is 1 cycle per second, and for Fig. 8.4(b), 2.5 cycles per second, and while for Fig. 8.4(c) the frequency will be 2 cycles per second.

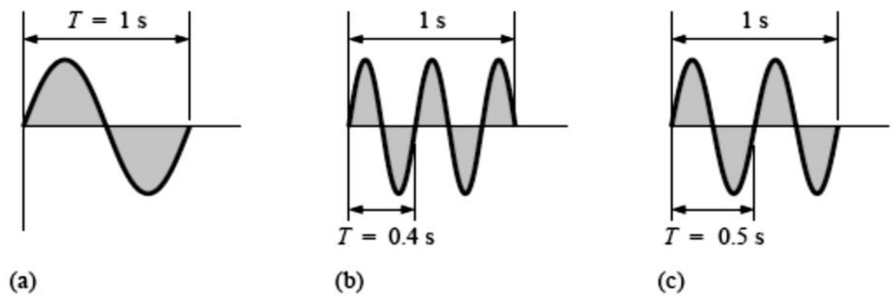


FIG. 8.4 Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

The unit of measure for frequency is the hertz (Hz), where

1 hertz (Hz) = 1 cycle per second (c/s)(8.1)

The unit hertz is derived from the surname of Heinrich Rudolph Hertz, who did original research in the area of alternating currents and voltages and their effect on the basic R, L, and C elements. Frequency spectrum from 1 Hz to 1000 GHz can be scaled off on the same axis, as shown in Fig. 8.5.

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$\mathbf{f} = \frac{\mathbf{1}}{\mathbf{T}}$(8.2)

where f = Hz ,T = seconds (s)

or$\mathbf{T} = \frac{\mathbf{1}}{\mathbf{f}}$(8.3)

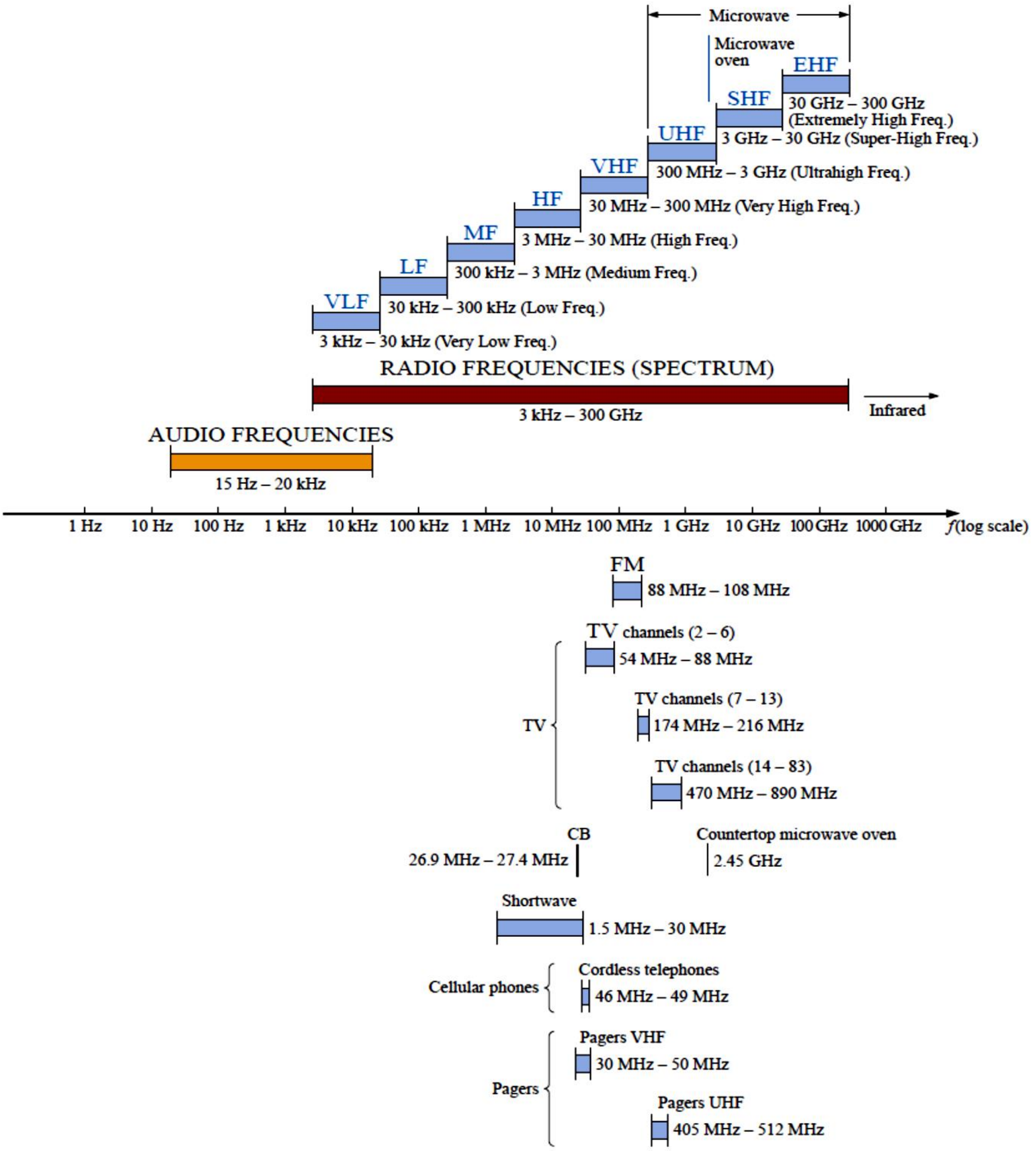


FIG. 8.5 Areas of application for specific frequency bands.

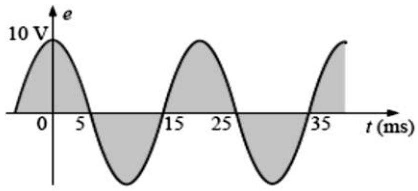
Example8.1: Find the period of a periodic waveform with a frequency of

a. 60 Hz. b. 1000 Hz.

Solutions: a. $\mathbf{T} = \frac{1}{f} = \frac{1}{6Hz} = 0.01667s\ or\ 16.67\ ms$

b. $\mathbf{T} = \frac{1}{f} = \frac{1}{1000Hz} = 10^{-3}s = 1\ ms$

Example8.2: Determine the frequency of the waveform of Figure below.



Solution: From the figure, T = (25 ms - 5 ms) = 20 ms, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}s} = 50\ Hz$$

Defined Polarities and Direction

In the following analysis, we will find it necessary to establish a set of polarities for the sinusoidal ac voltage and a direction for the sinusoidal ac current. In each case, the polarity and current direction will be for an instant of time in the positive portion of the sinusoidal waveform. This is shown in Fig. 8.6 with the symbols for the sinusoidal ac voltage and current.

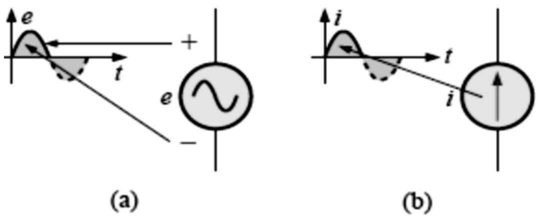


FIG. 8.6 (a) Sinusoidal ac voltage sources, (b) sinusoidal current sources.

8.3 THE SINE WAVE

The terms defined in the previous section can be applied to any type of periodic waveform, whether smooth or discontinuous. The sinusoidal waveform is of particular importance, however, since it lends itself readily to the mathematics and the physical phenomena associated with electric circuits. Consider the power of the following statement: ***The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.***

In other words, if the voltage across (or current through) a resistor, coil, or capacitor is sinusoidal in nature, the resulting current (or voltage, respectively) for each will also have sinusoidal characteristics, as shown in Fig. 8.7. If a square wave or a triangular wave were applied, such would not be the case.

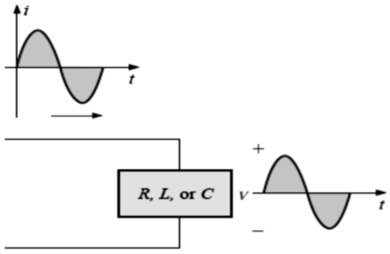


FIG. 8.7

The unit of measurement for the angle axis of is the degree. A second unit of measurement frequently used is the radian (rad). It is defined by a quadrant of a circle where the distance subtended on the circumference equals the radius of the circle.

2π rad = 360° (8.4)

With 1 rad = 57.296° ≅ 57.3° (8.5)

The quantity π is the ratio of the circumference of a circle to its diameter.

For 180° and 360°, the two units of measurement are related. The conversion equations between the two are the following:

Radians = $\left(\frac{\pi}{180^\circ}\right) \times$ (degrees)(8.6)

Degrees = $\left(\frac{180^\circ}{\pi}\right) \times$ (radians)(8.7)

The velocity with which the radius vector rotates about the center, called the angular velocity, can be determined from the following equation:

Angular velocity = $\frac{distance\ (degrees\ or\ radians)}{time\ (seconds)}$(8.8)

Substituting into Eq. (8.8) and assigning the Greek letter omega (ω) to the angular velocity, we have

$\omega = \alpha/t \quad \Rightarrow \alpha = \omega t$ (8.9)

Since q is typically provided in radians per second, the angle α obtained using Eq. (8.9) is usually in radians.

The angular velocity of the rotating radius vector is

$\boldsymbol{\omega = 2\pi f}$(8.10)

<u>**Example8.3:**</u> Determine the angular velocity of a sine wave having a frequency of 60 Hz.

<u>**Solution:**</u> $\omega = 2\pi f = (2\pi)(60\ Hz) = 377$ rad/s

Example8.4: Given ω= 200 rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

Solution: Eq. (8.9): α =ω t, and t = α /ω However, a must be substituted as π/2 (= 90°) since qis in radians per second: t = α/ ω = $\frac{\pi/2}{200}$= **7.85 ms**

Example8.5: Find the angle through which a sinusoidal waveform of 60 Hz will pass in a period of 5 ms.

Solution: Eq. (8.9): α=ω t, or α= 2π ft = (2π)(60 Hz)(5 × 10^{-3} s) = 1.885 rad = 108°

8.4 GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is.

$\mathbf{A_m \sin \alpha}$(8.11)

where $\mathbf{A_m}$ is the peak value of the waveform and **α** is the unit of measure for the horizontal axis, as shown in Fig. 8.8

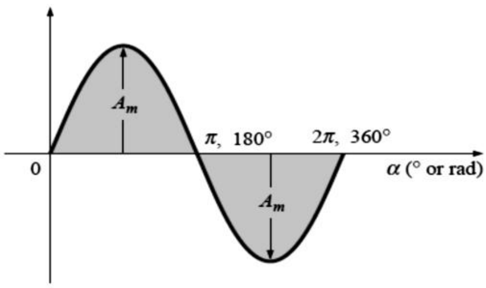


FIG. 8.8 Basic sinusoidal function.

Due to Eq. (8.9), the general format of a sine wave can also be written

$\mathbf{A_m \sin \omega}$t(8.12)

For electrical quantities such as current and voltage, the general format is

$\mathbf{i = I_m \sin \omega t = I_m \sin \alpha}$

$\mathbf{e = E_m \sin \omega t = E_m \sin \alpha}$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters i and e represent the instantaneous value of current or voltage, respectively, at any time t.

Example8.6: Given e = 5 sin α, determine e at α= 40° and α= 0.8π.

Solution: For α = 40°

e = 5 sin 40° = 5(0.6428) = 3.214 V

For α= 0.8π,

a (°) =(180/π)(0.8π) = 144°

and e = 5 sin 144° = 5(0.5878) = 2.939 V

8.5 PHASE RELATIONS

Thus far, we have considered only sine waves that have maxima at π/2 and 3π/2, with a zero value at 0, π, and 2π.Ifthe waveform is shifted to the right or left of 0°,the expression becomes

A_m sin(ωt ±θ)(8.13)

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive- going (increasing with time) slope before 0°, as shown in Fig. 8.9, the expression is

A_m sin(ωt + θ)(8.14)

At ωt =α= 0°, the magnitude is determined by A_m sin θ.

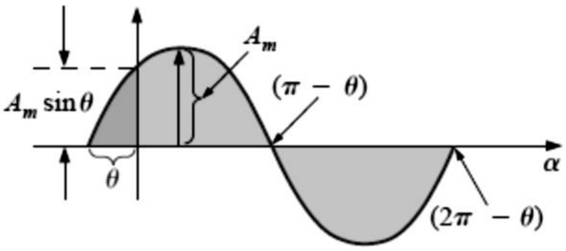


FIG. 8.9 Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

If the wave- form passes through the horizontal axis with a positive-going slope after 0°, as shown in Fig. 8.10, the expression is

A_m sin(ωt - θ)(8.15)

at ωt =α= 0°, the magnitude is $A_m$sin(-θ), which, by a trigonometric identity, is -A_m sin θ.

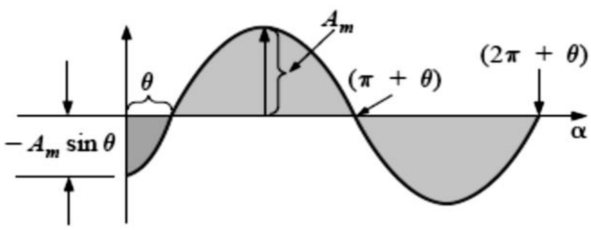


FIG. 8.10 Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.

The geometric relationship between various forms of the sine andcosine functions can be listed below:

cos α= sin(α+ 90°), sin α= cos(α- 90°)

-sin α= sin(α± 180°), -cos α= sin(α± 270°) = sin(α- 90°)etc.

sin(-α) = -sin α, cos(-α) = cos α

<u>Example8.7:</u> What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. v = 10 sin(ωt + 30°)d. i =-sin(ωt + 30°)

i = 5 sin(ωt + 70°)v = 2 sin(ωt + 10°)

b. i = 15 sin(ωt + 60°)e. i =-2 cos(ωt - 60°)

v = 10 sin(ωt - 20°)v = 3 sin(ωt - 150°)

c. i = 2 cos(ωt + 10°)

v = 3 sin(ωt - 10°)

<u>Solutions:</u>

a. See Fig. 8.11.

i leads v by 40°, or v lags i by 40°.

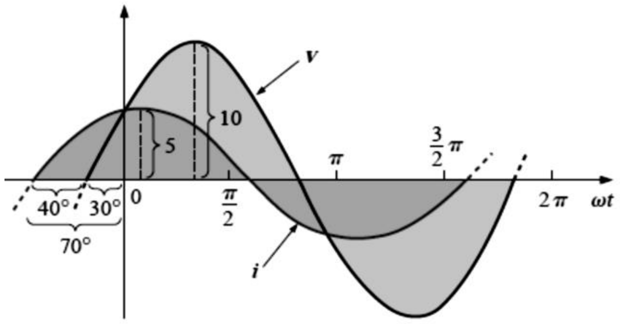


FIG. 8.11 Example 8.7; i leads v by 40°.

b. See Fig. 8.12.

i leads v by 80°, or v lags i by 80°.

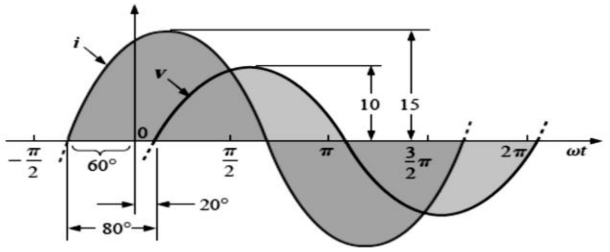


FIG. 8.12 Example 8.7; i leads v by 80°.

c. See Fig. 8.13.

i = 2 cos(ωt + 10°) = 2 sin(ωt + 10° + 90°)

= 2 sin(ωt + 100°)

i leads v by 110°, or v lags i by 110°.

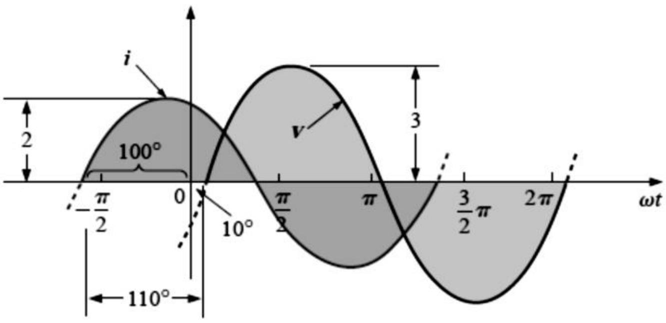


FIG. 8.13 Example 8.7; i leads v by 110°.

d. See Fig. 8.14.

Note

-sin(ωt + 30°) = sin(ωt + 30° - 180°)

= sin(ωt - 150°)

v leads i by 160°, or i lags v by 160°.

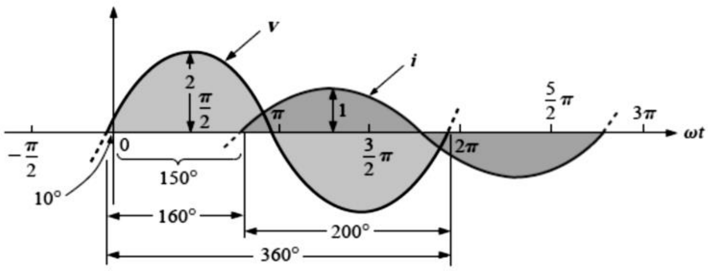


FIG. 8.14 Example 8.7; v leads i by 160°.

e. See Fig. 8.15.

i = -2 cos(ωt - 60°) = 2 cos(ωt - 60° - 180°)

= 2 cos(ωt - 240°)

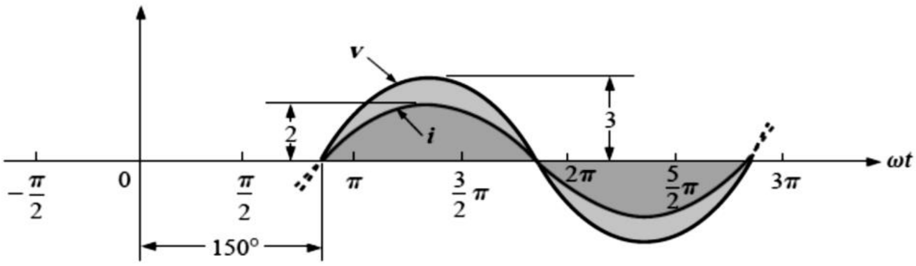


FIG. 8.15 Example 8.7; v and i are in phase.

8.7. EFFECTIVE ROOT-MEAN-SQUARE (R.M.S.) VALUE

The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time. It is also known as the effective or virtual value of the alternating current, the former term being used more extensively. For computing the r.m.s. value of symmetrical sinusoidal alternating currents, either mid-ordinate method or analytical method may be used, although for symmetrical but non- sinusoidal waves, the mid- ordinate method would be found more convenient.

8.7.1. Mid-ordinate Method

In Fig. 8.16 are shown the positive half cycles for both symmetrical sinusoidal and non-sinusoidal alternating currents. Divide time base't' into **n** equal intervals of time each of duration t/n seconds. Let the average values of instantaneous currents during these intervals be respectively $i_1, i_2, i_3, \ldots i_n$ (i.e. mid-ordinates in Fig. 8.16).

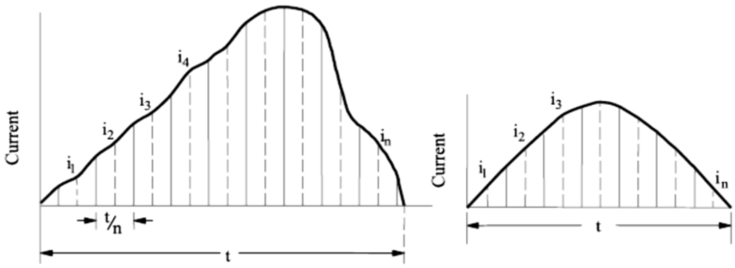


Fig. 8.16

The r.m.s. value of alternating current is

$$I^2 = \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$

$I = \sqrt{\frac{i_1^2+i_2^2+\cdots+i_n^2}{n}}$ = square root of the mean of the squares of the instantaneous currents

Similarly, the r.m.s. value of alternating voltage is given by the expression

$$V = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_n^2}{n}}$$

8.7.2 Analytical Method

The general form of r.m.s. value is

$$I^2 = \frac{1}{T}\int_0^T i^2 dt \quad \Longrightarrow \quad I = \sqrt{\frac{\int_0^T i^2 dt}{T}} \quad or\ I = \sqrt{\frac{\int_0^{2\pi} i^2 d\theta}{(2\pi - 0)}}$$

The standard form of a sinusoidal alternating current is $i = I_m \sin \omega t = I_m \sin \theta$.The mean of the squares of the instantaneous values of current over one complete cycle is (eventhe value over half a cycle will do).

$I = \sqrt{\frac{\int_0^{2\pi} i^2 d\theta}{(2\pi-0)}} = \sqrt{\frac{I_m^2}{2}}$

The square root of this value is

$$I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Hence, we find that for a symmetrical sinusoidal current

r.m.s. value of current = 0.707 × max. value of current

The r.m.s. value of an alternating current is of considerable importance in practice, because theammeters and voltmeters record the r.m.s. value of alternating current and voltage respectively. Inelectrical engineering work, *unless indicated otherwise, the values of the given current and voltageare always the r.m.s. values.*

It should be noted that the average heating effect produced during one cycle is

$$= I^2R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_m^2 R$$

8.8 AVERAGE VALUE

The average value $\mathbf{I_a}$ of an alternating current is expressed *by that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time*. In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

(i) Mid-ordinate Method

With reference to Fig. 8.16, $\boldsymbol{I_{av} = \frac{i_1 + i_2 + \cdots + i_n}{n}}$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(ii) Analytical Method

The general form of average value is

$$\boldsymbol{I_{av} = \frac{1}{T}\int_0^T i dt, \qquad or\ I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\, d\theta}$$

The standard equation of an alternating current is, $i = I_m \sin\theta$

$$\boldsymbol{I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m sin\theta\, d\theta}$$

∴ $\boldsymbol{I_{av} = 0.637\, I_m}$

∴ average value of current = 0.637 × maximum value

Note. R.M.S. value is always greater than average value except in the case of a rectangular wave whenboth are equal.

8.9. Form Factor

It is defined as the ratio, $K_f = \frac{r.m.s.value}{\text{average value}} = \frac{0.707\ I_m}{0.637\ I_m} = 1.11$ (for sinusoidal alternating currents only)

In the case of sinusoidal alternating voltage also, $K_f = 0.707\ E_m/0.637\ E_m = 1.11$

As is clear, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and vice-versa.

8.10. Crest or Peak or Amplitude Factor

It is defined as the ratio $K_a = \frac{\textbf{maximum value}}{\textbf{r.m.s.value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = \mathbf{1.414}$(for sinusoidal a.c. only)

For sinusoidal alternating voltage also, $K_a = \frac{E_m}{E_m/\sqrt{2}} = \mathbf{1.414}$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

Example 8.8: Compute the average and effective values of the square voltage wave shown inFig. 8.17.

Solution: As seen, for 0 < t < 0.1 i.e. for the time interval0 to 0.1 second, v = 20 V. Similarly, for 0.1 < t < 0.3,v = 0. Also time-period of the voltage wave is 0.3 second.

$$\therefore \boldsymbol{V_{av}} = \frac{1}{T}\int_0^T \boldsymbol{v dt} = \frac{1}{0.3}\int_0^{0.1} \mathbf{20}\boldsymbol{dt}$$

$$= \frac{1}{0.3}(20 \times 0.1) = \mathbf{6.6667}\ \boldsymbol{V}$$

$$V = \sqrt{\frac{\int_0^T v^2 dt}{T}} = \sqrt{\frac{\int_0^{0.1} 20^2 dt}{0.3}} = \sqrt{\frac{400 \times 0.1}{0.3}} = 11.5\ V$$

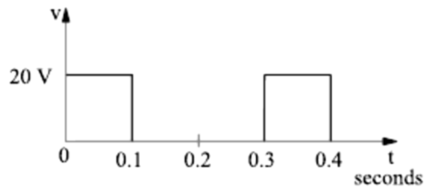
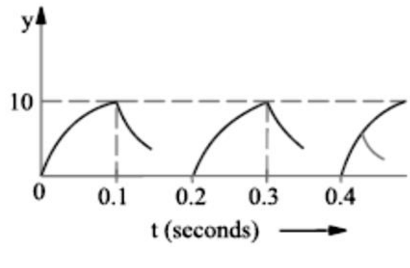


Fig. 8.17

Example 8.9: Calculate the RMS value of the function shownin Fig. 8.18 if it is given that for 0 < t < 0.1, $y = 10(1 - e^{-100t})$ and0.1 < t < 0.2, $y = 10\ e^{-50(t-0.1)}$

Solution:

$$Y^2 = \frac{1}{2}\left\{\int_0^{0.1} y^2 dt + \int_{0.1}^{0.2} y^2 dt\right\}$$



$$= \frac{1}{2}\left\{\int_0^{0.1} 10^2(1 - e^{-100t})^2 dt + \int_{0.1}^{0.2} 10^2\left(e^{-50(t-0.1)}\right)^2 dt\right\}$$

= 500 × 0.095 = 47.5 ∴ Y = $\sqrt{47.5}$= 6.9

Fig. 8.18

Example 8.10: Determinethe r.m.s. and average value of the waveform shown in Fig. 8.19?

Solution: The slope of the curve AB is BC/AC = 10/T.

Next, consider the function y at any time t. It is seen that

y = 10 + (10/T)t

This gives us the equation for the function for one cycle.

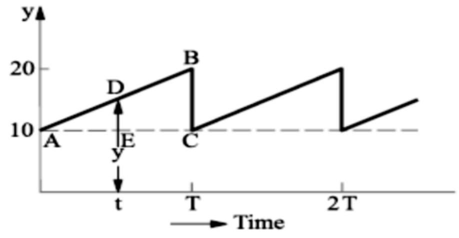


Fig. 8.18

$$Y_{av} = \frac{1}{T}\int_0^T y dt = \frac{1}{T}\int_0^T (10 + \frac{10t}{T}) dt = 15$$

Mean square value = $\frac{1}{T}\int_0^T y^2 dt = \frac{1}{2}\int_0^T \left(10 + \frac{10}{T}t\right)^2 dt = \frac{700}{3}$

or RMS value = 10 $\sqrt{7/3}$ = 15.2

CHAPTER 12

RESONANCE

12.1 INTRODUCTION

This chapter will introduce the very important resonant(or tuned)circuit, which is fundamental to the operation of a wide variety of electrical and electronic systems in use today. The resonant circuit is a combination of R, L, and C elements having a frequency response characteristic similar to the one appearing in Fig. 12.1. Note in the figure that the response is a maximum for the frequency f_r, decreasing to the right and left of this frequency. In other words, for a particular range of frequencies the response will be near or equal to the maximum. The frequencies to the far left or right have very low voltage or current levels and, for all practical purposes, have little effect on the system's response. The radio or television receiver has a response curve for each broadcast station of the type indicated in Fig. 12.1.

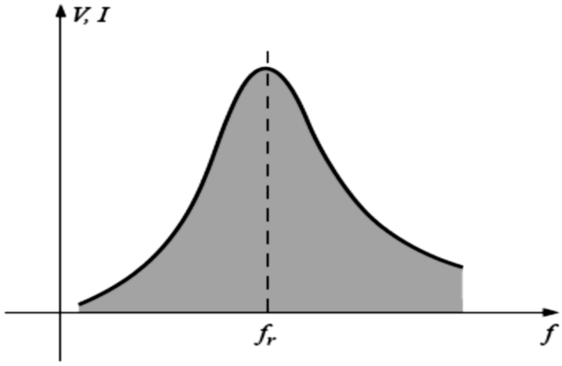


FIG. 12.1Resonance curve.

12.2 SERIES RESONANT CIRCUIT

A resonant circuit (series or parallel) must have an inductive and a capacitive element. A resistive element will always be present due to the internal resistance of the source (**R_s**), the internal resistance of the inductor (**R_l**), and any added resistance to control the shape of the response curve (**R_{design}**). The basic configuration for the series resonant circuit appears in Fig. 12.2(a) with the resistive elements listed above. The "cleaner" appearance of Fig. 12.2(b) is a result of combining the series resistive elements into one total value. That is,

$$R = Rs + R_l + Rd \quad (12.1)$$

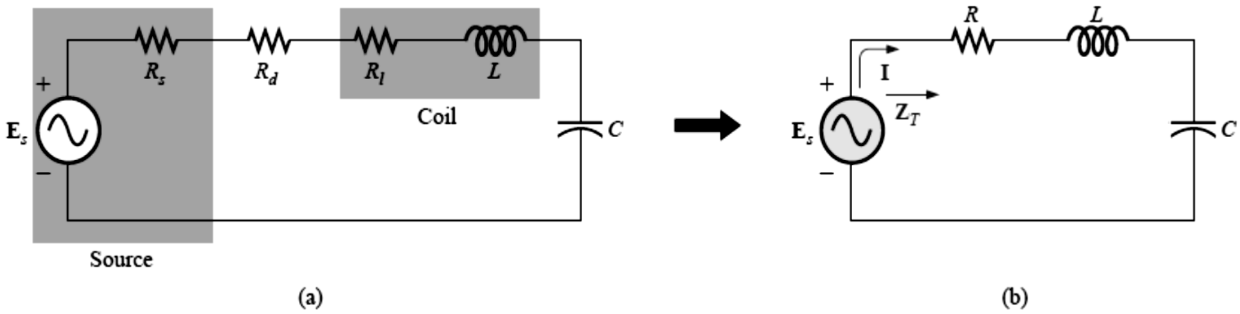


FIG. 12.2 Series resonant circuit.

The total impedance of this network at any frequency is determined by

$$Z_T = R + j X_L - j X_C = R + j (X_L - X_C)$$

The resonant conditions described in the introduction will occur when

$$X_L = X_C \quad (12.2)$$

removing the reactive component from the total impedance equation. The total impedance at resonance is then simply

$$Z_{Ts} = R$$

(12.3)

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance [Eq. (12.2)]:

$$\omega_s = \frac{1}{\sqrt{LC}}$$

(12.4)

or $$f_s = \frac{1}{2\pi\sqrt{LC}}$$

(12.5) ☐ L = henries (H), C = farads (F), f = hertz (Hz)

The current through the circuit at resonance is

$$I = \frac{E\angle 0^\circ}{R\angle 0^\circ} = \frac{E}{R}\angle 0^\circ$$

which you will note is the maximum current for the circuit of Fig. 12.2 for an applied voltage E since Z_T is a minimum value.

The average power to the resistor at resonance is equal to $\mathbf{I^2 R}$, and the reactive power to the capacitor and inductor are $\mathbf{I^2 X_C}$ and $\mathbf{I^2 X_L}$, respectively.

The total apparent power is equal to the average power dissipated by the resistor since $Q_L = Q_C$. The power factor of the circuit at resonance is

$$pF = \cos\theta = \frac{P}{S} = 1$$

(12.6) Plotting the power curves of each element on the same set of axes (Fig. 12.3), we note that, even though the total reactive power at any instant is equal to zero.

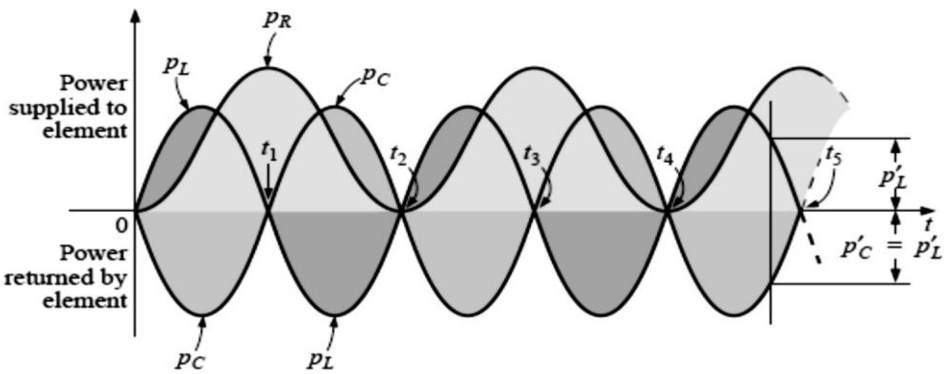


FIG. 12.3 Power curves at resonance for the series resonant circuit.

12.3 THE QUALITY FACTOR (Q)

The quality factor Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance. The quality factor is also an indication of how much energy is placed in storage (continual transfer from one reactive element to the other) compared to that dissipated.

$$Qs = \frac{XL}{R} = \frac{\omega sL}{R} = \frac{1}{\omega_s CR}$$

(12.7)

Also $$Qs = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(12.8)

By applying the voltage divider rule to the circuit of Fig. 12.2, we obtain

$$V_{Ls} = Qs\ E$$

$$V_{Cs} = Qs\ E$$

Since Qs is usually greater than 1, the voltage across the capacitor or inductor of a series resonant circuit can be significantly greater than the input voltage.

12.4 Z_T VERSUS FREQUENCY

The total impedance of the series R-L-C circuit of Fig. 12.2 at any frequency is determined by

$$Z_T = R + j\,X_L - j\,X_C \quad \text{or} \quad Z_T = R + j\,(X_L - X_C)$$

The magnitude of the impedance Z T versus frequency is determined by

$$Z_T = \sqrt{[R]^2 + [X_L - X_C]^2}$$

The total-impedance-versus-frequency curve for the series resonant circuit of Fig. 12.2 can be found by applying the impedance-versus- frequency curve for each element of the equation just derived, written in the following form:

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2} \quad or \quad Z_T(f) = \sqrt{[R(f)]^2 + [X(f)]^2}$$

(12.9)

where Z_T (*f*) "means" the total impedance as a function of frequency. For the frequency range of interest, we will assume that the resistance R does not change with frequency. The curve for the inductance, as determined by the reactance equation, is a straight line intersecting the origin with a slope equal to the inductance of the coil. Thus, for the coil,

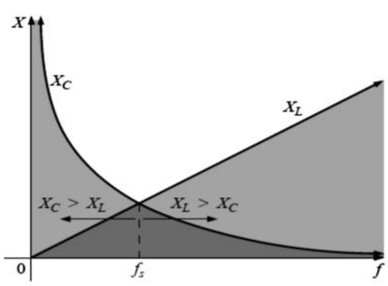


FIG. 12.4 Placing the frequency response of the inductive and capacitive reactance of a series R-L-C circuit on the same set of axes.

$$X_L = 2\pi L.f + 0$$

y = a . x + b

(where 2πL is the slope), producing the X_L results is straight line shown in Fig. 20.4.

For the capacitor,

$$X_C = \frac{1}{2\pi f C} \quad \text{or} \quad X_C f = \frac{1}{2\pi C}$$

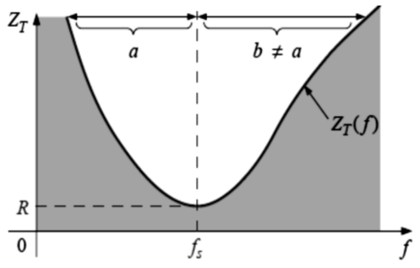


FIG. 12.5 Z_T versus frequency for the series resonant circuit.

which becomes y.x = k, the equation for a hyperbola, where

y (variable) = X_C, x (variable) = *f*, k (constant) = $\frac{1}{2\pi C}$

The hyperbolic curve for $X_C(f)$ is plotted in Fig. 12.4. In particular, note its very large magnitude at low frequencies and its rapid drop- off as the frequency increases.

The condition of resonance is now clearly defined by the point of intersection, where $X_L = X_C$. For frequencies less than fs , it is also quite clear that the network is primarily capacitive ($X_C > X_L$). For frequencies above the resonant condition, $X_L > X_C$, and the network is inductive.

Applying eq. (12.9) to the curves of Fig. 12.4, we obtain the curve for Z_T (*f*) as shown in Fig. 12.5. The minimum impedance occurs at the resonant frequency and is equal to the resistance R. Note that the curve is not symmetrical about the resonant frequency (especially at higher values of Z_T).

The phase angle associated with the total impedance is

$$\theta = \tan^{-1}\frac{(X_L - X_C)}{R}$$

(12.10)

At low frequencies, $X_C > X_L$, and v will approach −90° (capacitive), as shown in Fig. 12.6, whereas at high frequencies, $X_L > X_C$, and v will approach 90°In general, therefore, for a series resonant circuit:

f* < *fs: network capacitive; **I** leads **E**

f* > *fs: network inductive; **E** leads **I**

f* = *fs: network resistive; **E** and **I** are in phase.

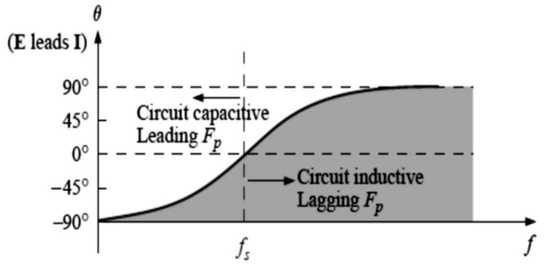


FIG. 12.6 Phase plot for the series resonant circuit.

12.5 SELECTIVITY

If we now plot the magnitude of the current $I = E/Z_T$ versus frequency for a fixed applied voltage E, we obtain the curve shown in Fig. 12.7, which rises from zero to a maximum value of E/R (where Z_T is a minimum) and then drops toward zero (as Z_T increases) at a slower rate than it rose to its peak value. The curve is actually the inverse of the impedance-versus-frequency curve. Since the Z_T curve is not absolutely symmetrical about the resonant frequency, the curve of the current versus frequency has the same property.

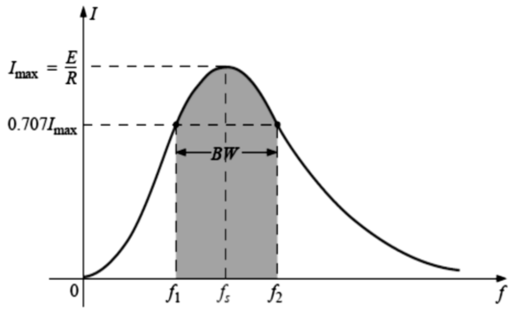
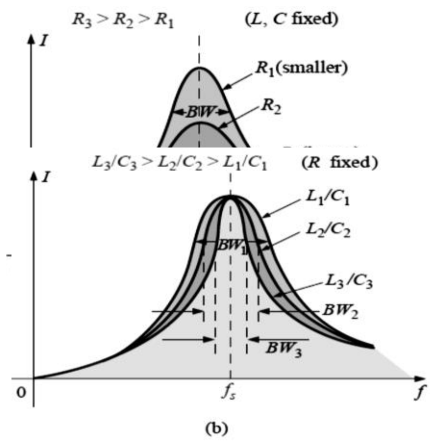


FIG. 12.7 I versus frequency for the series resonant circuit.

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of the maximum current are called the **band frequencies**, **cutoff frequencies**, or **half-power frequencies**. They are indicated by f_1 and f_2 in Fig. 12.7. The range of frequencies between the two is referred to as the bandwidth (abbreviated **BW**) of the resonant circuit. Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency; that is,

$$P_{HPF} = \frac{1}{2} P_{max} \tag{12.11}$$

Since the resonant circuit is adjusted to select a band of frequencies, the curve of Fig. 12.7 is called the **selectivity curve**. The term is derived from the fact that one must be selective in choosing the frequency to ensure that it is in the bandwidth. The smaller the bandwidth, the higher the selectivity. The shape of the curve, as shown in Fig. 12.8, depends on each element of the series R-L-C circuit.



Substituting $\sqrt{2}R$ into the equation for the magnitude of Z_T, we find that

$$Z_T = \sqrt{[R]^2 + [X_L - X_C]^2}$$

becomes $$\sqrt{2}R = \sqrt{[R]^2 + [X_L - X_C]^2}$$

or, squaring both sides, that

$$R^2 = (X_L - X_C)^2 \rightarrow R = X_L - X_C$$

Let us first consider the case where $X_L > X_C$, which relates to f_2 or ω_2. Substituting $\omega_2 L$ for X_L and $1/\omega_2 C$ for X_C.

FIG. 12.8
Effect of R, L, and C on the selectivity curve for the series resonant circuit.

can be written

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

Solving the quadratic, we have

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

(12.12) If we repeat the same procedure for $X_C > X_L$, which relates to ω_1 or f_1, the solution f_1 becomes

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

(12.13) The bandwidth (BW) is

$BW = f_2 - f_1 =$ Eq. (12.12) − Eq. (12.13)

and $$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

(12.14)

$$BW = \frac{f_s}{Q_s}$$

(12.15) The ratio BW/f_s is sometimes called the fractional bandwidth, providing an indication of the width of the bandwidth compared to the resonant frequency.

$$f_s = \sqrt{f_2 f_1}$$

(12.16)

12.6 V_R, V_L, AND V_C

Plotting the magnitude (effective value) of the voltages $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ and the current I versus frequency for the series resonant circuit on the same set of axes, we obtain the curves shown in Fig. 12.9. Note that the V_R curve has the same shape as the I curve and a peak value equal to the magnitude of the input voltage E. If Q <10 the capacitor max voltage at $f_{Cmax} < f_s$, while the inductor max voltage at $f_{Lmax} > f_s$.

The higher the Qs of the circuit, the closer f_{Cmax} will be to f_s, and the closer $V_{Cmax} \cong Q_sE$, and the closer f_{Lmax} will be to f_s, and the closer $V_{Lmax} \cong Q_sE$,

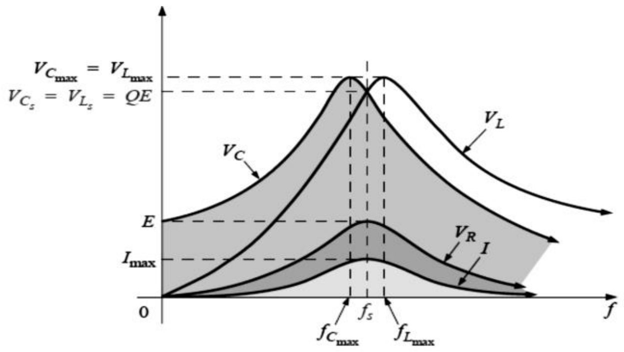


FIG. 12.9 VR, VL, VC, and I versus frequency for a series resonant circuit.

For the condition $Q_s \geq 10$, the curves of Fig. 12.9 will appear as shown in Fig. 12.10. Note that they each peak (on an approximate basis) at the resonant frequency and have a similar shape.

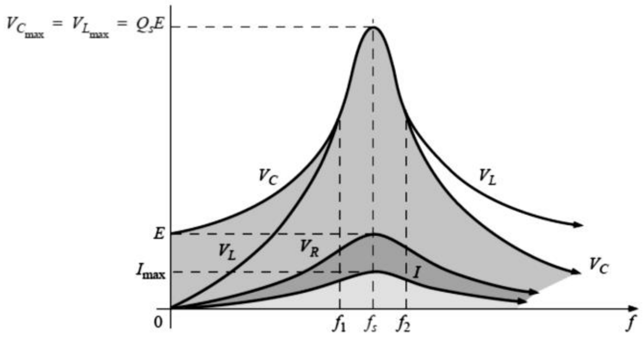


FIG. 12.10 V_R, V_L, V_C, and I for a series resonant circuit where $Q_s \geq 10$.

In review,

1. V_C and V_L are at their maximum values at or near resonance (depending on Qs).
2. At very low frequencies, V_C is very close to the source voltage and V_L is very close to zero volts, whereas at very high frequencies, V_L approaches the source voltage and V_C approaches zero volts.
3. Both V_R and I peak at the resonant frequency and have the same shape.

12.7 EXAMPLES (SERIES RESONANCE)

Example 12.1:

a. For the series resonant circuit of Fig. 12.11, find I, V_R, V_L, and V_C at resonance.

b. What is the Qs of the circuit?

c. If the resonant frequency is 5000 Hz, find the bandwidth.

d. What is the power dissipated in the circuit at the half-power frequencies?

Solutions:

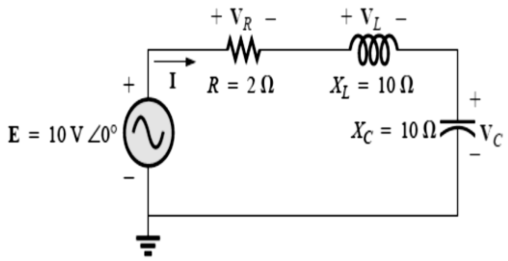


FIG. 12.11 *Example 12.1.*

a. $Z_{Ts} = R = 2\,\Omega$

$I = \frac{E}{Z_{Ts}} = 5\,A\,\angle 0°$

$V_R = E = 10\text{ V } \angle 0°$

$V_L = (I \angle 0°)(X_L \angle 90°) = (5 \angle 0°)(10 \angle 90°) = 50\text{ V } \angle 90°$

$V_C = (I \angle 0°)(X_C \angle -90°) = (5 \angle 0°)(10 \angle -90°) = 50\text{ V } \angle -90°$

b. $Q_s = \frac{X_L}{R} = \frac{10\Omega}{2\Omega} = 5$

c. $BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{5000Hz}{5} = 1000\text{ Hz}$

d. $P_{HPF} = \frac{1}{2} P_{max} = \frac{1}{2} I^2_{max} R = \frac{1}{2}(5\text{ A})^2(2\,\Omega) = 25\text{ W}$

Example 12.2: The bandwidth of a series resonant circuit is 400 Hz.

a. If the resonant frequency is 4000 Hz, what is the value of Qs?

b. If R = 10 Ω, what is the value of X_L at resonance?

c. Find the inductance L and capacitance C of the circuit.

Solutions:

a. $BW = \frac{f_s}{Q_s}$ or $Qs = \frac{f_s}{BW} = \frac{4000\ Hz}{400\ Hz} = 10$

b. $Qs = \frac{X_L}{R}$ or $XL = QsR = (10)(10\,\Omega) = 100\,\Omega$

c. $X_L = 2\pi f_s L$ or $L = \frac{X_L}{2\pi f_s} = \frac{100\,\Omega}{2\pi(4000\ Hz)} = 3.98\ mH$

$X_C = \frac{1}{2\pi f_s C}$ or $C = \frac{1}{2\pi f_s X_C} = 0.398\ \mu F$

Example 12.3: A series R-L-C circuit has a series resonant frequency of 12,000 Hz.

a. If R = 5 Ω, and if X_L at resonance is 300 Ω, find the bandwidth.

b. Find the cutoff frequencies.

Solutions:

a. $Qs = \frac{X_L}{R} = \frac{300}{5} = 60$

$BW = \frac{f_s}{Q_s} = \frac{12000\ Hz}{60} = 200\ Hz$

b. Since Qs ≥ 10, the bandwidth is bisected by *f*s. Therefore,

$f_2 = f_s + \frac{BW}{2} = 12{,}000\text{ Hz} + 100\text{ Hz} = 12{,}100\text{ Hz}$

and $f_1 = f_s - \frac{BW}{2}$ = 12,000 Hz − 100 Hz = 11,900 Hz

Example 12.4:

a. Determine the Qs and bandwidth for the response curve of Fig. 12.12.

b. For C = 101.5 nF, determine L and R for the series resonant circuit.

c. Determine the applied voltage.

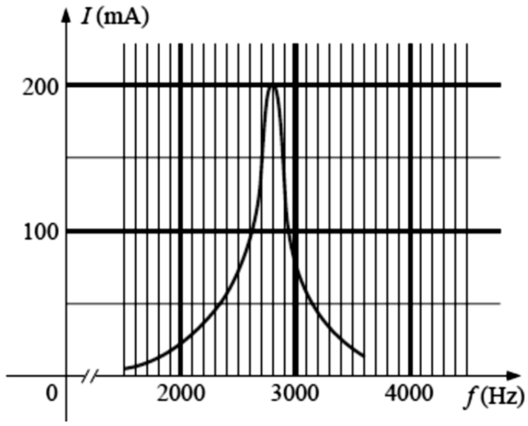


FIG. 12.12 ***Example 12.4.***

Solutions:

a. The resonant frequency is 2800 Hz. At 0.707 times the peak value,

$$BW = 200 \text{ Hz}$$

and $$Qs = \frac{f_s}{BW} = \frac{2800}{200} = 14$$

b. $$\boldsymbol{f_s = \frac{1}{2\pi\sqrt{LC}} \quad or \quad L = \frac{1}{4\pi^2 f_s^2 C}}$$

$$= \frac{1}{4\pi^2(2.8\times10^3\ Hz)^2(101.5\times10^{-9}\ F)} = 31.832 \text{ mH}$$

$$Qs = \frac{X_L}{R} \quad or \ R = \frac{X_L}{Q_S} = \frac{2\pi(2800\ Hz)(31.832\times10^{-3}\ H)}{14} = 40\ \Omega$$

c. $I_{max} = E/R$ or $E = I_{max}R = (200 \text{ mA})(40\ \Omega) = 8 \text{ V}$

Example 12.5: A series R-L-C circuit is designed to resonant at $\omega_s = 10^5$ rad/s, have a bandwidth of $0.15\omega_s$, and draw 16 W from a 120-V source at resonance.

a. Determine the value of R.

b. Find the bandwidth in hertz.

c. Find the nameplate values of L and C.

d. Determine the Qs of the circuit.

e. Determine the fractional bandwidth.

Solutions:

a. $$P = \frac{E^2}{R} \ and \ R = \frac{E^2}{P} = \frac{(120\ V)^2}{16} = 900\ \Omega$$

b. $$f_s = \frac{\omega_S}{2\pi} = 15{,}915.49\ Hz$$

$$BW = 0.15 f_s = 0.15(15{,}915.49 \text{ Hz}) = 2387.32 \text{ Hz}$$

c. Eq. (12.14):

$$BW = \frac{R}{2\pi L} \quad \text{and } L = \frac{R}{2\pi\ BW} = \frac{900\ \Omega}{2\pi(2387.32\ Hz)} = 60\ mH$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad or \quad C = \frac{1}{4\pi^2 f_S^2 L} = \frac{1}{4\pi^2(2387.32\ Hz)^2(60\times10^{-3}\ H)} = 1.67\ \text{nF}$$

d. $$Qs = \frac{X_L}{R} = \frac{2\pi f_S L}{R} = \frac{2\pi(15{,}915.49\ Hz)(60\ mH)}{900\ \Omega} = 6.67$$

e. $$\frac{f_2 - f_1}{f_s} = \frac{BW}{f_s} = \frac{1}{Q_s} = \frac{1}{6.67} = 0.15$$

Practice problems (SERIES RESONANCE)

1. For the series circuit of Fig. 12.13:

a. Find the value of X_C for resonance.

b. Determine the total impedance of the circuit at resonance.

c. Find the magnitude of the current I.

d. Calculate the voltages V_R, V_L, and V_C at resonance. How are V_L and V_C related? How does V_R compare to the applied voltage E?

e. What is the quality factor of the circuit? Is it a high or low-Q circuit?

f. What is the power dissipated by the circuit at resonance?

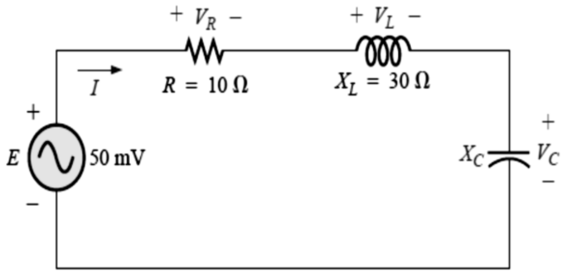


FIG. 12.13 ***Problem 1.***

2. For the series circuit of Fig. 12.14:

a. Find the value of X_L for resonance.

b. Determine the magnitude of the current I at resonance.

c. Find the voltages V_R, V_L, and V_C at resonance, and compare their magnitudes.

d. Determine the quality factor of the circuit. Is it a high or low-Q circuit?

e. If the resonant frequency is 5 kHz, determine the value of L and C.

f. Find the bandwidth of the response if the resonant frequency is 5 kHz.

g. What are the low and high cutoff frequencies?

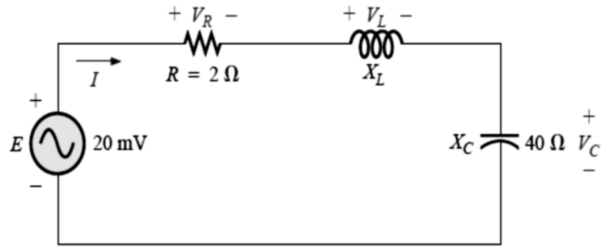


FIG. 12.14 *Problem 2.*

3. For the circuit of Fig. 12.15:

a. Find the value of L in millihenries if the resonant frequency is 1800 Hz.

b. Calculate X_L and X_C. How do they compare?

c. Find the magnitude of the current Irms at resonance.

d. Find the power dissipated by the circuit at resonance.

e. What is the apparent power delivered to the system at resonance?

f. What is the power factor of the circuit at resonance?

g. Calculate the Q of the circuit and the resulting bandwidth.

h. Find the cutoff frequencies, and calculate the power dissipated by the circuit at these frequencies.

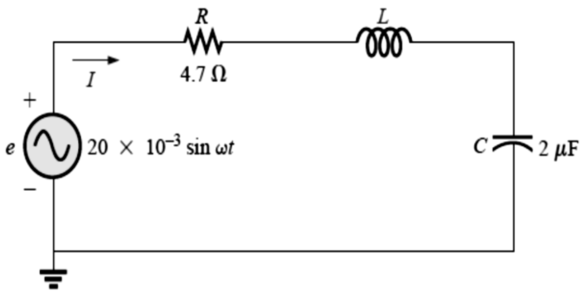


FIG. 12.15 ***Problem 3.***

4. The cutoff frequencies of a series resonant circuit are 5400 Hz and 6000 Hz.

a. Find the bandwidth of the circuit.

b. If Qs is 9.5, find the resonant frequency of the circuit.

c. If the resistance of the circuit is 2 Ω, find the value of X_L and X_C at resonance.

d. Find the value of L and C at resonance.

PARALLEL RESONANCE

12.8 PARALLEL RESONANT CIRCUIT

The basic format of the series resonant circuit is a series R-L-C combination in series with an applied voltage source. The parallel resonant circuit has the basic configuration of Fig. 12.16, a parallel R-L-C combination in parallel with an applied current source.

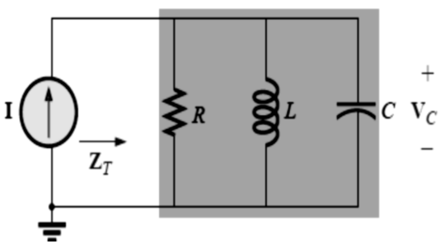


FIG. 12.16
Ideal parallel resonant network.

For the series circuit, the impedance was a minimum at resonance. For the parallel resonant circuit, the impedance is

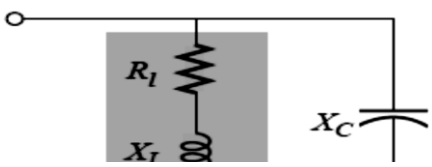


FIG. 12.17
Practical parallel L-C network.

relatively high at resonance. For the network of Fig. 12.16, resonance will occur when $X_L = X_C$, and the resonant frequency will have the same format obtained for series resonance.

In the practical world, the internal resistance of the coil R_l must be placed in series with the inductor, as shown in Fig. 12.17. Our first effort will be to find a parallel network equivalent (at the terminals) for the series R-L branch of Fig. 12.17. That is,

$$Z_{R\text{-}L} = R_l + j\,X_L \quad \rightarrow \quad Y_{R\text{-}L} = \frac{1}{Rp} + \frac{1}{jX_{LP}}$$

$$\mathbf{Rp} = \frac{R_l^2 + X_L^2}{R_l}, \qquad \mathbf{X_{Lp}} = \frac{R_l^2 + X_L^2}{X_L}$$

(12.17)

as shown in Fig. 12.18.

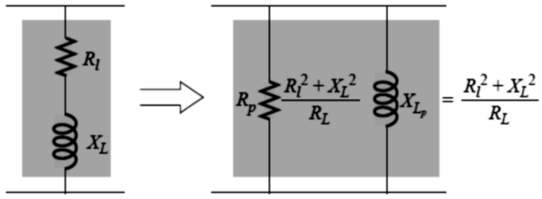


FIG. 12.18 Equivalent parallel network for a series R-L combination.

If we define the parallel combination of R_s and R_p by the notation

R = R_s || R_p

the network of Fig. 12.20 will result. It has the same format as the ideal configuration of Fig. 12.16.

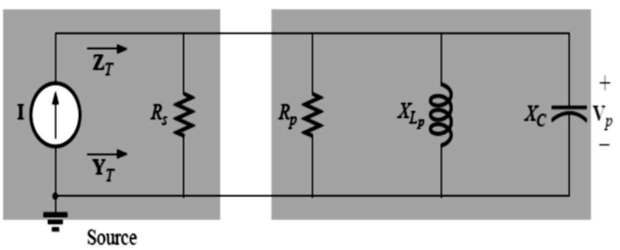


FIG. 12.19 ***Substituting the equivalent parallel network for the series R-L combination of Fig. 20.22.***

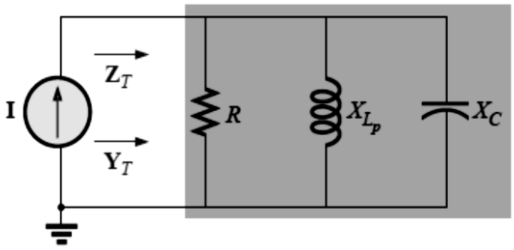


FIG. 12.20 Substituting R = Rs || Rp for the network of Fig. 12.19.

Unity Power Factor, f_p

For the network of Fig. 12.20,

$$\boldsymbol{Y_T = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{Lp}}\right)}$$

(12.18)

For unity power factor, the reactive component must be zero as defined by

$$\frac{1}{X_C} - \frac{1}{X_{Lp}} = 0$$

Therefore, $X_C = X_{Lp}$

(12.19)

Substituting for $\mathbf{X_{Lp}}$ yields

$$\frac{R_l^2+X_L^2}{X_L} = X_C$$

(12.20)

The resonant frequency, f_p, can now be determined from Eq. (12.20) as follows:

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1-\frac{R_l^2C}{L}}$$

(12.21)

$$f_p = f_s\sqrt{1-\frac{R_l^2C}{L}}$$

(12.22)

where f_p is the resonant frequency of a parallel resonant circuit (for pF = 1) and f_s is the resonant frequency as determined by $X_L = X_C$ for series resonance. Note that unlike a series resonant circuit, the resonant frequency f_p is a function of resistance (in this case R_l) and less than f_s. Recognize also that as the magnitude of R_l approaches zero, f_p rapidly approaches f_s.

Maximum Impedance, f_m

At $f = f_p$ the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of Rp. The frequency at which maximum impedance will occur is defined by fm and is slightly more than f_p, as demonstrated in Fig. 12.21. The resulting equation, however, is the following:

$$f_m = f_s\sqrt{1-\frac{1}{4}\left(\frac{R_l^2C}{L}\right)}$$

(12.23)

f_m is always closer to f_s and more than f_p. In general,

$$f_s > f_m > f_p$$

Once f_m is determined, the network of Fig. 12.20 can be used to determine the magnitude and phase angle of the total impedance at the resonance condition simply by substituting $f = f_m$ and performing the required calculations. That is,

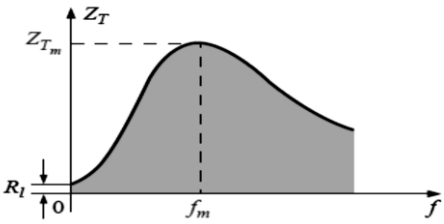


FIG. 12.21 Z_T **versus frequency for the parallel resonant circuit.**

$$Z_{Tm} = R \parallel X_{Lp} \parallel X_C \qquad f = f_m$$

(12.24)

12.9 SELECTIVITY CURVE FOR PARALLEL RESONANT CIRCUITS

The Z_T -versus-frequency curve of Fig. 12.21 clearly reveals that a parallel resonant circuit exhibits maximum impedance at resonance (f_m), unlike the series resonant circuit, which experiences minimum resistance levels at resonance. Note also that Z_T is approximately R_l at f = 0 Hz since $Z_T = R_s \parallel R_l \cong R_l$.

Since the current I of the current source is constant for any value of Z_T or frequency, the voltage across the parallel circuit will have the same shape as the total impedance Z_T.

$$V_C = V_p = I Z_T$$

(12.25)

The resonant value of V_C is therefore determined by the value of Z_{Tm} and the magnitude of the current source I. We can speak of the Q of the coil, where

$$Q_{coil} = Q_l = \frac{X_L}{R}$$

The quality factor of the parallel resonant circuit continues to be determined by the ratio of the reactive power to the real power. That is,

$$Qp = \frac{R}{X_{Lp}} = \frac{R}{X_C}$$

(12.26)

where $R = R_s \parallel R_p$, and V_p is the voltage across the parallel branches.

For the ideal current source ($R_s = \infty\Omega$) or when R_s is sufficiently large compared to R_p, we can make the following approximation:

$$R = R_s \parallel R_p \cong R_p$$

$$Q_p = \frac{R}{X_{Lp}} = Q_l \qquad R_s >> R_l$$

(12.27)

which is simply the quality factor Q_l of the coil.

In general, the bandwidth is still related to the resonant frequency and the quality factor by

$$BW = f_2 - f_1 = \frac{fr}{Qp}$$

(12.28)

The cutoff frequencies f1 and f2 can be determined using the equivalent network of Fig. 12.20 and the unity power condition for resonance. The half-power frequencies are defined by the condition that the output voltage is 0.707 times the maximum value.

Setting the input impedance for the network of Fig. 12.20 equal to this value will result in the following relationship:

$$Z = \frac{1}{\frac{1}{R}+j\left(\omega C-\frac{1}{\omega L}\right)} = 0.707R$$

will result in the following after a series of careful mathematical manipulations:

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2}+\frac{4C}{L}}\right]$$

(12.29a)

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2}+\frac{4C}{L}}\right]$$

(12.29b)

The effect of R_l, L, and C on the shape of the parallel resonance curve, as shown in Fig. 12.22 for the input impedance, is quite similar

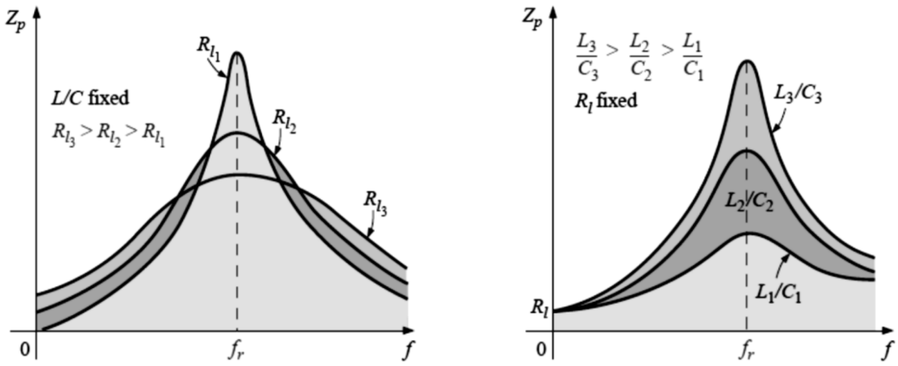


FIG. 12.22 ***Effect of R*_l_, L, and C on the parallel resonance curve.***

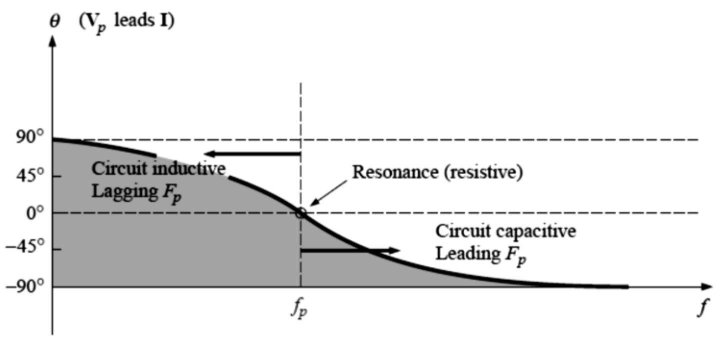


FIG. 12.23 Phase plot for the parallel resonant circuit.

12.10 EFFECT OF $Q_l \geq 10$

The quality factor of the coil $\boldsymbol{Q_l}$ is sufficiently large to permit a number of approximations that simplify the required analysis.

Inductive Reactance, X_{Lp}

$X_{Lp} \cong X_L \qquad Q_l \geq 10$

and since resonance is defined by $X_{Lp} = X_C$, the resulting condition for resonance is reduced to:

$X_L \cong X_C \qquad Q_l \geq 10$

Resonant Frequency, f_p (Unity Power Factor)

$$f_p = f_s\sqrt{1 - \frac{1}{Q_l^2}} \qquad Q_l \geq 10$$

$$f_p \cong fs = \frac{1}{2\pi\sqrt{LC}} \qquad Q_l \geq 10$$

Resonant Frequency, f_m (Maximum V_C)

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{1}{Q_l^2}\right)} \qquad Q_l \geq 10$$

$$f_m \cong fs = \frac{1}{2\pi\sqrt{LC}} \qquad Q_l \geq 10$$

R_p

$$Rp \cong Q_l^2 R_l$$

$$Rp \cong \frac{L}{R_l C} \qquad Q_l \geq 10$$

Z_{Tp}

The total impedance at resonance is now defined by

$$Z_{Tp} = R_s \parallel R_p = R_s \parallel Q_l^2 R_l \qquad Q_l \geq 10$$

$$Z_{Tp} \cong Q_l^2 R_l \qquad Q_l \geq 10 \quad Rs >> Rp$$

Q_p

The quality factor is now defined by

$$Qp = \frac{R}{X_{Lp}} \cong \frac{Rs \| Q_l^2 R_l}{X_L}$$

$$Qp \cong Q_l \qquad Q_l \geq 10 \quad Rs >> Rp$$

BW

The bandwidth defined by f_p is

$$BW = f_2 - f_1 = \frac{fp}{Qp} \cong \frac{1}{2\pi}\left[\frac{R_l}{L} + \frac{1}{R_S C}\right]$$

$$BW = f_2 - f_1 \cong \frac{R_l}{2\pi L} \qquad R_s = \infty\Omega$$

I_L and I_C

I_T defined as shown.

$$V_C = V_L = V_R = I_T Z_{Tp} = I_T Q_l^2 R_l$$

$$I_C \cong Q_l I_T \qquad Q_l \geq 10$$

$$I_L \cong Q_l I_T \qquad Q_l \geq 10$$

12.11 EXAMPLES (PARALLEL RESONANCE)

Example 12.6: Given the parallel network of Fig. 12.24 composed of "ideal" elements:

a. Determine the resonant frequency f_p.

b. Find the total impedance at resonance.

c. Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system.

d. Find the voltage V_C at resonance.

e. Determine the currents I_L and I_C at resonance.

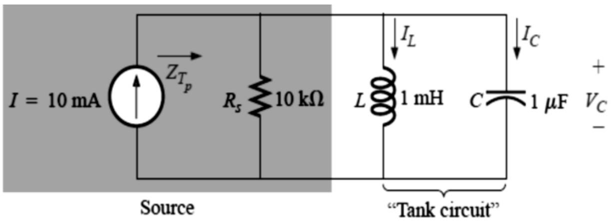


FIG. 12.24 ***Example 12.6.***

Solutions:

a. The fact that R_l is zero ohms results in a very high Q_l ($= X_L/R_l$), permitting the use of the following equation for fp:

$$f_p = f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1\text{ mH})(1\ \mu\text{F})}}$$
$$= \mathbf{5.03\ kHz}$$

b. For the parallel reactive elements:

$$\mathbf{Z}_L \parallel \mathbf{Z}_C = \frac{(X_L \angle 90^\circ)(X_C \angle -90^\circ)}{+j(X_L - X_C)}$$

but $X_L = X_C$ at resonance, resulting in a zero in the denominator of the equation and a very high impedance that can be approximated by an open circuit. Therefore,

$$Z_{T_p} = R_s \parallel \mathbf{Z}_L \parallel \mathbf{Z}_C = R_s = \mathbf{10\ k\Omega}$$

c. $$Q_p = \frac{R_s}{X_{L_p}} = \frac{R_s}{2\pi f_p L} = \frac{10\text{ k}\Omega}{2\pi(5.03\text{ kHz})(1\text{ mH})} = \mathbf{316.41}$$

$$BW = \frac{f_p}{Q_p} = \frac{5.03\text{ kHz}}{316.41} = \mathbf{15.90\ Hz}$$

Eq. (12.29a):

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$
$$= \frac{1}{4\pi(1\ \mu\text{F})}\left[\frac{1}{10\text{ k}\Omega} - \sqrt{\frac{1}{(10\text{ k}\Omega)^2} + \frac{4(1\ \mu\text{F})}{1\text{ mH}}}\right]$$
$$= \mathbf{5.025\ kHz}$$

Eq. (12.29b):

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$
$$= \mathbf{5.041\ kHz}$$

d. $V_C = IZ_{T_p} = (10\ \text{mA})(10\ \text{k}\Omega) = \mathbf{100\ V}$

e. $I_L = \frac{V_L}{X_L} = \frac{V_C}{2\pi f_p L} = \frac{100\ \text{V}}{2\pi(5.03\ \text{kHz})(1\ \text{mH})} = \frac{100\ \text{V}}{31.6\ \Omega} = \mathbf{3.16\ A}$

$I_C = \frac{V_C}{X_C} = \frac{100\ \text{V}}{31.6\ \Omega} = \mathbf{3.16\ A}\ (= Q_p I)$

<u>Example 12.7</u> For the parallel resonant circuit of Fig. 12.25 with $R_s = \infty\Omega$:

a. Determine f_s, f_m, and f_p, and compare their levels.

b. Calculate the maximum impedance and the magnitude of the voltage V_C at f_m.

c. Determine the quality factor Qp.

d. Calculate the bandwidth.

e. Compare the above results with those obtained using the equations associated with $Q_l \geq 10$.

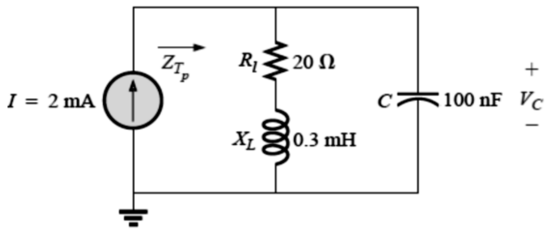


FIG. 12.25 Example 12.7.

Solutions:

a. $f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3\ \text{mH})(100\ \text{nF})}} = \mathbf{29.06\ kHz}$

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left[\frac{R_l^2 C}{L}\right]}$$
$$= (29.06\ \text{kHz})\sqrt{1 - \frac{1}{4}\left[\frac{(20\ \Omega)^2(100\ \text{nF})}{0.3\ \text{mH}}\right]}$$
$$= \mathbf{28.58\ kHz}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}} = (29.06\ \text{kHz})\sqrt{1 - \left[\frac{(20\ \Omega)^2(100\ \text{nF})}{0.3\ \text{mH}}\right]}$$
$$= \mathbf{27.06\ kHz}$$

Both fm and fp are less than fs, as predicted. In addition, fm is closer to fs than fp, as forecast. fm is about 0.5 kHz less than fs, whereas fp is about 2 kHz less. The differences among fs, fm, and fp suggest a low-Q network.

b. $\mathbf{Z}_{T_m} = (R_l + j\,X_L) \parallel -j\,X_C$ at $f = f_m$

$$X_L = 2\pi f_m L = 2\pi(28.58\text{ kHz})(0.3\text{ mH}) = 53.87\ \Omega$$

$$X_C = \frac{1}{2\pi f_m C} = \frac{1}{2\pi(28.58\text{ kHz})(100\text{ nF})} = 55.69\ \Omega$$

$$R_l + j\,X_L = 20\ \Omega + j\,53.87\ \Omega = 57.46\ \Omega\ \angle 69.63^\circ$$

$$\mathbf{Z}_{T_m} = \frac{(57.46\ \Omega\ \angle 69.63^\circ)(55.69\ \Omega\ \angle -90^\circ)}{20\ \Omega + j\,53.87\ \Omega - j\,55.69\ \Omega}$$

$$= \mathbf{159.34\ \Omega\ \angle -15.17^\circ}$$

$$V_{C_{\max}} = IZ_{T_m} = (2\text{ mA})(159.34\ \Omega) = \mathbf{318.68\ mV}$$

c. $R_s = \infty\ \Omega$; therefore,

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l}$$

$$= \frac{2\pi(27.06\text{ kHz})(0.3\text{ mH})}{20\ \Omega} = \frac{51\ \Omega}{20\ \Omega} = \mathbf{2.55}$$

The low Q confirms our conclusion of part (a). The differences among fs, fm, and fp will be significantly less for higher-Q networks.

d. $BW = \dfrac{f_p}{Q_p} = \dfrac{27.06\text{ kHz}}{2.55} = \mathbf{10.61\ kHz}$

e. For $Q_l \geq 10$, $f_m = f_p = f_s = \mathbf{29.06\ kHz}$

$$Q_p = Q_l = \frac{2\pi f_s L}{R_l} = \frac{2\pi(29.06\text{ kHz})(0.3\text{ mH})}{20\ \Omega} = \mathbf{2.74}$$

(versus 2.55 above)

$$Z_{T_p} = Q_l^2 R_l = (2.74)^2 \cdot 20\ \Omega = \mathbf{150.15\ \Omega\ \angle 0^\circ}$$

(versus 159.34 Ω ∠−15.17° above)

$$V_{C_{\max}} = IZ_{T_p} = (2\text{ mA})(150.15\ \Omega) = \mathbf{300.3\ mV}$$

(versus 318.68 mV above)

$$BW = \frac{f_p}{Q_p} = \frac{29.06\text{ kHz}}{2.74} = \mathbf{10.61\ kHz}$$

(versus 10.61 kHz above)

<u>Example 12.8</u>: For the network of Fig. 12.26 with *f*p provided:

a. Determine Q_l.

b. Determine Rp.

c. Calculate Z_{Tp}.

d. Find C at resonance.

e. Find Qp.

f. Calculate the BW and cutoff frequencies.

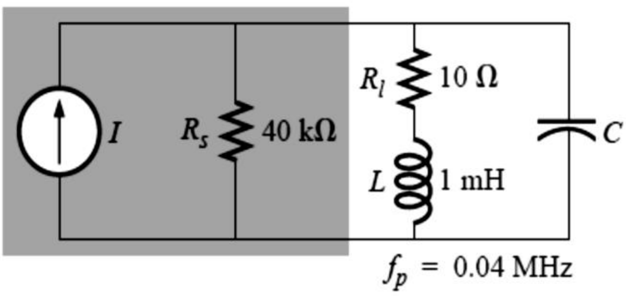


FIG. 12.26 Example 12.8.

Solutions:

a. $Q_l = \frac{X_L}{R_l} = \frac{2\pi f_p L}{R_l} = \frac{2\pi(0.04\text{ MHz})(1\text{ mH})}{10\ \Omega} = \mathbf{25.12}$

b. $Q_l \geq 10$. Therefore,

$$R_p \cong Q_l^2 R_l = (25.12)^2(10\ \Omega) = \mathbf{6.31\ k\Omega}$$

c. $Z_{T_p} = R_s \parallel R_p = 40\text{ k}\Omega \parallel 6.31\text{ k}\Omega = \mathbf{5.45\ k\Omega}$

d. $Q_l \geq 10$. Therefore,

$$f_p \cong \frac{1}{2\pi\sqrt{LC}}$$

and $C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(0.04\text{ MHz})^2(1\text{ mH})} = \mathbf{15.83\ nF}$

e. $Q_l \geq 10$. Therefore,

$$Q_p = \frac{Z_{T_p}}{X_L} = \frac{R_s \parallel Q_l^2 R_l}{2\pi f_p L} = \frac{5.45\text{ k}\Omega}{2\pi(0.04\text{ MHz})(1\text{ mH})} = \mathbf{21.68}$$

f. $BW = \frac{f_p}{Q_p} = \frac{0.04\text{ MHz}}{21.68} = \mathbf{1.85\ kHz}$

$$\begin{aligned}
f_1 &= \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right] \\
&= \frac{1}{4\pi(15.9\text{ mF})}\left[\frac{1}{5.45\text{ k}\Omega} - \sqrt{\frac{1}{(5.45\text{ k}\Omega)^2} + \frac{4(15.9\text{ mF})}{1\text{ mH}}}\right] \\
&= 5.005 \times 10^6[183.486 \times 10^{-6} - 7.977 \times 10^{-3}] \\
&= 5.005 \times 10^6[-7.794 \times 10^{-3}] \\
&= \mathbf{39.009\ kHz} \quad \text{(ignoring the negative sign)}
\end{aligned}$$

$$\begin{aligned}
f_2 &= \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right] \\
&= 5.005 \times 10^6[183.486 \times 10^{-6} + 7.977 \times 10^{-3}] \\
&= 5.005 \times 10^6[8.160 \times 10^{-3}] \\
&= \mathbf{40.843\ kHz}
\end{aligned}$$

Note that $f_2 - f_1$ = 40.843 kHz − 39.009 kHz = 1.834 kHz, confirming our solution for the bandwidth above. Note also that the bandwidth is not symmetrical about the resonant frequency, with 991 Hz below and 843 Hz above.

Example 12.9: The equivalent network for the transistor configuration of Fig. 12.27 is provided in Fig. 12.28.

a. Find fp.

b. Determine Qp.

c. Calculate the BW.

d. Determine Vp at resonance.

e. Sketch the curve of VC versus frequency.

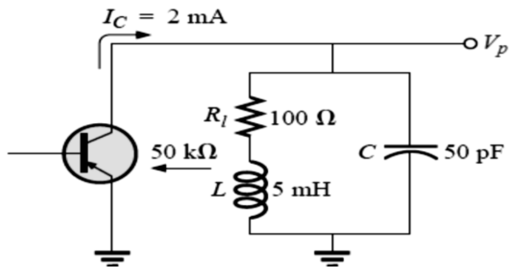


FIG. 12.27 Example 12.9.

Solutions:

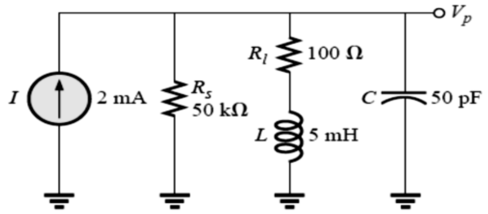


FIG. 12.28
Equivalent network for the transistor configuration of Fig. 12.27.

$$a.\ f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5\text{ mH})(50\text{ pF})}} = 318.31\text{ kHz}$$

$$X_L = 2\pi f_s L = 2\pi(318.31\text{ kHz})(5\text{ mH}) = 10\text{ k}\Omega$$

$$Q_l = \frac{X_L}{R_l} = \frac{10\text{ k}\Omega}{100\ \Omega} = 100 > 10$$

Therefore, $fp = fs$ = 318.31 kHz. Using Eq. (20.31) would result in ≅318.5 kHz.

$$b.\ Q_p = \frac{R_s \parallel R_p}{X_L}$$

$$R_p = Q_l^2 R_l = (100)^2 100\ \Omega = 1\text{ M}\Omega$$

$$Q_p = \frac{50\text{ k}\Omega \parallel 1\text{ M}\Omega}{10\text{ k}\Omega} = \frac{47.62\text{ k}\Omega}{10\text{ k}\Omega} = \mathbf{4.76}$$

Note the drop in Q from $Q_l = 100$ to $Q_p = 4.76$ due to R_s..

$$c.\ BW = \frac{f_p}{Q_p} = \frac{318.31\text{ kHz}}{4.76} = \mathbf{66.87\text{ kHz}}$$

On the other hand,

$$BW = \frac{1}{2\pi}\left(\frac{R_l}{L} + \frac{1}{R_s C}\right) = \frac{1}{2\pi}\left[\frac{100\ \Omega}{5\text{ mH}} + \frac{1}{(50\text{ k}\Omega)(50\text{ pF})}\right]$$
$$= \mathbf{66.85\text{ kHz}}$$

compares very favorably with the above solution.

d. $V_p = I Z_{Tp}$ = (2 mA)(Rs || Rp) = (2 mA)(47.62 kΩ)

= 95.24 V

e. See Fig. 12.29.

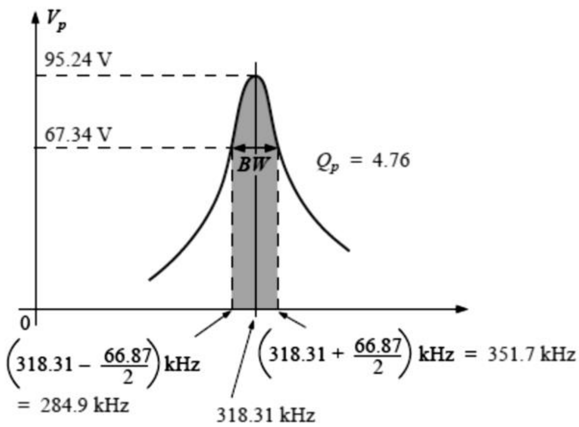


FIG. 12.29 ***Resonance curve for the network of Fig. 12.28.***

Example 12.10: Repeat Example 12.9, but ignore the effects of Rs, and compare results.

Solutions:

a. *f*p is the same, 318.31 kHz.

b. For Rs = ∞ Ω,

$$Q_p = Q_l = \mathbf{100} \qquad \text{(versus 4.76)}$$

c. $BW = \dfrac{f_p}{Q_p} = \dfrac{318.31\text{ kHz}}{100} = \mathbf{3.183\ kHz} \qquad \text{(versus 66.87 kHz)}$

d. $Z_{T_p} = R_p = 1\text{ M}\Omega \qquad \text{(versus 47.62 k}\Omega\text{)}$
$V_p = IZ_{T_p} = (2\text{ mA})(1\text{ M}\Omega) = \mathbf{2000\ V} \qquad \text{(versus 95.24 V)}$

The results obtained clearly reveal that the source resistance can have a significant impact on the response characteristics of a parallel resonant circuit.

Example 12.11: Design a parallel resonant circuit to have the response curve of Fig. 12.30 using a 1-mH, 10-Ω inductor and a current source with an internal resistance of 40 kΩ.

Solution:

BW _fp_Qp

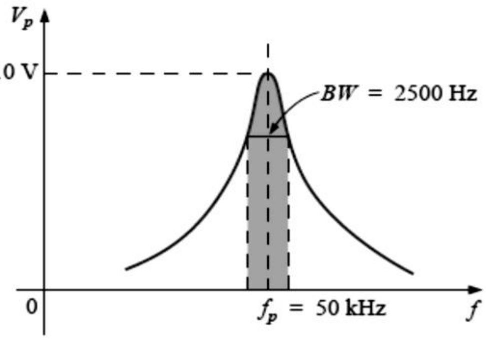


FIG. 12.30 ***Example 12.11.***

Therefore,

$$Q_p = \frac{f_p}{BW} = \frac{50{,}000\text{ Hz}}{2500\text{ Hz}} = \mathbf{20}$$

$$X_L = 2\pi f_p L = 2\pi(50\text{ kHz})(1\text{ mH}) = 314\ \Omega$$

and $\quad Q_l = \dfrac{X_L}{R_l} = \dfrac{314\ \Omega}{10\ \Omega} = \mathbf{31.4}$

$$R_p = Q_l^2 R = (31.4)^2(10\ \Omega) = \mathbf{9859.6\ \Omega}$$

$$Q_p = \frac{R}{X_L} = \frac{R_s \parallel 9859.6\ \Omega}{314\ \Omega} = 20 \qquad \text{(from above)}$$

so that $\quad \dfrac{(R_s)(9859.6)}{R_s + 9859.6} = 6280$

resulting in $\quad R_s = 17.298\text{ k}\Omega$

However, the source resistance was given as 40 kΩ. We must therefore add a parallel resistor (R′) that will reduce the 40 kΩ to approximately 17.298 kΩ; that is,

$$\frac{(40\text{ k}\Omega)(R')}{40\text{ k}\Omega + R'} = 17.298\text{ k}\Omega$$

Solving for R′:

R′= 30.481 kΩ

The closest commercial value is 30 kΩ. At resonance, $X_L = X_C$, and

$$X_C = \frac{1}{2\pi f_p C}$$

$$C = \frac{1}{2\pi f_p X_C} = \frac{1}{2\pi(50\text{ kHz})(314\ \Omega)}$$

and $$C \cong \mathbf{0.01\ \mu F} \quad \text{(commercially available)}$$

$$Z_{T_p} = R_s \parallel Q_l^2 R_l$$
$$= 17.298\text{ k}\Omega \parallel 9859.6\ \Omega$$
$$= 6.28\text{ k}\Omega$$

with $$V_p = I Z_{T_p}$$

and $$I = \frac{V_p}{Z_{T_p}} = \frac{10\text{ V}}{6.28\text{ k}\Omega} \cong \mathbf{1.6\ mA}$$

The network appears in Fig. 12.31.

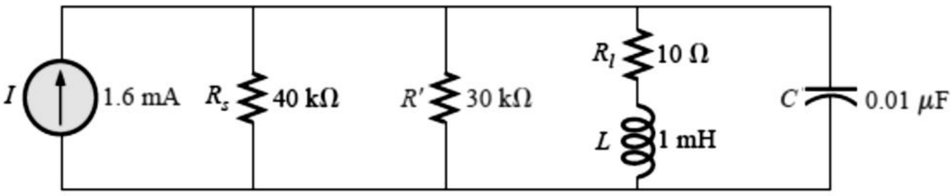


FIG. 12.31 ***Network designed to meet the criteria of Fig. 12.30.***

<u>Practice problems (Parallel Resonance)</u>

1. For the "ideal" parallel resonant circuit of Fig. 12.32:

a. Determine the resonant frequency (fp).

b. Find the voltage V_C at resonance.

c. Determine the currents I_L and I_C at resonance.

d. Find Qp.

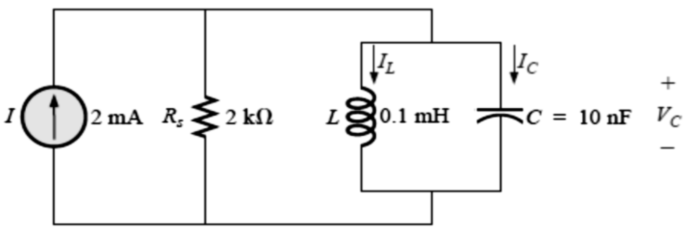


FIG. 12.32 ***Problem 1.***

2. For the parallel resonant network of Fig. 12.33:

a. Calculate f_s.

b. Determine Q_l using $f = f_s$. Can the approximate approach be applied?

c. Determine f_p and f_m.

d. Calculate X_L and X_C using f_p. How do they compare?

e. Find the total impedance at resonance (f_p).

f. Calculate V_C at resonance (f_p).

g. Determine Qp and the BW using f_p.

h. Calculate I_L and I_C at f_p.

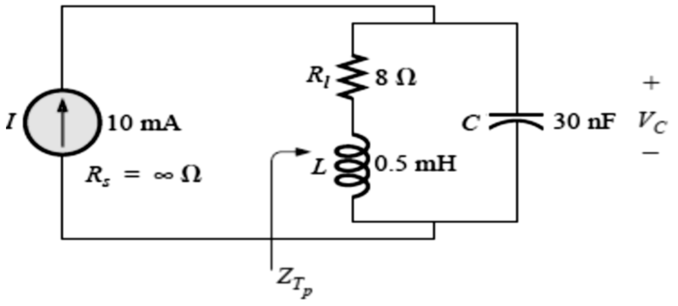


FIG. 12.33 Problem 2.

3. For the network of Fig. 12.34:

a. Find the value of X_C at resonance (f_p).

b. Find the total impedance Z_{Tp} at resonance (f_p).

c. Find the currents I_L and I_C at resonance (f_p).

d. If the resonant frequency is 20,000 Hz, find the value of L and C at resonance.

e. Find Qp and the BW.

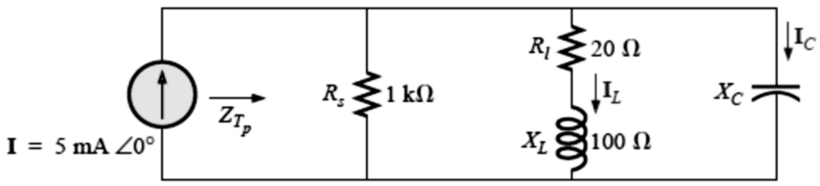


FIG. 12.34 *Problem 3.*

4. For the network of Fig. 12.35:

a. Find the resonant frequencies f_s, f_p, and f_m. What do the results suggest about the Qp of the network?

b. Find the values of X_L and X_C at resonance (f_p). How do they compare?

c. Find the impedance Z_{Tp} at resonance (f_p).

d. Calculate Qp and the BW.

e. Find the magnitude of currents I_L and I_C at resonance (f_p).

f. Calculate the voltage V_C at resonance (f_p).

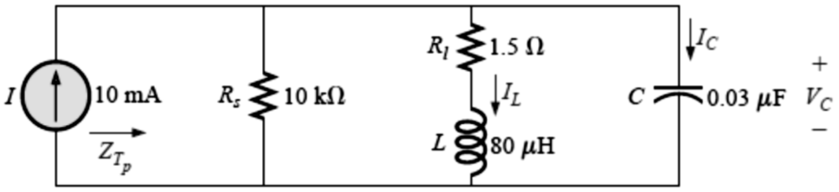


FIG. 12.35 ***Problem 4.***

5. For the network of Fig. 12.36:

a. Find the value of X_L for resonance.

b. Find Q_l.

c. Find the resonant frequency (f_p) if the bandwidth is 1 kHz.

d. Find the maximum value of the voltage V_C.

e. Sketch the curve of V_C versus frequency. Indicate its peak value, resonant frequency, and band frequencies.

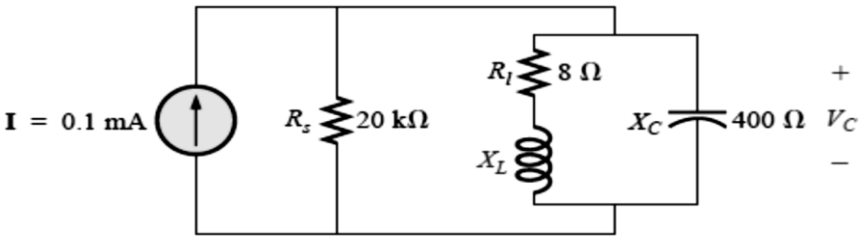


FIG. 12.36 ***Problem 5.***